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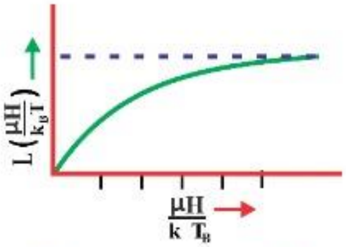

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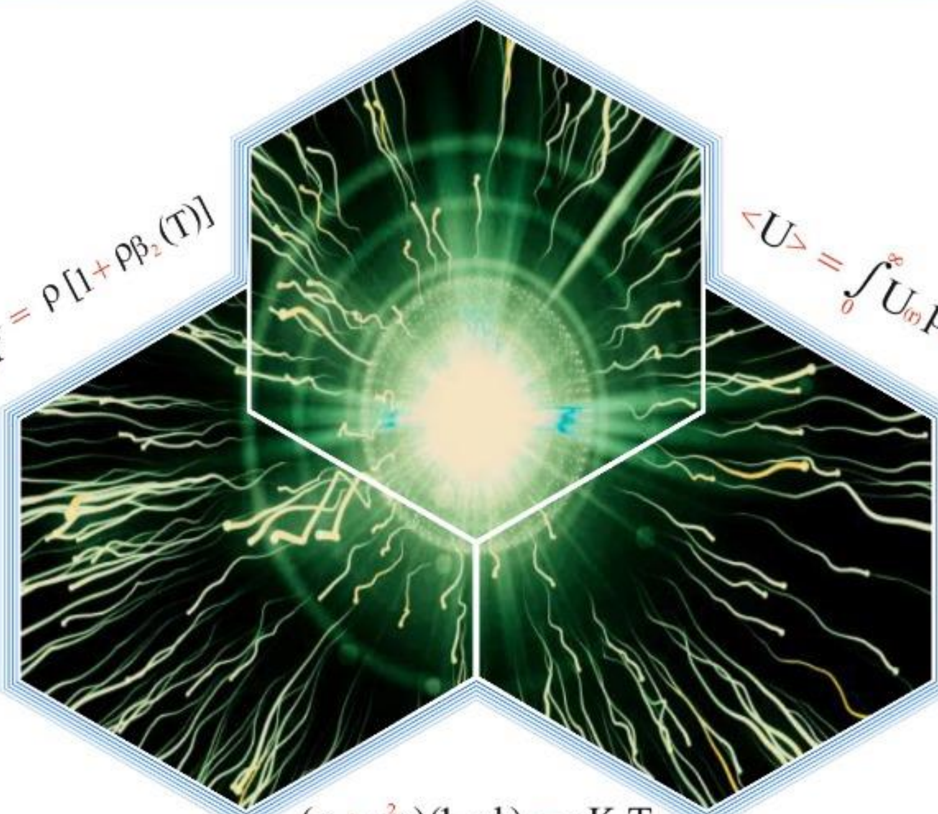
TEACH YOURSELF

THERMAL & STATISTICAL PHYSICS

For BS/M.Sc Physics Programme

3rd Edition

$\frac{p}{K_\beta T} = \rho\,[1+\rho\beta_2(T)]$



$\langle U \rangle = \int_0^\infty U_{(r)}\, p_{(r)}\, dr$

$(p+\rho^2 a)(1-\rho b) = \rho K_\beta T$

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Dr. Syed Hamad Bukhari

TEACH YOURSELF

THERMAL & STATISTICAL PHYSICS

2nd Edition

For **BS/M.Sc Physics** students of all Pakistani Universities/Colleges

•

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Quanta Publisher, 2660/6C Raza Abad, Shah Shamas, Multan.

Contents

Chapter 1

Equilibrium Thermodynamics

SOLVED PROBLEMS

Problem: 1.1- Rewrite the van der Waals equation in the form $P = G(\theta_G, V)$.

Solution

The van der Waals equation is given by

$$\Rightarrow \quad \theta_G R = \left(P + \frac{a}{V}\right)(V - b)$$

$$\frac{\theta_G R}{(V-b)} = P + \frac{a}{V}$$

$$\text{or} \quad P = \frac{\theta_G R}{V-b} - \frac{a}{V} \quad \text{or} \quad P = G(\theta_G, V)$$

Problem: 1.2-

By writing the internal energy as a function of state $U(T, V)$ show that

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$

Solution

$$U = U(T, V)$$

Differentiating it, we have

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \tag{1.1}$$

As, from first law of thermodynamics: $dQ = dU + PdV$, so that

$$dU = dQ - PdV \tag{1.2}$$

putting value of Eq.(1.2) and Eq.(1.1), we have

$$\begin{aligned} dQ - PdV &= \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial T}\right)_T dV \\ dQ &= \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial T}\right)_T dV + PdV \\ dQ &= \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial T}\right)_T + P\right] dV \end{aligned}$$

Hence proved.

Problem: 1.3- Which one of the following is the exact differential?

1. $dx = (10y + 6z)\, dy + 6ydz,$
2. $dx = (3y^2 + 4yz)\, dy + (2yz + y^2)\, dz,$
3. $dx = y^4 Z^{-1} dy + zdz$?

Solution

(1).

$$dx = (10y + 6z)\, dy + 6ydz$$

Let $dx = Ady + Bdz$, where $A = 10y + 6z$ and $B = 6y.$

$$\text{If} \quad \left(\frac{\partial A}{\partial z}\right)_y = \left(\frac{\partial B}{\partial y}\right)_z$$

then dx is an exact differential equation.

$$\begin{aligned} \left(\frac{\partial A}{\partial z}\right)_y &= \frac{\partial}{\partial z}(10y + 6z) = 6 \\ \left(\frac{\partial B}{\partial z}\right)_z &= \frac{\partial}{\partial z}(6y) = 6 \end{aligned}$$

Hence these are exact differentials.
(2).

$$dx = \left(3y^2 + 4yz\right) dy + \left(2yz + y^2\right) dz$$

Let $dx = Ady + Bdz$, where $A = (3y^2 + 4yz)$ and $B = 2yz + y^2$. The given equation is an exact differential.

$$\text{If} \quad \left(\frac{\partial A}{\partial z}\right)_y = \left(\frac{\partial B}{\partial y}\right)_z$$

Now

$$\left(\frac{\partial A}{\partial z}\right)_y = \frac{\partial}{\partial z}\left(3y^2 + 4yz\right) = 4y$$
$$\left(\frac{\partial B}{\partial y}\right)_z = \frac{\partial}{\partial y}\left(2yz + y^2\right) = 2z + 2y$$
$$\left(\frac{\partial A}{\partial z}\right)_y \neq \left(\frac{\partial B}{\partial y}\right)_z$$

Hence, this is not an exact differential.
(3).

$$dx = y^4 z^{-1} dy + z dz$$

Here $dx = Ady + Bdz$, where $A = y^4 z^{-1}$ and $B = z$. The given equation is an exact differential.

$$\text{If} \quad \left(\frac{\partial A}{\partial z}\right)_y = \left(\frac{\partial B}{\partial y}\right)_z$$

Let us check

$$\left(\frac{\partial A}{\partial z}\right)_y = \frac{\partial}{\partial z}\left(\frac{y^4}{z}\right) = -\frac{y^4}{z^2}$$
$$\left(\frac{\partial B}{\partial y}\right)_z = \frac{\partial}{\partial y}(z) = 0$$

So

$$\left(\frac{\partial A}{\partial z}\right)_y \neq \left(\frac{\partial B}{\partial y}\right)_z$$

is not an exact differential.

Problem: 1.4- The equation listed below are not exact differentials. Find for each equation an integrating factor $g(y,z) = y^{\alpha}z^{\beta}$, where α and β can be any number that will turn in into an exact differential.

1. $dx = 12z^2dy + 18yzdz$
2. $dx = 2e^{-z}dy - ye^{-z}dz$

Solution

(1).

$$\begin{aligned} dx &= g(y,z)\left[12z^2dy + 18yzdz\right] \\ \text{As} \qquad g(y,z) &= y^{\alpha}z^{\beta} \\ dx &= 12z^2y^{\alpha}z^{\beta}dy + 18yzy^{\alpha}z^{\beta}dz \\ dx &= 12y^{\alpha}z^{2+\beta}dy + 18y^{\alpha+1}z^{\beta+1}dz \end{aligned}$$

Here $dx = Ady + Bdz$ so $\quad A = 12y^{\alpha}z^{\beta+2} \quad$ and $\quad B = 18y^{\alpha+1}z^{\beta+1}$

$$\begin{aligned} \left(\frac{\partial A}{\partial z}\right)_y &= 12y^{\alpha}(\beta+2)z^{\beta+1} \\ \left(\frac{\partial B}{\partial y}\right)_z &= 18z^{\beta+1}(\alpha+1)y^{\alpha} \end{aligned}$$

dx is an exact differential, if $\left(\frac{\partial A}{\partial z}\right)y = \left(\frac{\partial B}{\partial y}\right)z$

$$\begin{aligned} \text{i.e.,} 12y^{\alpha}(\beta+2)z^{\beta+1} &= 18(\alpha+1)z^{\beta+1}y^{\alpha} \\ 2(\beta+2) &= 3(\alpha+1) \\ 2\beta+4 &= 3\alpha+3 \\ 2\beta-2\alpha+1 &= 0 \\ \alpha = \frac{2\beta+1}{3} \quad &\text{and} \quad \beta = \frac{3\alpha-1}{2} \end{aligned}$$

So, if $\beta = 1, \alpha = \frac{2+1}{3} = \frac{3}{3} = 1$. Hence, $\alpha = 1, \beta = 1$.

(2).

$$
\begin{aligned}
dx &= g\left(y,z\right)\left[2e^{-z}dy - ydze^{-z}\right] \\
dx &= y^{\alpha}z^{\beta}\left(2e^{-z}dy - ye^{-z}dz\right) \\
dx &= 2e^{-z}y^{\alpha}z^{\beta}dy - z^{\beta}y^{\alpha+1}e^{-z}dz = Ady + Bdz \\
A &= 2e^{-z}y^{\alpha}z^{\beta} \quad \text{and} \quad B \ = \ -e^{-z}y^{\alpha+1}z^{\beta}
\end{aligned}
$$

Now, let dx is an exact differential, so $\left(\frac{\partial A}{\partial z}\right)y \ = \ \left(\frac{\partial B}{\partial y}\right)z$.

$$
\begin{aligned}
\text{Now,} \qquad \left(\frac{\partial A}{\partial z}\right)_y &= 2y^{\alpha}\left(e^{-z}\left(-1\right)z^{\beta} + e^{-z}\beta z^{\beta-1}\right) \\
&= 2y^{\alpha}\left(e^{-z}\beta z^{\beta-1} - e^{-z}z^{\beta}\right) \\
\left(\frac{\partial B}{\partial y}\right)_z &= \frac{\partial}{\partial y}\left(-e^{-z}y^{\alpha+1}z^{\beta}\right) \\
&= \ -e^{-z}\left(\alpha+1\right)y^{\alpha}z^{\beta}
\end{aligned}
$$

Now

$$
\begin{aligned}
-2y^{\alpha}e^{-z}z^{\beta} + 2y^{\alpha}\left(e^{-z}\beta z^{\beta-1}\right) &= \ -e^{-z}\left(\alpha+1\right)y^{\alpha}z^{\beta} \\
2e^{-z}\left(\beta z^{\beta-1} - z^{\beta}\right) &= \ -e^{-z}\left(\alpha+1\right)z^{\beta} \quad \text{dividing by} y^{\alpha} \\
2\beta z^{\beta-1} - 2z^{\beta} &= \ -\left(\alpha+1\right)z^{\beta}\text{dividing by} e^{-z} \\
2\beta z^{\beta-1} &= \ -\left(\alpha+1\right)z^{\beta} + 2z^{\beta} \\
2\beta &= \frac{-\left(\alpha+1\right)z^{\beta} + 2z^{\beta}}{z^{\beta-1}} \\
2\beta &= \ -\frac{\left(\left(\alpha+1\right)+2\right)z^{\beta}}{z^{\beta-1}} \\
2\beta &= \frac{\left(-\alpha-1+2\right)z^{\beta}}{z^{\beta-1}}
\end{aligned}
$$

$$\begin{aligned} 2\beta &= z\left(1-\alpha\right) \\ \frac{2\beta}{z} &= 1-\alpha \\ \frac{2\beta}{z}-1 &= -\alpha \\ \alpha &= 1-\frac{2\beta}{z} \end{aligned}$$

$$\alpha = \frac{z-2\beta}{z}$$

So, if $\alpha = 1, \beta = 0$

Problem: 1.5- Differentiate

$$x = z^2 e^{y^2 z}$$

to get expression for $dx = Ady + Bdz$. Now divided by $ze^{y^2 z}$. Is the resulting equation an exact differential?

Solution

$$\begin{aligned} dx &= Ady + Bdz \\ A = \left(\frac{dx}{dy}\right)_z &= z^2 e^{y^2 z}\left(2zy\right) = 2yz^3 e^{y^2 z} \\ B = \left(\frac{dx}{dz}\right)_y &= 2ze^{y^2 z} + z^2 e^{y^2 z}\left(y^2\right) = 2ze^{yz^2} + y^2 z^2 e^{y^2 z} \\ dx &= 2yz^3 e^{y^2 z} dy + (2ze^{yz^2} + y^2 z^2 e^{y^2 z})dz \end{aligned}$$

Now divided by $ze^{y^2 z}$, we get

$$dx = 2yz^2 dy + \left(2 + y^2 z\right) dz = A^{'} dy + B^{'} dz$$

Now, $A^{'} = 2yz^2, B^{'} = 2 + y^2 z$

$$\left(\frac{\partial A^{'}}{\partial z}\right)_y = 4yz$$

$$\left(\frac{\partial B'}{\partial y}\right)_z = 2yz$$

It is not an exact differential, because $\left(\frac{\partial A'}{\partial z}\right)y \neq \left(\frac{\partial B'}{\partial y}\right)z.$

Chapter 2

Elements of Probability Theory

SOLVED PROBLEMS

Problem: 2.1- Two drunks start out together at the origin each having equal probability of making step to the left or right along x-axis. Find the probability they meet again after N-step. It is to be understood that the men make their steps simultaneously.

Solution

We consider the relative motion of two drunks, with each simultaneous step, they have probability of $\frac{1}{4}$ of decreasing their separation and $\frac{1}{4}$ of increasing their separation. Let the number of times each case occurs n_1, n_2 and n_3 respectively.

$$W(n_1, n_2, n_3) = \frac{N!}{n_1!n_2!n_3!} \left(\frac{1}{4}\right)^{n_1} \left(\frac{1}{4}\right)^{n_2} \left(\frac{1}{4}\right)^{n_3}$$

Where $n_1 + n_2 + n_3 = N$.

The drunk meets if $n_1 = n_2$ the probability that they meet after N-steps irrespective of the number of step n_3 which leave their separation unchanged,

$$P = \sum_{n_3=0}^{N} \frac{N!}{n_1!n_2!n_3!} \left(\frac{1}{4}x\right)^{n_1} \left(\frac{1}{4}x\right)^{n_2} \left(\frac{1}{2}\right)^{n_3}$$

Where we have inserted a parameter x which cancels if $n_1 = n_2$.

By Binomial expansion

$$P' = \left(\frac{1}{4}x + \frac{1}{4}x + \frac{1}{2}\right)^N = \left(\frac{1}{2}\right)^{2N} \left(x^{1/2} + x^{-1/2}\right)^{2N}$$

Now

$$P' = \left(\frac{1}{2}\right)^{2N} \sum_{n}^{2N} \frac{2N!}{n!(2N-n)!} \left(x^{1/2}\right)^n \left(x^{-1/2}\right)^{2N-n}$$

Since x cancel, we choose the term where $n = 2N - n$ or $N = n$.

$$\boxed{P = \left(\frac{1}{2}\right)^{2N} \frac{(2N)!}{(N!)^2}}$$

Problem: 2.2- A penny is tossed 400 times. Find the probability of getting 220 heads.

Solution

We use the Gaussian approximation to Binomial distribution.

$$\begin{aligned} W(n) =& \frac{1}{\sqrt{2\pi \frac{400}{4}}} e^{\frac{-(220-200)}{2(400/4)}} \\ W(n) =& (0.0399)e^{-(20/200)} \\ W(n) =& (0.0399)e^{-(0.1)} \\ W(n) =& (0.0399)(0.9048) \end{aligned}$$

$$\boxed{W(n) = 0.0361}$$

Problem: 2.3- Consider a random walk problem in $1D$, the probability of displacement between S and $S + dS$ being

$$w(S)dS = (2\pi S^2)^{-\frac{1}{2}} e^{-(S-l)^2/2S^2}$$

After N-steps. What is dispersion $(x - \hat{x})^2$?

Solution

$$(x - \bar{x})^2 = \sum_i^N \overline{(S_i - l)}^2 + \sum_{j+i}^N \sum_i^N \overline{(S_i - l)}\,\overline{(S_i - l)}$$

$$\text{Where} \qquad \overline{(S_i - l)} = l - l = 0$$

To find the dispersion $\overline{(S_i - l)}^2$ we note the probability that step length is between S and $S + ds$ in the $\frac{ds}{2b}$.

$$\overline{(S_i - l)}^2 = \int_{l-b}^{l+b} \frac{\overline{(S_i - l)}^2 ds_i}{2b} = \frac{b^2}{3}$$

$$\text{Since} \qquad (x - \bar{x})^2 = \sum_i^N S^2 = NS^2$$

Problem: 2.4- Suppose that preceding problem the volume V under consideration such that $0 << \frac{V}{V_o} << 1$. What is the probability that the number of molecules in the volume between N and $N + dN$.

Solution

Since $\frac{V}{V_o} << 1$ and N_o is large. We use Gaussian distribution

$$P(N)dN = \frac{1}{\sqrt{2\pi\overline{\Delta N}^2}} \exp\left(\frac{-(N - \bar{N})^2}{2\Delta\bar{N}^2}\right) dN$$

Problem: 2.5- A pair of six faced dice with faces marked form 1 to 6 each thrown simultaneously. What is the probability that sum of numbers which shown up is 6.

Solution

No. of ways in which 1st dice can fall $n_1 = 6$.
No. of ways in which 2nd dice can fall $n_2 = 6$.
Total number of equally likely ways in which dice can fall.

$$N = n_1 \times n_2$$
$$N = 6 \times 6 = 36$$

These 36 ways given by

$$(1,1)(1,2)(1,3)(1,4)\underline{(1,5)}(1,6)$$
$$(2,1)(2,2)(2,3)\underline{(2,4)}(2,5)(2,6)$$
$$(3,1)(3,2)\underline{(3,3)}(3,4)(3,5)(3,6)$$
$$(4,1)\underline{(4,2)}(4,3)(4,4)(4,5)(4,6)$$
$$\underline{(5,1)}(5,2)(5,3)(5,4)(5,5)(5,6)$$
$$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)$$

It is clear form above number of ways, in which two dice can fall with sum of numbers equal to 6 is number of favorable ways is equal to 5. Required probability

$$P = \frac{m}{n}$$
$$P = \frac{5}{36}$$

Problem: 2.6- Specific heat of 4.25kg/k Boltzmann constant $k = 1.3851$k. Calculate the ratio of numbers of accessible microstates to 1gm of water at 300K and 300.001K.

Solution

$$\text{Mass of water } m = 1gm$$

$$\begin{aligned}
\text{Specific heat of water} =& 4.25\text{J/mol/K} \\
\text{Initial temperature} =& 300\text{K} \\
\text{Final temperature } T + dT =& 300.001\text{K} \\
\text{Rise of temperature} =& (T + dT) - T = dT \\
=& 300.001 - 300 = 0.0001 = 10^{-4}\text{K}
\end{aligned}$$

Heat gained by water

$$\begin{aligned}
\Delta Q =& mCdT \\
=& 1 \times 4.2 \times 10^{-4} \\
=& 4.2 \times 10^{-4} J
\end{aligned}$$

Change in entropy

$$\begin{aligned}
\Delta S =& \frac{\Delta Q}{T} \\
=& \frac{4.2 \times 10^{-4}}{300} \\
=& 1.4 \times 10^{-6} Jk^{-1}
\end{aligned}$$

As

$$\begin{aligned}
\ln\left(\frac{W_2}{W_1}\right) =& \frac{\Delta S}{k} \\
\ln\left(\frac{W_2}{W_1}\right) =& \frac{1.4 \times 10^{-6}}{1.38 \times 10^{-23}} \\
\ln\left(\frac{W_2}{W_1}\right) =& 1.01 \times 10^{17}
\end{aligned}$$

Chapter 3

Formulation of Statistical Methods

SOLVED PROBLEMS

Problem: 3.1- Write an expression for partition function z if the particle obeys Maxwell-Boltzmann distribution. Consider a system having two particles, each one can be in any one of three Quantum states $0, \varepsilon$ and 3ε. The system is in contact with heat reservoir.

Solution

To find the Maxwell-Boltzmann statics, the table for configuration is

Configuration			**No. of states**
0	ε	3ε	**MB**
xx			1
	xx		1
		xx	1
x	x		2
x		x	2
	x	x	2

From table the partition function for Maxwell-Boltzmann statistics will be:

$$Z_{MB} = 1 + e^{-2\varepsilon\beta} + e^{-6\varepsilon\beta} + 2e^{-\varepsilon\beta} + 2e^{-3\varepsilon\beta}$$

Problem: 3.2- The calculation involving Fermi-Dirac statistics gives rise to integral I_m. Show that all these integral can be evaluated

$$J(k) = \int_{-\infty}^{\infty} \frac{e^{ikx}dx}{(e^x+1)(e^{-x}+1)}$$

Since power series $J(k)$ yields

$$J(k) = \sum_{n=0}^{\infty} \frac{(ik)^m}{m!} I_m$$

Solution

We expand e^{ikx} in the integral

$$\begin{aligned} J(k) &= \int_{-\infty}^{\infty} \frac{e^{ikx}dx}{(e^x+1)(e^{-x}+1)} \\ &= \int_{-\infty}^{\infty} \frac{e^x e^{ikx}dx}{(e^x+1)^2} \\ &= \int_{-\infty}^{\infty} \frac{e^x dx}{(e^x+1)^2} \sum_{m=0}^{\infty} \frac{(ik)^m}{m!} x^m \end{aligned}$$

Integrating the sum the yields

$$\begin{aligned} J(k) =& \frac{(ik)^m}{m!} \int_{-\infty}^{\infty} \frac{e^x}{(e^x+1)^2} x^m dx \\ =& \sum_{m=0}^{\infty} \frac{(ik)^m}{m!} I_m \end{aligned}$$

Problem: 3.3- Radiation from the Big Bang has been doppler shifted to longer wave-length by expansion of universe today has a spectrum corresponding to that of a black-body at 2.7K. Find the wave-length.

Solution

From Wein's displacement law, we have

$$\begin{aligned}\lambda_{max} =& \frac{2.898 \times 10^{-3} mK}{T} \\ =& \frac{2.898 \times 10^{-3} mK}{2.7K} \qquad \text{Given } T = 2.7K \\ =& 1.1 \times 10^{-3} m\end{aligned}$$

Problem: 3.4- A dielectric solid has an index of refraction n_o. Which can be assumed to be constant up to infrared frequency. Calculate the contribution of black body radiation in solid heat capacity at temperature $T = 300^o$K. Compare this result with the classical lattice heat capacity of $3R$ per mole.

Solution

The energy of a black body radiation in dielectric

$$\bar{F} = V\bar{\mu} = V\frac{\pi^2}{15}\frac{(k_BT)^4}{(c\hbar)^3}$$

Where c is the velocity of light in the material c.

$$\begin{aligned}\bar{F} =& V\frac{\pi^2}{15}\frac{(k_BT)^4}{(c\hbar)^3}n_o^3 \\ =& \frac{4\sigma n_o^3 V T^4}{c}\end{aligned}$$

Thus Stephen-Boltzmann constant

$$\sigma = \frac{\pi^2 k^4}{60c^2\hbar^3}$$

Thus

$$C_v = \left(\frac{\partial \bar{F}}{\partial T}\right)_v$$
$$= \frac{16\sigma n_o^3 V T^3}{c}$$

Taking its ratio with $C_V = 3R$ gives

$$\frac{C_{v'}}{C_v} = \frac{16\sigma n_o^3 V T^3}{3R_C}$$

For an order of magnitude calculation we can let $V = 10 cm^3/mol$ and $n_o = 1.5$. At $300K$ we find

$$\frac{C_{v'}}{C_v} \approx 10^{-13}$$

Problem: 3.5- An electron in one dimensional infinite potential well defined by $V(x) = 0$ for $-a \leq x \leq a$ and $V(x) = \infty$ otherwise goes from the $n = 4, n = 2$ level. The frequency of emitted photon is 3.43×10^{14}Hz. Find the width of box.

Solution

$$E_n = \frac{\pi^2 \hbar^2 n^2}{8ma^2}$$

Given that $n = 1, n = 2$

$$E_1 - E_2 = \frac{12\pi^2\hbar^2}{8ma^2}$$
$$h\nu = \frac{12\pi^2\hbar^2}{8ma^2} \qquad \because E_1 - E_2 = h\nu$$
$$a^2 = \frac{12\pi^2\hbar^2}{8m\hbar\nu}$$
$$a^2 = \frac{3\hbar^2}{8m\hbar\nu} = \frac{3\hbar}{8m\nu}$$
$$a^2 = \frac{3(6.626 \times 10^{-34} Js)}{8(9.1 \times 10^{-31})(3.43 \times 10^{14})}$$
$$a^2 = 79.6 \times 10^{-20} m^2$$
$$a = 8.92 \times 10^{-10} m$$

Chapter 4

Partition Function

SOLVED PROBLEMS

Problem: 4.1- A system consists of three energy levels i.e., ground level $E_o = 0J$, $E_1 = 0.25k_BT$J and $E_2 = 0.77k_BTJ$. Calculate the partition function, also calculate the probability of 2nd energy level.

Solution

Partition function: Let z is the partition function

$$z = \sum_i e^{-\beta E_i}$$

Where $\beta = \frac{1}{k_BT}$, Now

$$z = \exp\left(\frac{-1}{k_BT}(0)\right) + \exp\left(\frac{-1}{k_BT}(0.25)k_BT\right) + \exp\left(\frac{1}{k_BT}(0.77)k_BT\right)$$

$$z = e^0 + e^{-0.25} + e^{-0.77}$$

$$z = 1 + 1.2418 \qquad e^o = 1$$

$$z = 2.2418$$

Which is partition function.

Probability:

$$
\begin{aligned}
P(E) \propto & e^{-E/k_BT} \\
P(E) = & \frac{e^{-E_2/k_BT}}{e^{-E_o/k_BT} + e^{-E_1/k_BT} e^{-E_2/k_BT}}
\end{aligned}
$$

Putting the values

$$
\begin{aligned}
P(E) = & \frac{e^{\frac{-0.77k_BT}{k_BT}}}{e^{\frac{-0k_BT}{k_BT}} + e^{\frac{-0.25k_BT}{k_BT}} + e^{\frac{-0.77k_BT}{k_BT}}} \\
P(E) = & \frac{e^{-0.77}}{e^0 + e^{-0.25} + e^{-0.77}} \\
P(E) = & \frac{0.46301}{1 + 0.77880 + 0.46301} \\
P(E) = & \frac{0.46301}{2.24184} \;=\; 0.20653
\end{aligned}
$$

Problem: 4.2- Determine the probability of an energy state above E_F occupied by an electron. Determine the probability that energy level $3k_BT$ above the Fermi-energy level is occupied by an electron.

Solution

$$
f_E(E) = \frac{1}{1 + e^{(E - E_F/k_BT)}}
$$

Putting values

$$
\begin{aligned}
f_E = & \frac{1}{1 + e^{(3k_BT/k_BT)}} \\
f_E = & \frac{1}{1 + e^3} \\
f_E = & \frac{1}{1 + 20.08554} \\
f_E = & \frac{1}{21.08554} \;=\; 0.04743
\end{aligned}
$$

An energies above E_F the probability of state being occupied by an electron can becomes significantly less then unity.

Problem: 4.3- Assume a Fermi-energy level exactly in the center of bandgap energy of a Semi-conductor at $T = 300K$. Calculate the probability that energy of a state in bottom.

Solution

Since

$$\begin{aligned} E - E_F &= E_c - E_F \\ 0.56 &>> k_B T \end{aligned}$$

We can use the Boltzmann approximation

$$\begin{aligned} f(E) =& \frac{1}{1 + e^{(E-E_F)/k_B T}} \\ \approx& \frac{1}{e^{(E-E_F)/k_B T}} \\ =& e^{-(E-E_F)/k_B T} \\ =& e^{(E+E_F)/k_B T} \\ =& e^{-\frac{0.56eV}{0.002580}} \\ =& 3.938 \times 10^{-10} \end{aligned}$$

Problem: 4.4- Harmonic oscillator/canonical ensemble. Consider a system of N harmonic oscillators. Which are indistinguishable, one dimensional and having same frequency ω.

(a) Compute partition function. (b) Find free energy.

Solution

A system of N non-integrating independent distinguishable, in thermal equilibrium at absolute temperature,

$$H = \frac{P^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

Partition function:

$$
\begin{aligned}
z_1(T,V) =& \frac{1}{n}\int e^{-\beta P^2/2m} e^{-\beta m\omega^2 x^2/2} dx dp \\
z_1(T,V) =& \frac{1}{n}\int_{-\infty}^{\infty} e^{-\beta P^2/2m} dp \int_{-\infty}^{\infty} e^{-\beta m\omega^2 x^2/2} dx \\
z_1(T,V) =& \frac{4}{n}\left[\frac{\sqrt{\pi}}{2}\frac{1}{(\beta/2m)^{1/2}}\right]\left[\frac{\sqrt{\pi}}{2}\frac{1}{(\beta m\omega^2/2)^{1/2}}\right] \\
z_1(T,V) =& \frac{k_B T}{\hbar\omega}
\end{aligned}
$$

Partition function for whole system

$$
z(T,V) = (z_1)^N = \left(\frac{k_B T}{\hbar\omega}\right)^N
$$

Free energy:

$$
\begin{aligned}
A =& -k_B T \ln z \\
A =& -k_B T \ln\left[\frac{k_B T}{\hbar\omega}\right] \\
A =& N k_B T \ln\left(\frac{\hbar\omega}{k_B T}\right)
\end{aligned}
$$

Problem: 4.5- Calculate the probability of Harmonic oscillator $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$ in state with n odd number if the oscillator is in contact with heat bath at temperature T.

Solution

The probability that the harmonic oscillator is in state with n-odd numbers is given by

$$
\begin{aligned}
P\left(n+\frac{1}{2}\right) =& \frac{\sum_{n=1}^{\infty} e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}} \\
=& \frac{\sum_{n=1}^{\infty} e^{-\beta\left(n+\frac{1}{2}\right)\hbar\omega}}{\sum_{n=0}^{\infty} e^{-n\beta\hbar\omega}}
\end{aligned}
$$

By expansion

$$
\begin{aligned}
&= \frac{e^{-\beta\hbar\omega/2} + e^{-3\beta\hbar\omega/2} + e^{-5\beta\hbar\omega/2} + e^{-7\beta\hbar\omega/2} + \cdots + \infty}{1 + e^{-\beta\hbar\omega} + e^{-2\beta\hbar\omega} + e^{-3\beta\hbar\omega} + \cdots + \infty} \\
&= \frac{e^{-\beta\hbar\omega} / \left(1 + e^{-2\beta\hbar\omega}\right)}{1/\left(1 + e^{-\beta\hbar\omega}\right)} \\
&= \frac{e^{-\beta\hbar\omega}\left(1 - e^{-\beta\hbar\omega}\right)}{1 - e^{-2\beta\hbar\omega}} \\
&= \frac{e^{-\beta\hbar\omega} - e^{-2\beta\hbar\omega}}{1 - e^{-2\beta\hbar\omega}}
\end{aligned}
$$

By simplifying it can be written as

$$P = \frac{e^{\beta\hbar\omega} - 1}{e^{2\beta\hbar\omega}}$$

$$P\left(n + \frac{1}{2}\right) = \frac{e^{\beta\hbar\omega} - 1}{e^{2\beta\hbar\omega} - 1}$$

Chapter 5

Statistical System

SOLVED PROBLEMS

Problem: 5.1- N non-interacting bosons are in an infinite potential well defined by $V(x) = 0$ for $0 < x < a$; $V(x) = \infty$ for $x < 0$ and $x > a$. Find the ground state energy of system. What would be the ground state energy if the particles are fermions.

Solution

The energy eigenvalue of a particle in infinite square well is given by

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

As the particles are Bosons N-particles will be in $n = 1$ state, total energy

$$E_{n_1,n_2,n_3,\cdots n_\gamma} = \frac{\pi^2\hbar^2}{2a^2m}(1^2 + 1^2 + 1^2 + \cdots 1^2)$$
$$E = \frac{\pi^2\hbar^2}{2a^2m}$$

If particle are fermions a state can have only two of them one spin up and another spin down. Therefore, lowest state $N/2$ will be filled.

$$2E_1, 2E, 2E_3, \cdots, E_{N/2} = E' = \frac{\pi^2\hbar^2}{2ma}(1^2 + 1^2) + (2^2 + 2^2 + \cdots)\cdots$$
$$E^o = \frac{\pi^2\hbar^2 N^3}{24ma^2}$$

Problem: 5.2- Calculate the root mean square speed if nitrogen at 27^oC. Given $N = 6 \times 10^{23}$ molecules/mole, $k_B = 1.38 \times 10^{-16}$ergs/k.

Solution

$$\text{Temperature} = T = 27 + 273 = 300K$$
$$\text{Mass of Nitrogen molecule} = m = \frac{\text{Mol.Wt}}{N} = \frac{28}{6 \times 10^{23}}$$

$$m = 4.66 \times 10^{-23} gm$$
$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

Putting the values, where k is Boltzmann constant

$$V_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-16} \times 300}{4.66 \times 10^{-23}}}$$
$$= \sqrt{26.65 \times 10^8}$$
$$= 5.16 \times 10^4 \text{cm/sec}$$

Problem: 5.3- Draw the energy levels, including the spin orbit interaction for $n = 3$ and $n = 2$, state of Hydrogen atom and calculate the spin orbit double separation of the $2p, 3p$ and $3d$ state. The Redberg constant of Hydrogen is $1.097 \times 10^7 m^{-1}$.

Solution

As we know that the energy levels for $n = 3$ and $n = 2$ states of Hydrogen ($z = 1$) including the spin orbit interaction.
The double separation

$$\Delta E = \frac{z^4 a^2 R}{n^3(l+1)}$$

For $2p$ state, $n = 2l, \ l = 1$

$$(\varDelta E)_{2p} = \frac{(1/137)^2(1.097 \times 10^7)}{8 \times 2}$$
$$(\varDelta E)_{2p} = 36.53m^{-1}$$

For $3p$ state $n = 3,\ l = 1$

$$(\varDelta E)_{3p} = \frac{(1/137)^2(1.097 \times 10^7)}{27 \times 2}$$
$$= 10.82m^{-1}$$

For $3d$ state $n = 3,\ l = 2$

$$(\varDelta E)_{3d} = \frac{(1/137)^2(1.097 \times 10^7)}{27 \times 3 \times 2}$$
$$= 3.61m^{-1}$$

Problem: 5.4- Consider an ideal gas of N-electrons in volume V at absolute zero.

(a) Calculate the total mean energy $\bar{E}$ of this gas.

(b) Express $\bar{E}$ in term of Fermi energy μ.

Solution

(a)

At $T = 0$ all states are filled up the energy μ. Hence mean number of particle per state is just 1. We have

$$\bar{E} = \int_0^\mu En(E)dE$$

$$\text{As} \quad n(E)dE = \frac{2V}{4\pi^2}\frac{(2m)^{3/2}}{\hbar^2}e^{1/2}dE$$

Where the factor 2 is introduced since electrons have two spin states

$$\bar{E} = \frac{V}{2\pi^2}\frac{(2m)^{3/2}}{\hbar^2}\int_0^\mu e^{3/2}dE$$
$$\bar{E} = \frac{V}{2\pi^2}\frac{(2m)^{3/2}}{\hbar^2}\mu^{5/2}$$

(b)

Since

$$\mu = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{\gamma}\right)^{3/2}$$

So we have

$$\bar{E} = \frac{3}{5}N\mu$$

Which is our required result.

Problem: 5.5- A quark having one-third the mass of a proton is confined in a cubical box of side $1.8 \times 10^{-15}m$. Find the excitation energy in Mev form the first excited state to the second excited state.

Solution

$$E_{n_1n_2n_3} = \frac{\pi^2\hbar^2}{2ma^2}(n_1^2 + n_2^2 + n_3^2)$$

And

$$\text{First excited state } E_{211} = E_{121} = E_{112} = \frac{6\pi^2\hbar^2}{2ma^2}$$
$$\text{First excited state } E_{221} = E_{212} = E_{122} = \frac{9\pi^2\hbar^2}{2ma^2}$$

$$m = \frac{1.67262 \times 10^{-27}}{3}$$
$$m = 0.55754 \times 10^{-27}kg$$
$$\Delta E = \frac{3\pi^2\hbar^2}{2ma^2}$$
$$\Delta E = \frac{3(3.14)^2(1.05 \times 10^{-34})^2}{2(0.55754 \times 10^{-27}kg)(1.8 \times 10^{-15})}$$

$$\Delta E = 9.0435 \times 10^{-11} J$$

$$\Delta E = \frac{9.0435}{1.6 \times 10^{-19} J/eV}$$

$$\Delta E = 565.2 MeV$$

Chapter 6

Statistical Mechanics of Interacting System

SOLVED PROBLEMS

Problem: 6.1- Use the Debye approximation to find the following thermodynamics functions of a solid as a function of absolute temperature T.

(a) $\ln z$, where z is partition function.

(b) The mean energy.

Solution

Partition function:

As

$$\ln z = \beta N_\eta - \int_0^\infty \ln\left(1 - e^{\beta \frac{1}{2} m\omega}\right) \delta_D(\omega) d\omega$$

Where

$$\delta_D(\omega) \begin{cases} \frac{3V}{2\pi^2} e^3 \omega^2 & \text{For } \omega_D > \omega \\ 0 & \text{For } \omega > \omega_D \end{cases}$$

Put

$$V = 6\pi^2 N \left(\frac{C}{\omega_D}\right)$$

We find

$$\ln z = \beta N_\eta - 9\frac{N}{\omega_D}\int_0^{\omega_D} \ln\left(1 - e^{\beta t\omega}\right)\omega^2 d\omega$$

In terms of dimensionless variables $x = \beta\hbar\omega$ and $y = \beta\hbar\omega$, this gives

$$\begin{aligned}
\ln z =& \frac{\partial N_\eta}{\hbar\omega_D} - \frac{9N}{Y^3}\int_0^y \ln(1-e^x)x^2 dx \\
\ln z =& Y\frac{N_\eta}{\hbar\omega_D} - \frac{9N}{Y^3}\left[\ln(1-e^{-x})\frac{x^3}{3}\right]^y - \frac{1}{3}\int_0^y \frac{x^3 dx}{e^x - 1} \\
\ln z =& y\frac{N_\eta}{\hbar\omega_D} - 3N\ln\left(1 - e^{-y}\right) + N_D(y) \\
\ln z =& \frac{N_\eta}{\hbar\omega_D} - 3N\ln\left(1 - e^{-\theta_D/T}\right) + N_D\left(\frac{\theta_D}{T}\right)
\end{aligned}$$

Where $k\theta_D = \hbar\omega_D$.

Free energy:

$$\bar{E} = -\frac{\partial}{\partial\beta}\ln z = -\hbar\omega_D\frac{\partial}{\partial y}\ln z$$

Here

$$\begin{aligned}
\bar{E} =& -N_\eta + \frac{3N}{\beta}Dy \qquad \because Dy = D\left(\frac{\theta_D}{T}\right) \\
\bar{E} =& -N_\eta + 3NkTD\left(\frac{\theta_D}{T}\right)
\end{aligned}$$

Problem: 6.2- Using heat of vaporization for water in J/g. Calculate the energy needed to boil $50.0g$ of water at its boiling point of 100^oC

Solution

$$\text{The mass of water} = m = 50.0g$$
$$\text{Heat of vaporization} = Q = 2259\ J/g$$

We have to find the energy is required to boil the involved amount of water

$$E = Qm$$
$$E = (50)(2259)$$
$$E = 112950J$$
$$E = 113 \times 10^3 J$$
$$E = 113\ kJ$$

Problem: 6.3- What is the measured pressure of 10 moles of O_2 gas in SL container at 30^oC by using van dar Waals equation.

Solution

$$PV = nRT$$
$$P = \frac{nRT}{V}$$

Given

$$n = 10\ moles$$
$$T = 30^oC = 303K$$
$$V = 5L$$

Putting values

$$P = \frac{(10)(303)(8.314)}{5}$$
$$P = \frac{25191.42}{5}$$
$$P = 5038.2\ Pa$$
$$P = 5.03 \times 10^3\ Pa$$

Problem: 6.4- One mole of a gas obeys van dar Waals equation of state

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

and its internal energy is $U = cT - \frac{a}{V}$. Where a, b, c and R are constants. Calculate C_V

Solution

$$U = \text{function of } (T, V)$$
$$dU = \left(\frac{\partial U}{\partial T}\right)_v dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$
$$dU = dQ - PdV$$

Therefore

$$dQ = \left(\frac{\partial U}{\partial T}\right)_v dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$
$$C_V = \left(\frac{\partial U}{\partial T}\right) = C$$
$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{a}{V^2}$$
$$C_V = \left(\frac{\partial U}{\partial T}\right)_P = \frac{P}{\left(P - \frac{2a}{V^3}\right)(V - b) - b\left(P - \frac{a}{V^2}\right)}$$

Problem: 6.5- Determine the 2^{nd} nearest neighbor distance for N_i at 100^oC of its density at temp is $8.83km^3$.

Solution

$$N_i, \; n = 4$$
$$\text{atomic weighted} = 58.70g/mol$$
$$\rho = 8.83gkm^3$$

Now

$$\frac{\text{Atomic weight}}{\rho} \times 10^{-6} = \frac{N_A}{n} \times a^3$$
$$a^3 = \frac{(58.7)(10^{-6})(4)}{(6.023 \times 10^{23})(8.83)}$$
$$a = 4.41 \times 10^{-29} m^3$$
$$a = 3.61 \times 10^{-10} \times \frac{10^{12}}{m} pm$$
$$a = 3.61 \times 10^2 pm$$

Chapter 7

Advanced Topics

SOLVED PROBLEMS

Problem: 7.1- Use density matrix and trace to calculate the probability of obtaining state measurement.

Solution

If we perform a Von.Neumann measurement of state $\{(q_k|\psi_k\rangle)\}$ w.r.t a basis containing $|\phi\rangle$ the probability obtaining $|\phi\rangle$ is

$$\sum_k q_k|\langle\psi_k|\phi\rangle|^2 = \sum_k q_k T_r(|\psi_k\rangle\langle\phi_k||\phi\rangle\langle\phi|)$$
$$\sum_k q_k|\langle\psi_k|\phi\rangle|^2 = T_r\left\{\sum_k q_k|\psi_k\rangle\langle\phi_k||\phi\rangle\langle\phi|\right\}$$
$$\sum_k q_k|\langle\psi_k|\phi\rangle|^2 = T_r(\rho|\phi\rangle\langle\phi|)$$

The same state.

Problem: 7.2- show that

$$\hat{\rho} = \frac{1}{2}|+n\rangle\langle+n| + \frac{1}{2}|-n\rangle\langle-n| = \frac{1}{2}|+z\rangle\langle+z| + \frac{1}{2}|-z\rangle\langle-z|$$

Where

$$
\begin{aligned}
|+n\rangle &= \cos\left(\frac{\theta}{2}\right)|+z\rangle + e^{i\theta}\sin\left(\frac{\theta}{2}\right)|-z\rangle \\
|-n\rangle &= \sin\left(\frac{\theta}{2}\right)|+z\rangle - e^{-i\theta}\cos\left(\frac{\theta}{2}\right)|-z\rangle
\end{aligned}
$$

Solution

Given density operator is

$$
\hat{\rho} = \frac{1}{2}|+n\rangle\langle +n| + \frac{1}{2}|-n\rangle\langle -n|
$$

Substituting the value of $|+n\rangle$ and $|1-n\rangle$ yield,

$$
\begin{aligned}
\hat{\rho} =& \frac{1}{2}\left[\left(\cos\left(\frac{\theta}{2}\right)|+z\rangle + e^{i\theta}\sin\left(\frac{\theta}{2}\right)|-z\rangle\right)\cdot\left(\langle +n|\cos\left(\frac{\theta}{2}\right) + \langle -z|ee^{i\theta}\sin\left(\frac{\theta}{2}\right)\right)\right] \\
&+ \frac{1}{2}\left[\left(\sin\left(\frac{\theta}{2}\right)|+z\rangle - e^{-i\theta}\cos\left(\frac{\theta}{2}\right)|-z\rangle\right)\cdot\left(\langle +n|\sin\left(\frac{\theta}{2}\right) + \langle -z|e^{i\theta}\cos\left(\frac{\theta}{2}\right)\right)\right] \\
\hat{\rho} =& \left[\frac{1}{2}\right]\cos^2\left(\frac{\theta}{2}\right)|+z\rangle\langle +z| + e^{-i\theta}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)|+z\rangle\langle -z| \\
&+ e^{i\theta}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)|-z\rangle\langle +z| + e^{i\theta}e^{-i\theta}\sin^2\left(\frac{\theta}{2}\right) \\
&+ \sin^2\left(\frac{\theta}{2}\right)|+z\rangle\langle +z| - e^{-i\theta}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)|+z\rangle\langle +z| \\
&- e^{i\theta}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)|+z\rangle\langle +z| + e^{i\theta}e^{-i\theta}\cos^2\left(\frac{\theta}{2}\right)|+z\rangle\langle +z|
\end{aligned}
$$

Now we are left with following expression

$$
\begin{aligned}
\hat{\rho} =& \frac{1}{2}\left[\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)\right]|+z\rangle\langle +z| \\
&+ \frac{1}{2}\left[\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)\right]|-z\rangle\langle -z| \\
\hat{\rho} =& \frac{1}{2}|+z\rangle\langle +z| + \frac{1}{2}|-z\rangle\langle -z|
\end{aligned}
$$

That is required result.

Problem: 7.3- Evaluate the behavior of internal energy and specific heat of Bosons gas in vicinity of Einstein condensation.

Solution

Let us define the function $g_\alpha z$ for $\alpha > 0$ and $|z| < |$ by the series.

$$g_\alpha z = \sum_{k=1}^{\infty} \frac{z^k}{k^\infty}$$

The function has the integral representation

$$g_\alpha z = \frac{1}{\Gamma(\alpha)} \int_0^\infty dx x^{\alpha-1} \frac{ze^{-x}}{1 - ze^{-x}}$$

T_α is Eular gamma function

$$\rho = \frac{1}{\lambda_B^3(T_o)}$$

$$\lambda_B(T) = \left(\frac{h^2}{2\pi k_B T} \right)^{1/2}$$

The thermal De-Broglie length

$$p(T) = T \left(\frac{T}{T_o} \right)^{3/2} g_{5/2}(z(T))$$

Where $z(T)$ satisfies the equation

$$\left(\frac{T}{T_o} \right)^{3/2} g_{3/2}(z(T)) = 1$$

The energy of the per particle

$$E(T) = \frac{3}{2} p(T) = \frac{3}{2} T \left(\frac{T}{T_o} \right)^{3/2} g_{5/2}(z(T))$$

if $T > T_c$

$$\left(\frac{T_c}{T_o} \right)^{3/2} g_{3/2}(1) = 1$$

Riemann zeta function

$$z(R) = \sum_{k=1}^{\infty} \frac{1}{k^{\alpha}}$$
$$T(z) = g_{3/2}(z)^{-2/3}$$

Where T is function of fugacity

$$T(z) = g_{3/2}(z)^{-2/3}$$

if $T < T_c$, $z = 1$. Therefore one has

$$E = \frac{3}{2}T\left(\frac{T}{T_o}\right)^{3/2} g_{3/2}(1)$$

Problem: 7.4- Consider a system of N quantum particles of spin zero and mass m on d dimensions subject to a harmonic potential form

$$U(r) = \frac{1}{2}m\omega_o^2 r^2$$

(a) Give expression of grand canonical function

(b) Give expression of number N' of particles

Solution

(a)

The grand canonical function

$$E_k = \hbar\omega_o\left(\sum_{k=1}^{d} k_i + \frac{d}{z}\right)$$

Thus grand canonical function at temperature T

$$\ln z = -\sum_k \ln\left(1 - e^{-(E_k - U)/k_B T}\right)$$

The sum

$$
\begin{aligned}
N' &= \sum_{k'} \frac{1}{\frac{e^{E_k/k_BT}}{z-1}} \\
&= \sum_{n=1}^{\infty} \frac{Nd(n)}{\frac{e^{E_k/k_BT}}{z-1}}
\end{aligned}
$$

$Nd(n)$ is a polynomial in n of degree 1, we have

$$\ln z = -\sum_{n=0}^{\infty} Nd(n) \ln(1 - ze^{-kn})$$

Where we have introduced the factuality

$$z = e^{-(U_o - U)/k_BT}$$

(b)
The number of particles in excited state

$$N = \sum_{n=0}^{\infty} \frac{Nd(n)}{s^{kn}/(z-1)}$$

Problem: 7.5- For a semi-conductor without impurities and with an energy gap E_g show

$$U_e = \frac{E_\theta}{2} + \frac{k_BT}{2} \ln\left(n\frac{Q_k}{nQ_s}\right)$$

Where the subscripts e and h refers to electron and holes.

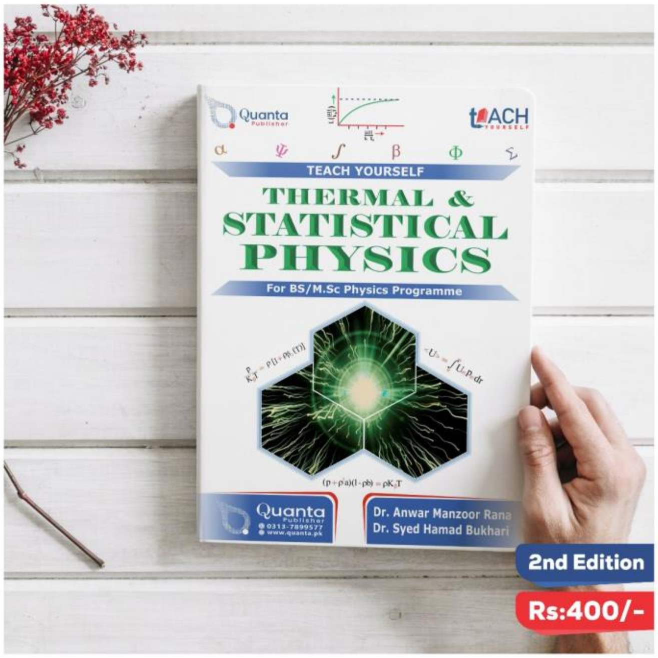
Solution

In equilibrium

$$
\begin{aligned}
U_e + U_h =& 0 \\
ne =& nh
\end{aligned}
$$

In the limits of a low density non-interacting gas at high temperature

$$
\begin{aligned}
U =& \mathit{\Delta} + k_B T \ln\left(\frac{n}{n_Q}\right)\\
U =& E_g + k_B T \ln\left(\frac{n_e}{n_Q^{'}}\right)\\
nh =& nQk^{'} e^{Un/k_B T}\\
nh =& 2n_Q e^{-U_o/k_B T} = ne\\
U_e =& \frac{E_g}{2} + \frac{k_B T}{2}\ln\left(n\frac{Q_n}{n_Q}\right)
\end{aligned}
$$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