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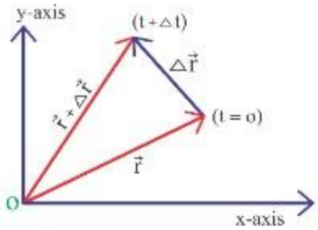

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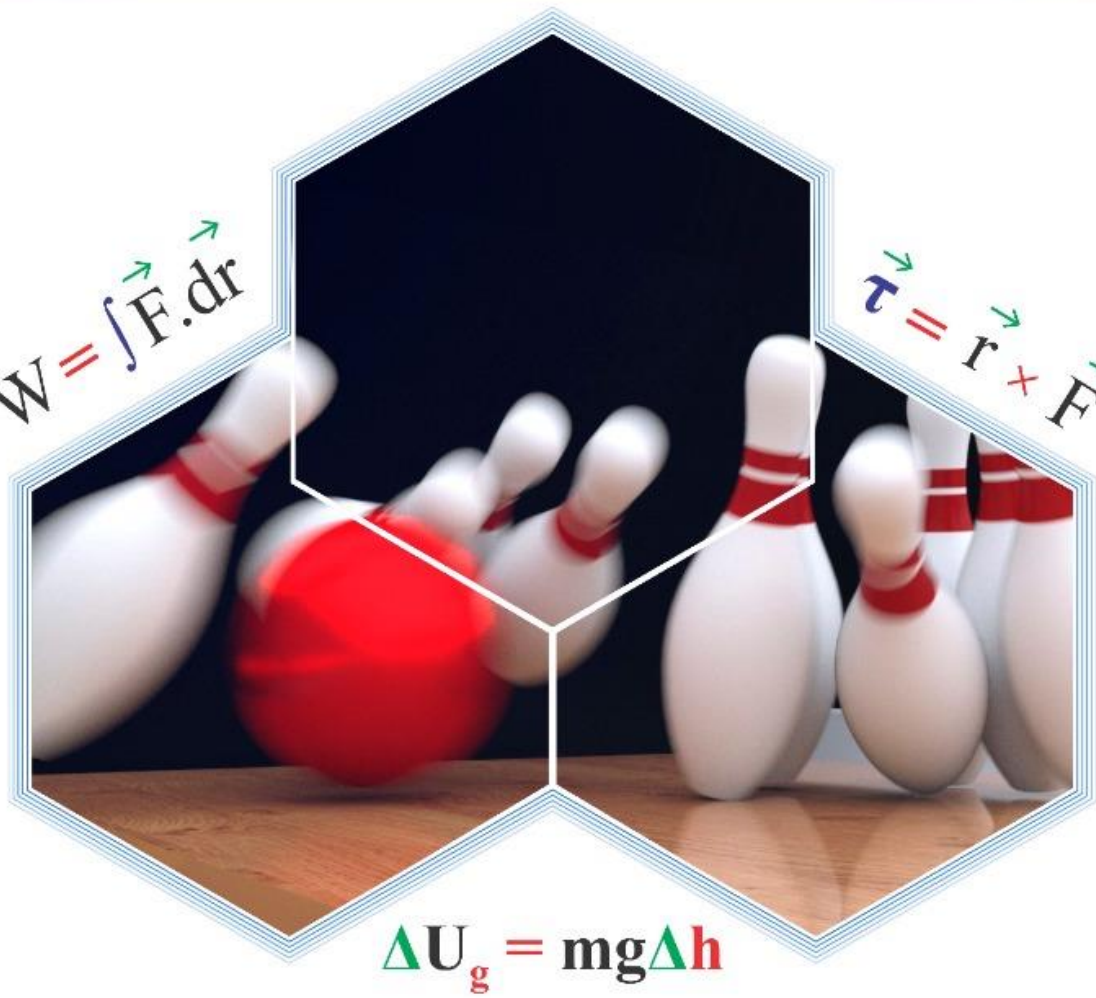
TEACH YOURSELF

MECHANICS -I

For BS Physics Programme

$W = \int \vec{F}.\vec{dr}$



$\vec{\tau} = \vec{r} \times \vec{F}$

$\Delta U_g = mg\Delta h$

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TEACH YOURSELF

MECHANICS-I

1st Edition

For **BS/ADS, Physics/Mathematics & Engineering** students

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Contents

Chapter 1

Vector Analysis

SOLVED PROBLEMS

Problem: 1.1- Show that vectors $\vec{A} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{B} = \hat{i} - \hat{j} - \hat{k}$ are perpendicular.

Solution

Two vectors are perpendicular if their dot product is zero. It is given that

$$\begin{aligned}\vec{A} &= \hat{i} + 3\hat{j} - 2\hat{k}\\ \vec{B} &= \hat{i} - \hat{j} - \hat{k}\end{aligned}$$

Now, taking the dot product on both sides, we get

$$\begin{aligned}\vec{A}\cdot\vec{B} &= \left(\hat{i} + 3\hat{j} - 2\hat{k}\right)\cdot\left(\hat{i} - \hat{j} - \hat{k}\right)\\ \vec{A}\cdot\vec{B} &= 1(\hat{i}\cdot\hat{i}) - 3(\hat{j}\cdot\hat{j}) + 2(\hat{k}\cdot\hat{k})\\ \vec{A}\cdot\vec{B} &= 1(1) - 3(1) + 2(1) \qquad \because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\\ \vec{A}\cdot\vec{B} &= 1 - 3 + 2\\ \vec{A}\cdot\vec{B} &= 3 - 3 = 0\end{aligned}$$

Hence $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Problem: 1.2- If $\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$, find divergence and curl of vector $\vec{A}$ at $(1, -1, 1)$.

Solution

$$\begin{aligned}
\text{div}\vec{A} &= \vec{\nabla} \cdot \vec{A} \\
\vec{\nabla} \cdot \vec{A} &= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}\right) \\
\vec{\nabla} \cdot \vec{A} &= \frac{\partial}{\partial x}\left(xz^3\right) - \frac{\partial}{\partial y}\left(2x^2yz\right) + \frac{\partial}{\partial z}\left(2yz^4\right) \\
\vec{\nabla} \cdot \vec{A} &= z^3\frac{\partial x}{\partial x} - 2x^2z\frac{\partial y}{\partial y} + 2y\frac{\partial z^4}{\partial z} \\
\vec{\nabla} \cdot \vec{A} &= z^3(1) - 2x^2z(1) + 8yz^3 \\
\vec{\nabla} \cdot \vec{A} &= z^3 - 2x^2z + 8yz^3
\end{aligned}$$

Now, at $(x, y, z) = (1, -1, 1)$, we get

$$\begin{aligned}
\vec{\nabla} \cdot \vec{A} &= (1)^3 - 2(1)^2(1) + 8(-1)(1)^3 \\
\vec{\nabla} \cdot \vec{A} &= 1 - 2 - 8 \\
\vec{\nabla} \cdot \vec{A} &= -9
\end{aligned}$$

$$\begin{aligned}
\text{Curl}\,\vec{A} &= \vec{\nabla} \times \vec{A} \\
\vec{\nabla} \times \vec{A} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^3 & -2x^2yz & 2yz^4 \end{vmatrix} \\
\vec{\nabla} \times \vec{A} &= \left[\frac{\partial}{\partial y}(2yz^4) - \frac{\partial}{\partial z}(-2x^2yz)\right]\hat{i} + \left[\frac{\partial}{\partial z}(xz^3) - \underbrace{\frac{\partial}{\partial x}(2yz^4)}_{0}\right]\hat{j} \\
&+ \left[\frac{\partial}{\partial x}(-2x^2yz) - \frac{\partial}{\partial y}(xz^3)\right]\hat{k} \\
\vec{\nabla} \times \vec{A} &= \left(2z^4 + 2x^2y\right)\hat{i} + 3xz^2\hat{j} - 4xyz\hat{k}
\end{aligned}$$

Now, at $(x, y, z) = (1, -1, 1)$,

$$\begin{aligned}\vec{\nabla} \times \vec{A} &= (2-2)\hat{i} + 3\hat{j} + 4\hat{k} \\ \vec{\nabla} \times \vec{A} &= 3\hat{j} + 4\hat{k}\end{aligned}$$

Problem: 1.3- If $\phi = 2x^3y^2z^4$, find div(grad)ϕ.

Solution

$$\begin{aligned}\text{div(grad)}\phi &= \vec{\nabla} \cdot \vec{\nabla}\phi \\ \vec{\nabla} \cdot \vec{\nabla}\phi &= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi \\ \vec{\nabla} \cdot \vec{\nabla}\phi &= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}\right)\end{aligned}$$

$$\begin{aligned}\vec{\nabla} \cdot \vec{\nabla}\phi &= \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \\ \vec{\nabla} \cdot \vec{\nabla}\phi &= \frac{\partial^2}{\partial x^2}(2x^3y^2z^4) + \frac{\partial^2}{\partial y^2}(2x^3y^2z^4) + \frac{\partial^2}{\partial z^2}(2x^3y^2z^4) \\ \vec{\nabla} \cdot \vec{\nabla}\phi &= 2y^2z^4\frac{\partial^2}{\partial x^2}(x^3) + 2x^3z^4\frac{\partial^2}{\partial y^2}(y^2) + 2x^3y^2\frac{\partial^2}{\partial z^2}(z^4) \\ \vec{\nabla} \cdot \vec{\nabla}\phi &= 2y^2z^4(6x) + 2x^3z^4(2) + 2x^3y^2(12z^2) \\ \vec{\nabla} \cdot \vec{\nabla}\phi &= 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2\end{aligned}$$

Problem: 1.4- Find a unit vector which is perpendicular to both vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}$.

Solution

Let $\hat{n}$ be a unit vector which is perpendicular to $\vec{A}$ and $\vec{B}$, is given as:
The vector $\vec{A} \times \vec{B}$ is always perpendicular to both $\vec{A}$ and $\vec{B}$, thus the unit vector can be defined as (since $\hat{n}$ is perpendicular to $\vec{A} \times \vec{B}$):

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{\left|\vec{A} \times \vec{B}\right|}$$

Now,

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{i}(-1-3) - \hat{j}(-1-2) + \hat{k}(3-2)$$

$$\vec{A}\times\vec{B} = -4\hat{i} + 3\hat{j} + \hat{k}$$

Now, the magnitude of $\vec{A}\times\vec{B}$ is

$$\left|\vec{A}\times\vec{B}\right| = \sqrt{(-4)^2 + (3)^2 + (1)^2}$$

$$\left|\vec{A}\times\vec{B}\right| = \sqrt{16+9+1}$$

$$\left|\vec{A}\times\vec{B}\right| = \sqrt{26}$$

So, the unit vector $\hat{n}$ is

$$\hat{n} = \frac{-4\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{26}}$$

Problem: 1.5- A particle moves along a curve whose parametric equations are $x = 2e^{-t}$, $y = 2\cos 3t$ and $z = 2\sin 3t$, t being time. Determine the velocity and acceleration at any time t. And calculate the magnitudes of velocity and acceleration at $t = 0$.

Solution

The position vector of a moving particle at any time can be expressed as:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = 2e^{-t}\hat{i} + 2\cos 3t\hat{j} + 2\sin 3t\hat{k} \qquad \text{By substituting given value of } x, y \text{ and } z$$

Velocity is defined as the time derivative of position vector and is given as:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d}{dt}\left(2e^{-t}\hat{i} + 2\cos 3t\hat{j} + 2\sin 3t\hat{k}\right)$$

$$\vec{v} = \frac{d}{dt}(2e^{-t}\hat{i}) + \frac{d}{dt}(2\cos 3t\hat{j}) + \frac{d}{dt}(2\sin 3t\hat{k})$$
$$\vec{v} = 2e^{-t}(-1)\hat{i} - 2\sin 3t(3)\hat{j} + 2\cos 3t(3)\hat{k}$$
$$\vec{v} = -2e^{-t}\hat{i} - 6\sin 3t\hat{j} + 6\cos 3t\hat{k} \quad (1.1)$$

And the acceleration is defined as the time derivative of velocity and is given as:

$$\vec{a} = \frac{d\vec{v}}{dt}$$
$$\vec{a} = \frac{d}{dt}\left(-2e^{-t}\hat{i} - 6\sin 3t\hat{j} + 6\cos 3t\hat{k}\right)$$
$$\vec{a} = \frac{d}{dt}(-2e^{-t}\hat{i}) - \frac{d}{dt}(6\sin 3t\hat{j}) + \frac{d}{dt}(6\cos 3t\hat{k})$$
$$\vec{a} = -2e^{-t}(-1)\hat{i} - 6\cos 3t(3)\hat{j} - 6\sin 3t(3)\hat{k}$$
$$\vec{a} = 2e^{-t}\hat{i} - 18\cos 3t\hat{j} - 18\sin 3t\hat{k} \quad (1.2)$$

From Eqs.(1.1) and (1.2), we can write:
At $t = 0$, $\vec{v} = -2\hat{i} + 6\hat{k}$ and $\vec{a} = 2\hat{i} - 18\hat{j}$. So,

$$|\vec{v}| = \sqrt{(-2)^2 + (6)^2}$$
$$|\vec{v}| = \sqrt{4 + 36}$$
$$|\vec{v}| = \sqrt{40} \text{ units.}$$

And,

$$|\vec{a}| = \sqrt{(2)^2 + (-18)^2}$$
$$|\vec{a}| = \sqrt{4 + 324}$$
$$|\vec{a}| = \sqrt{328} \text{ units.}$$

Chapter 2

Particle Dynamics

SOLVED PROBLEMS

Problem: 2.1- The coefficient of static friction between tires of a car and dry road is 0.62. The mass of the car is $1500kg$. What maximum braking force is obtained on level road and on an 8.6° downgrade?

Solution

It is given that

$$m = 1500kg$$
$$\mu_s = 0.62$$
$$\theta = 8.6^\circ$$

Since we know that

$$\mu_s = \frac{f_s}{N}$$
$$\mu_s = \frac{f_s}{mg} \qquad \because N = mg$$
$$f_s = \mu_s mg$$
$$f_s = 0.62 \times 1500 \times 9.8$$
$$f_s = 9114\ N$$

For downgrade,

$$f_s = \mu_s mg \cos\theta$$
$$f_s = 0.62 \times 1500 \times 9.8 \cos(8.6)^\circ$$
$$f_s = 9011\ N$$

Problem: 2.2- Consider a rotor of radius $2m$. It is given that coefficient of friction between material of clothing and rotor wall is 0.40. Find speed of object, time period and frequency of rotor.

Solution

Given that:

$$\mu_s = 0.40$$
$$R = 2m$$
$$v = ?$$
$$T = ?$$
$$f = ?$$

Using the relation for the speed

$$v = \sqrt{\frac{gR}{\mu_s}}$$
$$v = \sqrt{\frac{9.8 \times 2}{0.40}}$$
$$v = 7\ ms^{-1}$$

Also,

$$T = \frac{2\pi R}{v}$$
$$T = \frac{2 \times 3.14 \times 2}{7}$$
$$T = 1.80s$$

Also, frequency f is the reciprocal of time period T. So,

$$f = \frac{1}{T}$$
$$f = \frac{1}{1.80}$$
$$f = 0.56 rev.s^{-1}$$

Problem: 2.3- A circular curve of highway is designed for traffic moving at 60 kmh^{-1}. If radius of curve is 150 m, what is correct angle of banking of the road?

Solution

Given data:

$$R = 150m$$
$$v = 60kmh^{-1}$$
$$v = \frac{60 \times 1000}{3600} ms^{-1}$$
$$v = 16.7ms^{-1}$$
$$\text{Angle of banking} = \theta = ?$$

Using the relation

$$\tan\theta = \frac{v^2}{gR}$$
$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$
$$\theta = \tan^{-1}\left(\frac{(16.7)^2}{9.8 \times 150}\right)$$
$$\theta = 10.74^\circ$$

Problem: 2.4- A crate of mass $360kg$ rests on the bed of truck that is moving at $120kmh^{-1}$. The driver applies the brakes and slows to a speed of $62kmh^{-1}$ in $17s$. What force acts on crate during this time?

Solution

Given data:

$$\begin{aligned}
\text{Initial speed of truck} &= v_\circ = 120kmh^{-1} \\
\text{Final speed of truck} &= v = 62kmh^{-1} \\
\text{Mass of crate} &= m = 360kg \\
\text{Force on crate} &= F = ?
\end{aligned}$$

Now, using the relation

$$\begin{aligned}
v &= v_\circ + at \\
at &= v - v_\circ \\
a &= \frac{v - v_\circ}{t} \\
a &= \frac{62 - 120}{17} \\
a &= -3.41kmh^{-1}s^{-1} \\
a &= -\frac{3.41 \times 1000}{3600} ms^{-2} \\
a &= -0.95ms^{-2}
\end{aligned}$$

Now, force on a crate is given by Newton's 2^{nd} law of motion

$$\begin{aligned}
F &= ma \\
F &= 360 \times -0.95 \\
F &= -340N
\end{aligned}$$

Problem: 2.5- A long jumper leaves the ground at an angle of 20° above the horizontal and at a speed of $11ms^{-1}$. How far does he jump in the horizontal direction? What is the maximum height reached?

Solution

$$\theta = 20^o$$
$$v_o = 11ms^{-1}$$

Since the horizontal range is given as

$$R = \frac{v_\circ^2}{g}\sin 2\theta$$
$$R = \frac{(11)^2}{9.8}\sin 2(20^\circ)$$
$$R = \frac{121}{9.8}\sin(40^\circ)$$
$$R = 7.94m$$

Also, maximum height is given by

$$H = R = \frac{v_\circ^2 \sin^2\theta}{2g}$$
$$H = R = \frac{(11)^2 \sin^2(20^\circ)}{2 \times 9.8}$$
$$H = R = 0.722m$$

Chapter 3

Work, Power and Energy

SOLVED PROBLEMS

Problem: 3.1- Suppose a neutron travels a distance $6.2m$ in a time $160 \times 10^{-6}s$. Calculate its kinetic energy?

Solution

$$\begin{aligned}
\text{Mass of neutron } &= m = 1.67 \times 10^{-27}kg \\
\text{Distance traveled by neutron } &= s = 6.2m \\
\text{Time } &= t = 160 \times 10^{-6}s \\
\text{Kinetic energy } &= K.E = ?
\end{aligned}$$

Since, we know that

$$\begin{aligned}
K.E &= \frac{1}{2}mv^2 \\
K.E &= \frac{1}{2}m\left(\frac{s}{t}\right)^2 \qquad \because v = \frac{s}{t} = \frac{\text{distance}}{\text{time}} \\
K.E &= \frac{1}{2} \times 1.67 \times 10^{-27}\left(\frac{6.2}{160 \times 10^{-6}}\right)^2 \\
K.E &= 1.26 \times 10^{-18}J
\end{aligned}$$

$$K.E = \frac{1.26\times10^{-18}}{1.6\times10^{-19}}\ eV \qquad 1ev = 1.6\times10^{-19}J$$
$$K.E = 7.9\ eV$$

Problem: 3.2- The hydrogen filled airship could cruise at 77 knots with engine providing 4800 hp. Calculate the air drag force in Newton on the airship at this speed.

Solution

$$\text{Speed of airship} = v = 77 \text{ knots}$$
$$\text{Speed of airship} = v = 77\times1.688\times0.3048\ ms^{-1}$$
$$\text{Speed of airship} = v = 39.62\ ms^{-1}$$

$$\text{Power given by engine} = P = 4800\ hp = 4800\times746W \qquad \because 1hp = 746W$$
$$\text{Power given by engine} = P = 3.5\times10^{6}\ W$$
$$\text{Air drag force} = F = ?$$

Since, the power is defined as:

$$P = Fv$$
$$F = \frac{P}{v}$$
$$F = \frac{3.5\times10^{6}}{39.62}$$
$$F = 9.0386\times10^{4}\ N$$

Problem: 3.3- Find the potential energy of a system consisting of a 65 kg man on a 3 m high diving board. Let us take water level as reference level.

Solution

Given data:

$$\begin{aligned}\text{Height form water level} &= h = 3\ m\\ \text{Mass} &= m = 65\ kg\\ \text{Gravitational acceleration} &= g = 9.8\ ms^{-2}\\ \text{Potential energy} &= P.E = ?\end{aligned}$$

Using the relation

$$\begin{aligned}P.E &= mgh\\ P.E &= 65 \times 9.8 \times 3\\ P.E &= 1900\ J\end{aligned}$$

Problem: 3.4- How much work is required for a 74 kg sprinter to accelerate it from rest to 2.2 ms^{-1}.

Solution

Given data:

$$\begin{aligned}v &= 2.2\ ms^{-1}\\ v_\circ &= 0\ ms^{-1} \qquad \text{(As spring is at rest)}\\ m &= 74\ kg\end{aligned}$$

By work-energy theorem

$$\begin{aligned}W &= \frac{1}{2}mv^2 - \frac{1}{2}mv_\circ^2\\ W &= \frac{1}{2} \times 74 \times (2.2)^2 - \frac{1}{2} \times 74 \times (0)^2\\ W &= \frac{1}{2} \times 74 \times 4.84 - \frac{1}{2} \times 74 \times (0)\end{aligned}$$

$$W = \frac{1}{2} \times 358.16 - 0$$
$$W = 179.08\ J$$

Problem: 3.5- Approximately $5.5 \times 10^2\ kg$ of water drops 30 m over Mangla falls every second. Find power generated by electric plant that would convert all of potential energy into electrical energy. If company sold this energy at a rate of 8 rupee per kWh, what would be its yearly income from this source?

Solution

Given data:

$$m = 5.5 \times 10^2\ kg$$
$$g = 9.8\ ms^{-1}$$
$$h = 30\ m$$
$$t = 1\ s$$

Since,

$$\text{Work done} = W = Fd = Fh \quad \text{and} \quad F = mg$$
$$\text{So} \quad W = mgh$$

$$P = \frac{W}{t}$$
$$P = \frac{Fh}{t} = \frac{mgh}{t}$$
$$P = \frac{5.5 \times 10^2 \times 9.8 \times 30}{1}$$
$$P = 1.617 \times 10^5\ W$$
$$P = 1.617 \times 10^2\ kW$$

Now, the total energy generated is:

$$E = Pt = 1.617 \times 10^2\ kW \times 8760\ h$$

$\because$ In 1 year, there are $365 \times 24 = 8760\ hr$

$$E = 1416492\ kWh$$

So, yearly income will be:

$$= 1416492 \times 8\ Rs. = 11331936 Rs. = 11.33\ MRs$$

Chapter 4

Systems of Particles

SOLVED PROBLEMS

Problem: 4.1- How far is the center of mass of the earth-moon system from center of the earth?

Solution

It is given that

$$\begin{aligned} \text{Mass of earth} &= m_1 = 6 \times 10^{24}\ kg \\ \text{Mass of moon} &= m_2 = 7.36 \times 10^{22}\ kg \\ \text{Coordinate of earth} &= x_1 = 0 \\ \text{Coordinate of moon} &= x_2 = 3.84 \times 10^{8}\ m \\ \text{Center of mass} &= x_{cm} = ? \end{aligned}$$

Since, we know that

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{cm} = \frac{6 \times 10^{24} \times (0) + 7.36 \times 10^{22} \times 3.84 \times 10^{8}}{(6 \times 10^{24}) + (7.36 \times 10^{22})}$$

$$x_{cm} = \frac{0 + 28.2624 \times 10^{30}}{(6 \times 10^{24} + 7.36 \times 10^{22})}$$

$$x_{cm} = \frac{28.2624 \times 10^{30}}{10^{22}(6 \times 10^2 + 7.36)}$$
$$x_{cm} = \frac{28.2624 \times 10^{8}}{(6 \times 10^2 + 7.36)}$$
$$x_{cm} = \frac{28.2624 \times 10^{8}}{(600 + 7.36)}$$
$$x_{cm} = \frac{28.2624 \times 10^{8}}{(607.36)}$$
$$x_{cm} = 0.04653 \times 10^{8}$$
$$x_{cm} = 4.653 \times 10^{6}\ m$$

Problem: 4.2- A 60 kg archer stands at rest on frictionless ice and fires a 0.03 kg arrow horizontally at 85 ms^{-1}. With what velocity does the archer move across the ice after firing the arrow?

Solution

Given data:

$$m_1 = 60\ kg$$
$$m_2 = 0.03\ kg$$
$$v_2 = 85\ ms^{-1}$$
$$v_1 = ?$$

By law of conservation of linear momentum

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$
$$v_1 = -\frac{m_2}{m_1}v_2$$
$$v_1 = -\frac{0.03}{60} \times 85$$

$$v_1 = -\frac{2.55}{60}$$
$$v_1 = -0.042\ ms^{-1}$$

The archer will move backward with a velocity of 0.042 m/s

Problem: 4.3- Suppose a rod is non-uniform such that its mass per unit length varies linearly with x according to the expression $\lambda = ax$, where a is a constant. Find the x-coordinate of the center of mass as a fraction of L.

Solution

x-coordinate of center of mass is:

$$x_{cm} = \frac{\int x dm}{\int dm}$$
$$x_{cm} = \frac{\int_0^L x a x dx}{\int_0^L a x dx}$$
$$x_{cm} = \frac{a\int_0^L x^2 dx}{a\int_0^L x dx} = \frac{\int_0^L x^2 dx}{\int_0^L x dx}$$
$$x_{cm} = \frac{\left|\frac{x^3}{3}\right|_0^L}{\left|\frac{x^2}{2}\right|_0^L} = \frac{2}{3}\frac{L^3}{L^2}$$
$$x_{cm} = \frac{2}{3}L$$

Problem: 4.4- A 2000 kg truck traveling north at 40 kmh^{-1} turns east and accelerates to 50 kmh^{-1}. What is change in kinetic energy of truck and also calculate the magnitude and direction of change of momentum of truck.

Solution

$$
\begin{aligned}
m &= 2000\ kg \\
v_1 &= 40\ kmh^{-1} = \frac{40 \times 1000}{3600}\ ms^{-1} = 11.11\ ms^{-1} \\
v_2 &= 50\ kmh^{-1} = \frac{50 \times 1000}{3600}\ ms^{-1} = 13.88\ ms^{-1}
\end{aligned}
$$

Since the change in kinetic energy is

$$
\begin{aligned}
\Delta K.E &= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \\
\Delta K.E &= \frac{1}{2}m\left(v_2^2 - v_1^2\right) \\
\Delta K.E &= \frac{1}{2} \times 2000 \times \left((13.88)^2 - (11.11)^2\right) \\
\Delta K.E &= 1000 \times (192.65 - 123.43) \\
\Delta K.E &= 1000 \times 69.22 = 692222\ J
\end{aligned}
$$

Now, the change in momentum is given as:
As $\vec{P}_1$ is along north (y-axis), so, $\vec{P}_1 = P_1 j$ and momentum $\vec{P}_2$ is in the direction of East (along x-axis).
Thus

$$
\begin{aligned}
\vec{P}_2 &= P_2\hat{i} \\
\Delta\vec{P} &= \vec{P}_2 - \vec{P}_1 \\
\Delta\vec{P} &= P_2\hat{i} - P_1\hat{j}
\end{aligned}
$$

So magnitude of ΔP is

$$
\begin{aligned}
\Delta P &= \sqrt{P_1^2 + P_2^2} \\
\Delta P &= \sqrt{(mv_1)^2 + (mv_2)^2}
\end{aligned}
$$

$$\Delta P = \sqrt{(2000 \times 11.11)^2 + (2000 \times 1.88)^2}$$
$$\Delta P = 3.5 \times 10^4 \; N - s$$

Direction of momentum is $\tan\theta = \frac{P_1}{P_2}$

$$\tan\theta = \frac{mv_1}{mv_2}$$

$$\theta = \tan^{-1}\left(\frac{v_1}{v_2}\right)$$
$$\theta = \tan^{-1}\left(\frac{11.11}{13.88}\right)$$
$$\theta = \tan^{-1}(0.800)$$
$$\theta = 38.6^\circ$$

Problem: 4.5- A rocket of total mass $1.11 \times 10^5 \; kg$ of which $8.70 \times 10^4 \; kg$ fuel is to be launched vertically. The fuel will be burnt at the constant rate of 820 kgs^{-1}. Relative to the rocket, what is minimum exhaust speed that allows lift off at launch?

Solution

Given data:

$$m_\circ = 1.11 \times 10^5 \; kg$$
$$\frac{dm}{dt} = 820 \; kgs^{-1}$$
$$g = 9.8 \; ms^{-2}$$

Thrust on rocket is:

$$F = v_\circ \frac{dm}{dt}$$
$$m_\circ g = v_\circ \frac{dm}{dt}$$

$$
\begin{aligned}
v_\circ &= \frac{m_\circ g}{\frac{dm}{dt}} \\
v_\circ &= \frac{1.11 \times 10^5 \times 9.8}{820} \\
v_\circ &= 1326 \ ms^{-1}
\end{aligned}
$$

Chapter 5

Collisions

SOLVED PROBLEMS

Problem: 5.1- How fast must a 816 kg vehicle travel to have same momentum as a 2650 kg Suzuki goes 16 kmh^{-1}?

Solution

$$\text{Mass of vehicle} = m_1 = 816\ kg$$
$$\text{Mass of suzuki} = m_2 = 2650\ kg$$
$$\text{Velocity of vehicle} = v_1 = ?$$
$$\text{Velocity of suzuki} = v_2 = 16\ kmh^{-1}$$

Since,

$$\text{Momentum of vehicle} = \text{Momentum of suzuki}$$

$$m_1 v_1 = m_2 v_2$$
$$v_1 = \frac{m_2 v_2}{m_1}$$
$$v_1 = \frac{2650 \times 16}{816}$$
$$v_1 = \frac{42400}{816}$$
$$v_1 = 51.96\ kmh^{-1}$$

Problem: 5.2- A golfer hits golf ball of mass 46 g, imparting to it an initial speed of 52.2 ms^{-1} directed at some angle to horizontal. The club and ball are in contact for 1.20 ms, find average force exerted on ball by club.

Solution

$$\begin{aligned}
\text{Mass of golf ball} &= m \;=\; 46 \times 10^{-3}\ kg \\
\text{Velocity of golf ball} &= v_f \;=\; 52.2\ ms^{-1} \\
\text{Time} &= t \;=\; 1.20\ ms \;=\; 1.20 \times 10^{-3} s \\
v_i &= 0\ ms^{-1} \\
\text{Average force} &= F \;=\; ?
\end{aligned}$$

Using the relation

$$\begin{aligned}
F &= \frac{dP}{dt} \\
F &= \frac{mv_f - mv_i}{dt} \\
F &= \frac{46 \times 10^{-3} \times 52.2 - 0}{1.20 \times 10^{-3}} = \frac{4.6 \times 52.2}{1.2} \\
F &= 2001\ N
\end{aligned}$$

Problem: 5.3- Two titanium spheres approach each other head on with same speed and collide elastically. After collision, one of the spheres, whose mass is 300 g remains at rest. What is mass of other sphere?

Solution

$$\begin{aligned}
\text{Mass of first sphere} &= m_1 \;=\; 300\ g \\
\text{Mass of second sphere} &= m_2 \;=\; ? \\
\text{Velocity of } m_1 \text{ before collision} &= u_1 \\
\text{Velocity of } m_2 \text{ before collision} &= u_2 \;=\; -u_1 \\
\text{Velocity of } m_1 \text{ after collision} &= v_1 \;=\; 0
\end{aligned}$$

Using the relation

$$
\begin{aligned}
v_1 &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2 \\
0 &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) (-u_1) \\
0 &= \left(\frac{m_1 - m_2 - 2m_2}{m_1 + m_2}\right) u_1 \\
0 &= m_1 - m_2 - 2m_2 \qquad \text{as } u_1 \neq 0, So \ \frac{m_1 - m_2 - 2m_2}{m_1 + m_2} = 0
\end{aligned}
$$

$$
\begin{aligned}
m_1 &= 3m_2 \\
m_2 &= \frac{m_1}{3} = \frac{300}{3} = 100 \ g
\end{aligned}
$$

Problem: 5.4- A 5.18 g bullet moving at 672 ms^{-1} strikes a 715 g wooden block at rest on a frictionless surface. The bullet emerges with its speed reduced to 428 ms^{-1}. Find resulting speed of block.

Solution

$$
\begin{aligned}
\text{Mass of bullet} &= m_1 = 5.18 \times 10^{-3} \ kg \\
\text{Mass of wooden block} &= m_2 = 715 \times 10^{-3} \ kg \\
\text{Velocity of } m_1 \text{ before collision} &= u_1 = 672 \ ms^{-1} \\
\text{Velocity of } m_2 \text{ before collision} &= u_2 = 0 \ ms^{-1} \\
\text{Velocity of } m_1 \text{ after collision} &= v_1 = 428 \ ms^{-1} \\
\text{Velocity of } m_2 \text{ after collision} &= v_2 = ?
\end{aligned}
$$

Since the law of conservation of momentum

$$
\begin{aligned}
\text{Total initial momentum} &= \text{Total final momentum} \\
m_1 u_1 + m_2 u_2 &= m_1 v_1 + m_2 v_2 \\
m_2 v_2 &= m_1 u_1 + m_2 u_2 - m_1 v_1 \\
v_2 &= \frac{m_1 u_1 + m_2 u_2 - m_1 v_1}{m_2}
\end{aligned}
$$

$$v_2 = \frac{(5.18\times10^{-3}\times672)+(715\times10^{-3}+0)-(5.18\times10^{-3}\times428)}{715\times10^{-3}}$$

$$v_2 = 2.76\ ms^{-1}$$

Problem: 5.5- In calculations of energy, we can ignore the kinetic energy of the earth, when considering the energy of a system consisting of the earth and a dropped ball. Prove it.

Solution

Suppose a ball is dropped from a certain height at the earth, the ball falls while the earth remains stationary. By Newton's 3rd law of motion, the earth exerts an upward force and therefore an upward acceleration while the ball falls. Let m_1 be mass of earth and m_2 be mass of ball. Let v_1 be velocity of earth and v_2 be velocity of ball. From law of conservation of linear momentum,

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

$$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

Ratio of kinetic energy of earth to ball is:

$$\frac{K.E_1}{K.E_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2}$$

$$\frac{K.E_1}{K.E_2} = \frac{m_1v_1^2}{m_2v_2^2}$$

$$\frac{K.E_1}{K.E_2} = \frac{m_1}{m_2}\left(\frac{v_1^2}{v_2^2}\right)$$

$$\frac{K.E_1}{K.E_2} = \frac{m_1}{m_2}\left(-\frac{m_2}{m_1}\right)^2$$

$$\frac{K.E_1}{K.E_2} = \frac{m_1}{m_2}\times\frac{m_2^2}{m_1^2}$$

$$\frac{K.E_1}{K.E_2} = \frac{m_2}{m_1}$$

If we take the mass of ball $m_2 = 0.25\ kg$, then

$$\frac{K.E_1}{K.E_2} = \frac{0.25}{6 \times 10^{24}}$$

$$\frac{K.E_1}{K.E_2} = 0.041 \times 10^{-24} = 4.1 \times 10^{-26}$$

$$\frac{K.E_1}{K.E_2} \cong 10^{-26}$$

This relation indicates that K.E of earth is 10^{-26} times the K.E of the ball so K.E. of earth can be neglected.

Chapter 6

Gravitation

SOLVED PROBLEMS

Problem: 6.1- Calculate the potential energy of the moon-earth system relative to the potential energy at infinite separation.

Solution

As we know that:

$$\begin{aligned} \text{Mass of earth} &= M = 6 \times 10^{24}\ kg \\ \text{Mass of moon} &= m = 7.36 \times 10^{22}\ kg \\ \text{Separation distance} &= r = 3.82 \times 10^{8}\ m \\ \text{Gravitational constant} &= G = 6.67 \times 10^{-11}\ Nm^2kg^{-2} \end{aligned}$$

Since,

$$U(r) = -\frac{GMm}{r}$$

$$U(r) = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.36 \times 10^{22}}{3.82 \times 10^{8}}$$

$$U(r) = -\frac{294.55 \times 10^{35}}{3.82 \times 10^{8}}$$

$$U(r) = -77.10 \times 10^{27}$$
$$U(r) = -7.71 \times 10^{28}\ J$$

Problem: 6.2- Calculate the gravitational force between two 7.3 kg bowling balls separated by 0.65 m between their centers.

Solution

$$\text{Mass of each bowling ball} = m_1 = m_2 = 7.3\ kg$$
$$\text{Distance between centers of balls} = r = 0.65\ m$$
$$\text{Gravitational constant} = G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$$
$$\text{Gravitational force} = F = ?$$

Since we know that:

$$F = G\frac{m_1 m_2}{r^2}$$
$$F = 6.67 \times 10^{-11}\frac{7.3 \times 7.3}{(0.65)^2}$$
$$F = \frac{355.44 \times 10^{-11}}{0.4225}$$
$$F = 841.28 \times 10^{-11}$$
$$F = 8.41 \times 10^{-9}\ N$$

Problem: 6.3- A satellite orbits at a height of 230 km above the surface of earth. Calculate the period of satellite.

Solution

$$\text{Mass of earth} = M = 6 \times 10^{24}\ kg$$
$$\text{Height} = h = 230\ km = 230 \times 10^3\ m$$
$$\text{Radius of satellite orbit} = r = R + h = 6400 + 230 = 6630 \times 10^3\ m$$
$$\text{Gravitational constant} = G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$$

Now according to law of periods, we have

$$T^2 = \frac{4\pi^2 r^3}{GM}$$
$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$
$$T = \sqrt{\frac{4 \times (3.14)^2 \times (6630 \times 10^3)^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}}}$$
$$T = \sqrt{\frac{39.44 \times 2.91 \times 10^{11} \times 10^9}{40.02 \times 10^{13}}}$$
$$T = \sqrt{\frac{114.77 \times 10^{20}}{40.02 \times 10^{13}}}$$
$$T = \sqrt{2.8678 \times 10^7}$$
$$T = \sqrt{28.678 \times 10^6}$$
$$T = 5.355 \times 10^3$$
$$T = 5355\ s$$

Problem: 6.4- A reconnaissance spacecraft circles the moon at very low altitude. Calculate its speed.

Solution

$$\text{Mass of the moon} = M = 7.36 \times 10^{22}\ kg$$
$$\text{Radius of orbit} = r = 1.74 \times 10^6\ m$$
$$\text{Gravitational constant} = G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$$
$$\text{Speed of spacecraft} = v = ?$$

Since the relation is:

$$v = \sqrt{\frac{GM}{r}}$$
$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 7.36 \times 10^{22}}{1.74 \times 10^6}}$$
$$v = \sqrt{\frac{49.091 \times 10^{11}}{1.74 \times 10^6}}$$

$$v = \sqrt{28.213 \times 10^5}$$
$$v = \sqrt{282.13 \times 10^4}$$
$$v = 16.79 \times 10^2 \ ms^{-1}$$
$$v = 1.67 \times 10^3 \ ms^{-1}$$

Problem: 6.5- Find the mass of Mars having radius $3.39 \times 10^6 \ m$. Given that acceleration due to gravity on surface of Mars is $3.73 \ ms^{-2}$.

Solution

$$\text{Radius of Mars} = R = 3.39 \times 10^6 \ m$$
$$\text{Gravitational acceleration} = g = 3.73 \ ms^{-2}$$
$$\text{Gravitational constant} = G = 6.67 \times 10^{-11} \ Nm^2kg^{-2}$$
$$\text{Mass of Mars} = M = ?$$

As we know that:

$$g = \frac{GM}{R^2}$$
$$\Rightarrow M = \frac{gR^2}{G}$$
$$M = \frac{3.73 \times (3.39 \times 10^6)^2}{6.67 \times 10^{-11}}$$
$$M = 6.42 \times 10^{23} \ kg$$


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