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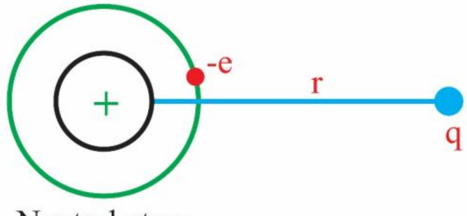


Neutral atom

tEACH YOURSELF

α Ψ $\int$ β Φ Σ

TEACH YOURSELF

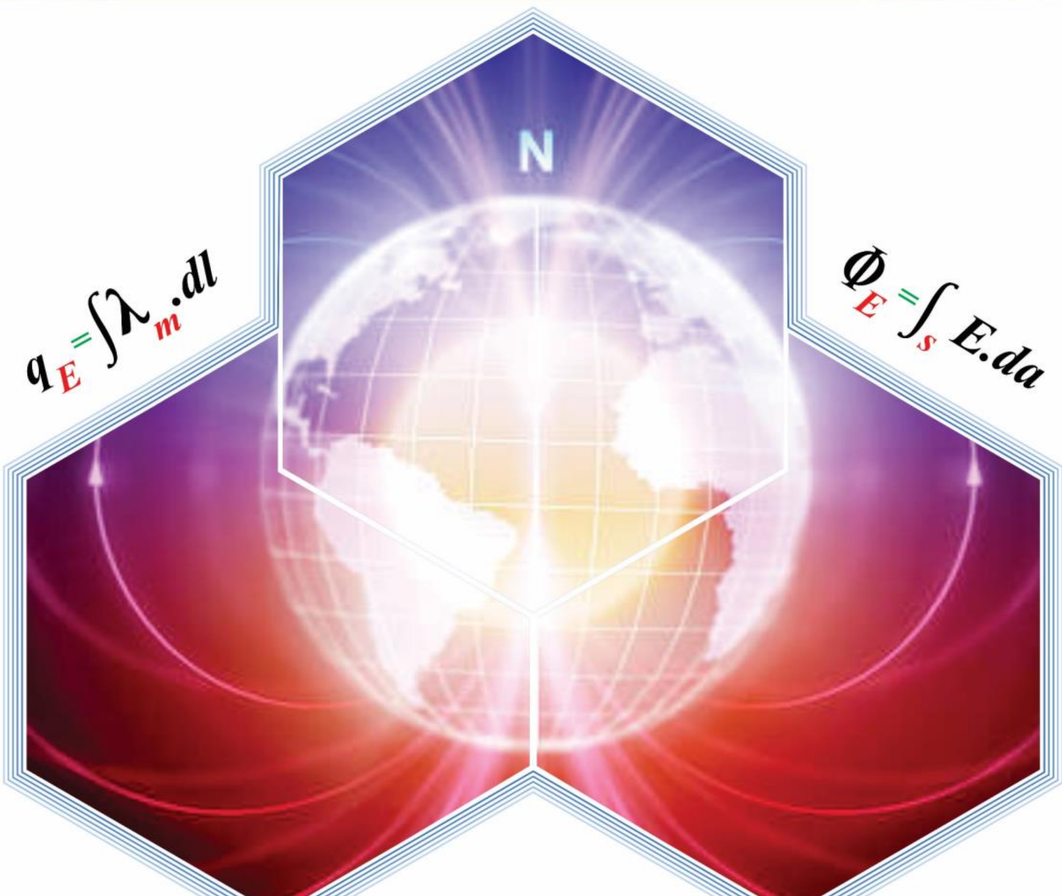
ELECTROMAGNETIC THEORY-II

an approach to

ELECTRODYNAMICS-II

For BS/M.Sc Physics Programme

1st Edition



$q_E = \int \lambda_m . dl$

$\phi_E = \int_s E.da$

$L = N(d\Phi/dt)$

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Dr. Syed Hamad Bukhari

TEACH YOURSELF

ELECTROMAGNETIC THEORY - II

an approach to

ELECTRODYNAMICS - II

1st Edition

For **BS/M.Sc Physics** students of all Pakistani Universities/Colleges

•

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Contents

Chapter 1

Magnetic Field of Steady Currents

SOLVED PROBLEMS

Problem: 1.1- A strip of copper 150 μm thick is placed in a magnetic field 0.65 T perpendicular to the plane of strip, and a current 23 A is set up in the strip. What Hall potential difference would appear across the width of strip of there were one charge carries per atom?

Solution

For copper strip

$$
\begin{aligned}
n &= 8.49 \times 10^{28} \text{ electrons}/ms \\
B &= 0.65\ T \\
t &= 150\ \mu m \\
t &= 150 \times 10^{-6}\ m \\
I &= 23\ A \\
e &= 1.6 \times 10^{-19}\ C \\
V &= ?
\end{aligned}
$$

As, we know that

$$V = \frac{IB}{nte}$$

$$V = \frac{23 \times 0.65}{8.49 \times 10^{28} \times 150 \times 10^{-6} \times 1.6 \times 10^{-19}}$$
$$V = 7.3 \times 10^{-6}\ V$$

Problem: 1.2- What is magnetic dipole moment of the coil which is 2.1 cm high and 1.2 cm wide with 250 turns and assuming that it carries current of 85 μA. The magnetic dipole moment of the coil is lined up with external magnetic field of 0.85 T. How much work would be done by external agent to rotate the coil through and angle of 180°.

Solution

$$A = 2.1 \times 1.2\ cm^2$$
$$A = 2.1 \times 1.2 \times 10^{-4}\ m^2$$
$$A = 2.52 \times 10^{-4}\ m^2$$
$$N = 250$$
$$I = 85\ \mu A$$
$$I = 85 \times 10^{-6}\ A$$
$$B = 0.85\ T$$
$$\mu = ?$$
$$W = ?$$

Since, we have to know that

$$\mu = NIA$$
$$\mu = 250 \times 85 \times 10^{-6} \times 2.52 \times 10^{-4}$$
$$\mu = 5.36 \times 10^{-6}\ J/T$$

Also,

$$W = \mu B(\cos\theta - \cos\theta_\circ)$$
$$W = \mu B(\cos 180^\circ - \cos 0^\circ)$$
$$W = \mu B((-1) - (1))$$
$$W = \mu B(-2)$$

$$W = -2\mu B$$
$$W = -2 \times 5.36 \times 10^{-6} \times 0.85$$
$$W = -9.1 \times 10^{-6}\ J$$
$$W = -9.1\ \mu J$$

Problem: 1.3- A 1.15 kg copper rod rests on two horizontal rails 95 m apart and carries a current of 53.2 A from one rail to other. The coefficient of static friction is 0.58. Find the smallest magnetic field that would cause the bar to slide.

Solution

$$m = 1.15\ kg$$
$$L = 95\ m$$
$$\mu_s = 0.58$$
$$I = 53.2\ A$$
$$B = ?$$

Since,

$$F_m = F$$
$$ILB = \mu_s mg$$
$$B = \frac{\mu_s mg}{IL}$$
$$B = \frac{0.58 \times 1.15 \times 9.8}{53.2 \times 95}$$
$$B = 1.3 \times 10^{-3}\ T$$

Problem: 1.4- A circular coil of 160 turns has a radius of 1.93 cm. Calculate the current that results in a magnetic moment of 2.3 Am^2. Find the maximum torque on the coil that can be experienced in field of 34.6 mT.

Solution

$$
\begin{aligned}
N &= 160\\
r &= 1.93\ cm\ =\ 1.93\times 10^{-2}\ m\\
\mu &= 2.3\ Am^2\\
B &= 34.6\ mT\\
I &= ?\\
\tau &= ?
\end{aligned}
$$

Since, we know that

$$
\begin{aligned}
\mu &= NIA\\
\mu &= NI\pi r^2\\
\text{or}\quad I &= \frac{\mu}{N\pi r^2}\\
I &= \frac{2.3}{160\times 3.14\times (1.93\times 10^{-2})^2}\\
I &= 12.4\ A
\end{aligned}
$$

Since, the torque is maximum when angle is 90°. So,

$$
\begin{aligned}
\tau &= \mu B\sin\theta\\
\tau &= NIAB\sin\theta\\
\tau &= 160\times 12.4\times 3.14(1.93\times 10^{-2})^2\\
&\times 34.6\sin 90^\circ\\
\tau &= 80\ Nm
\end{aligned}
$$

Problem: 1.5- A circular wire loop having radius 8 cm carries a current of $0.2A$. A unit vector parallel to the dipole moment μ of the loop is given by $\left(0.6\hat{i}-0.8\hat{j}\right)$. If the loop is located in $\vec{B}=\left(0.25\ \hat{i}+0.3\hat{k}\right)\ T$. Find the torque on the loop and also find the magnetic potential energy of the loop.

Solution

Given data:

$$\begin{aligned}
\vec{B} &= \left(0.25\ \hat{i}+0.3\hat{k}\right)\ T \\
\hat{\mu} &= \left(0.6\hat{i}-0.8\hat{j}\right) \\
I &= 0.2\ A \\
r &= 8\ cm\ =\ 8\times10^{-2}\ m \\
N &= 1 \\
\vec{\tau} &= ? \\
U_m &= ?
\end{aligned}$$

Since, we have to know that

$$\begin{aligned}
\vec{\mu} &= \mu\hat{\mu} \\
\vec{\mu} &= NIA\hat{\mu} \qquad \because \mu = NIA \\
\vec{\mu} &= NI\pi r^2\hat{\mu} \qquad \because A=\pi r^2 \\
\vec{\mu} &= 1\times0.2\times3.14\times(8\times10^{-2})^2\left(0.6\hat{i}-0.8\hat{j}\right) \\
\vec{\mu} &= 4.01\times10^{-3}\left(0.6\hat{i}-0.8\hat{j}\right)\ J/T \\
\vec{\mu} &= \left(2.41\times10^{-3}\hat{i}-3.20\times10^{-3}\hat{j}\right)\ J/T
\end{aligned}$$

Now, the torque acting on the loop is

$$\begin{aligned}
\vec{\tau} &= \vec{\mu}\times\vec{B} \\
\vec{\tau} &= \left(2.41\times10^{-3}\hat{i}-3.20\times10^{-3}\hat{j}\right)\times\left(0.25\ \hat{i}+0.3\hat{k}\right) \\
\vec{\tau} &= 9.60\times10^{-3}\hat{i}-7.23\times10^{-3}\hat{j}-80.0\times10^{-3}\hat{k}
\end{aligned}$$

Also, we know that

$$U_m = -\vec{\mu}\cdot\vec{B}$$
$$U_m = -\left(2.41\times10^{-3}\hat{i} - 3.20\times10^{-3}\hat{j}\right)\cdot\left(0.25\ \hat{i} + 0.3\hat{k}\right)$$
$$U_m = -\left(-2.41\times10^{-3}\times0.25\right)$$
$$U_m = 2.41\times10^{-3}\times0.25$$
$$U_m = 6.025\times10^{-4}\ Nm$$

Chapter 2

Magnetic Properties of Matter

SOLVED PROBLEMS

Problem: 2.1- A magnetic slab of infinite extent in the x and y directions is placed within a uniform magnetic field $H_\circ i_x$. Find the $\vec{H}$ field within the slab when it is,
(a) Permanently magnetized with magnetization $M_\circ i_x$. **(b)** A linear permeable material with permeability μ.

Solution

For both cases, the $\vec{B}$ field across the boundaries be continuous as it is normally incident.
(a) For the permanently magnetized slab, this requires that

$$\mu_\circ H_\circ = \mu_\circ(H + M_\circ) \Rightarrow H = H_\circ - M_\circ$$

Note that when there is no externally applied field ($H_\circ = 0$), the resulting field within the slab is oppositely directed to the magnetization so that $\vec{B} = 0$.
(b) For a linear permeable medium,

$$\mu_\circ H_\circ = \mu H \Rightarrow H = \frac{\mu_\circ}{\mu} H_\circ$$

For $\mu > \mu_\circ$ the internal magnetic field is reduced. If $\mathbf{H}_\circ$ is set to zero, the magnetic field within the slab is also zero.

Problem: 2.2- Find the magnetic field of a uniformly magnetized sphere.

Solution

Choosing the z axis along the direction of M, we have

$$J_b = \Delta \times M = 0, \quad K_b = M \times \hat{n} = M sin\theta \hat{\phi}$$

Now, a rotating spherical shell, of uniform surface charge σ, corresponds to a surface current density.

$$K = \sigma v = \sigma\omega R sin\theta \hat{\phi}$$

It follows, therefore, that the field of a uniformly magnetized sphere is identical to the field of a spinning spherical shell, with the identification $\sigma R\omega \to M$, we conclude that

$$B = \frac{2}{3}\mu_\circ M$$

Inside the sphere, whereas the field outside is the same s that of pure dipole,

$$m = \frac{4}{3}\pi R^3 M$$

Notice that the internal field is uniform, like the electric field inside a uniformly polarized sphere, although the actual formulas for the two cases are curiously different (($\frac{2}{3}$ in place of $-\frac{1}{3}$).The external fields are also analogous; pure dipole in both cases.

Problem: 2.3- Calculate the torque exerted on the square loop shown in the figure, due to the circular loop(assume r is much larger than$a$ and b). If the square loop is free to rotate, what will its equilibrium orientation be?

Solution

$$\begin{aligned}
N &= m_2 \times B_1; \\
B_1 &= \frac{\mu_\circ}{4\pi}\frac{1}{r^3}[3(m_1.\hat{r})\hat{r} - m_1]; \\
\hat{r} &= \hat{y}; \\
m_1 &= m_1\hat{z};
\end{aligned}$$

$$
\begin{aligned}
m_2 =& m_2\hat{y};\\
B_1 =& \frac{\mu_\circ}{4\pi}\frac{m_1}{r^3}\hat{z}\\
N =& -\frac{\mu_\circ}{4\pi}\frac{m_1 m_2}{r^3}(\hat{y}\times\hat{z}) = -\frac{\mu_\circ}{4\pi}\frac{m_1 m_2}{r^3}\hat{X}
\end{aligned}
$$

Here,

$m_1 = \pi a^2 I, \qquad m_2 = b^2 I$

So,

$$N = -\frac{\mu_\circ}{4}\frac{(abI)^2}{r^3}\hat{X}$$

Final orientation: Downward $(-\hat{z})$.

Problem: 2.4- Starting from Lorentz force law, in the form of Eq.5.16, show that the torque on any steady current distribution (Not just a square loop) in a uniform field $\vec{B}$ is $\mathbf{m}\times\vec{B}$.

Solution

$$
\begin{aligned}
dF =& I dl \times B;\\
dN =& r\times dF = Ir\times(dl\times B)
\end{aligned}
$$

Now,

$$
\begin{aligned}
r\times(dl\times B) + dl\times(B\times r)&\\
+B\times(r\times dl) &= 0
\end{aligned}
$$

But,

$$
\begin{aligned}
d[r\times(r\times B)] =& dr\times(r\times B)\\
&+r\times(dr\times B)
\end{aligned}
$$

Since $\vec{B}$ is constant, and $dr = dl$ So,

$$dl \times (B \times r) = r \times (dl \times B) \\ -d[r \times (r \times B)]$$

Hence,

$$2r \times (dl \times B) = d[r \times (r \times B)] \\ -B \times (r \times dl)$$

$$dN = \frac{1}{2} I \{ d[r \times (r \times B)] \\ -B \times (r \times dl) \}$$

therefore

$$N = \frac{1}{2} I \{ \oint d[r \times (r \times B)] - B \times \oint (r \times dl) \}$$

But the first term is zero ($\oint d(\cdots) = 0$), and the second term is $2a$. So,

$$N = -I(B \times a) = m \times B. \qquad qed$$

Problem: 2.5- A straight solenoid ha 50 turns per cm in primary and total 200 turns in secondary. The area of cross-section of the solenoid is $4cm^2$. Calculate the mutual inductance. Primary is tightly kept inside the secondary.

Solution

The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current i_1 is given by the relation,

$$B = \mu_\circ n_1 i_1$$

The magnetic flux through the secondary of N_2 turns each of area S is given s

$$N_2\phi_2 = N_2(BS) = \mu_\circ n_1 N_2 i_1 S$$

$$\therefore \qquad M = \frac{N_2\phi_2}{i_1} = \mu_\circ n_1 N_2 S$$

Substituting the values, we get

$$
\begin{aligned}
M =& (4\pi \times 10^{-7})(\frac{50}{10^{-2}})(200)(4 \times 10^{-4}) \\
=& 5.0 \times 10^{-4} \quad H
\end{aligned}
$$

Problem: 2.6- Two solenoids A and B spaced close to each other and sharing the same cylindrical axis, have 400 and 700 turns, respectively. A current of $3.50A$ in coil A produced an average flux of $300\mu T - m^2$ through each turn of A and flux of $900\mu T - m^2$ through each turn of B

(a) Calculate the mutual inductance of the two solenoids.

(b) What is the self-inductance of A?

(c) What emf is induced in B when the current is A increases at the rate of 0.5 A/s?

Solution

(a)

$$
\begin{aligned}
M =& \frac{N_B \phi_B}{i_A} \\
=& \frac{(700)(90 \times 10^{-6})}{3.5} \\
=& 1.8 \times 10^{-2} \quad H
\end{aligned}
$$

(b)

$$
\begin{aligned}
L_A =& \frac{N_A \phi_A}{i_A} \\
=& \frac{(400)(300 \times 10^{-6})}{3.5} \\
=& 3.43 \times 10^{-2} \quad H
\end{aligned}
$$

(C)

$$
\begin{aligned}
e_B =& M(\frac{di_A}{dt}) \\
=& (1.8 \times 10^{-2})(0.5) \\
=& 9.0 \times 10^{-3} \quad V
\end{aligned}
$$

Chapter 3

Maxwell's Equations

SOLVED PROBLEMS

Problem: 3.1- Write down Maxwell's equation assuming that no dielectric or magnetic materials are present state your system of units.

Solution

Use MKS system of units in the absence of dielectric or magnetic materials Maxwell's equations are:

$$\begin{aligned}
\vec{\nabla}.\vec{E} &= \frac{\rho}{\epsilon_o} \\
\vec{\nabla}.\vec{E} &= \frac{-\partial\vec{B}}{\partial t} \\
\vec{\nabla}.\vec{B} &= 0 \\
\vec{\nabla}\times\vec{B} &= \mu_o j + \frac{1}{c^2}\frac{-\partial\vec{E}}{\partial t}
\end{aligned}$$

Problem: 3.2- Write down Maxwell's equation in a non conducting with constant probability ($P = j = 0$) show that the E and B each satisfies the wave function, find an expression for wave velocity.

Solution

The Maxwell'd equation in a source free, hydrogens and non conducting medium are:

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \tag{3.1}$$

$$\vec{\nabla} \times \vec{H} = \frac{-\partial \vec{D}}{\partial t} \tag{3.2}$$

$$\vec{\nabla} \times \vec{D} = 0 \tag{3.3}$$

$$\vec{\nabla} . \vec{B} = 0 \tag{3.4}$$

Where $\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$, $\epsilon . \mu$ being constant as:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla}.\vec{E}) - \vec{\nabla}^2 \vec{E} = -\vec{\nabla}^2 \vec{E}$$

and can be written as:

$$\vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t} \right) = \mu \epsilon \frac{\partial \vec{E}}{\partial t^2}$$

Eq.(3.1), gives:

$$\vec{\nabla}^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Similarly one finds,

$$\vec{\nabla}^2 \vec{B} - \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

This each of the field vector $\vec{E}$ and $\vec{B}$ satisfies at the wave equation. A compassion with the standard wave equation.

$$\vec{\nabla}^2\vec{E} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

Shows that the wave velocity is:

$$v = \frac{1}{\sqrt{\epsilon\mu}}$$

Problem: 3.3- In the region of empty space, negative field is described by

$$\vec{B} = B_o e^{ax} e^{x} \hat{z} \sin w$$

Where $w = ky - \omega t$ calculate $\vec{E}$.

Solution

Express $\vec{B}$ as $I_m(B_o e^{ax} e^{iw})\hat{z}e$ using Maxwell equation:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial E}{\partial t} B_o e^{ax} e^{x} \hat{z} \sin w$$

And the definition $k = \frac{w}{c}$ for empty pace we:

$$\vec{E} = \frac{ic}{w}\vec{\nabla} \times \vec{B}$$

$$\vec{E} = \frac{i}{k}\begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & B_2 \end{vmatrix}$$

Where $\frac{\partial}{\partial z} = 0$as B does not depend on z. Hence:

$$\begin{aligned} E_x &= I_m(\frac{i}{k} B_o e^{ax} ik e^{iw}) = -B_o e^{ax} \sin w \\ E_y &= I_m(-\frac{i}{k} B_o e^{ax} e^{iw}) = -\frac{ac}{w} B_o e^{ax} \cos w \\ E_z &= 0 \end{aligned}$$

Problem: 3.4- Given the plane wave characterized by an E_x by poynting in the positive z direction

$$\vec{E} = \hat{i} E_o \sin \frac{2\pi}{\lambda}(z - ct)$$

Show that it is possible to take the scaler potential $\phi = 0$ and find a possible vector potential $\vec{A}$ for which the lorentz conductor is satisfied.

Solution

consider:

$$\vec{E} = \hat{i} E_o \sin \frac{2\pi}{\lambda}(z - ct) \tag{3.5}$$

We know that:

$$\vec{E} = \text{grad}\,\vec{\phi} - \frac{\partial \vec{A}}{\partial t} \tag{3.6}$$

Assuming that $\phi = 0$ Eq.(3.6), reduces to:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

$$-\frac{\partial \vec{A}}{\partial t} = \hat{i} E_o \sin \frac{2\pi}{\lambda}(z - ct)$$

Integrating the above equation w.r.t time:

$$\vec{A} = -E_o \cos\left[\frac{2\pi}{\lambda}(z - ct)\right](\frac{\lambda}{2\pi c})\hat{i}$$

$$\vec{A} = \left[(\frac{\lambda}{2\pi c}) E_o \cos \frac{2\pi}{\lambda}(z - ct)\right]\hat{i} \tag{3.7}$$

By the lorentz condition:

$$\vec{\nabla}.\vec{A} + \epsilon\mu \frac{\partial \phi}{\partial t} = 0$$

$$\vec{\nabla}.\left[(\frac{\lambda}{2\pi c}) E_o \cos \frac{2\pi}{\lambda}(z - ct)\hat{i}\right] + \epsilon\mu \frac{\partial \phi}{\partial t} = 0$$

Because all terms in the first term of above equation are constant, therefore its divergence is zero. Thus we are left with:

$$\epsilon\mu\frac{\partial\phi}{\partial t} = 0$$
$$\epsilon\mu \neq 0$$
$$\frac{\partial\phi}{\partial t} = 0$$

$$\phi = 0 \tag{3.8}$$

This is the required result along with Eq.(3.6)

Problem: 3.5- Given the electromagnetic wave

$$\vec{E} = iE_o\cos w(\sqrt{\epsilon\mu}z - t) + \hat{j}\sin w(\sqrt{\epsilon\mu}z - t)$$

Where E_o is a constant . Find the corresponding magnetic field and Poynting vector.

Solution

Let us consider that:

$$\vec{E} = iE_o\cos w(\sqrt{\epsilon\mu}z - t) + \hat{j}\sin w(\sqrt{\epsilon\mu}z - t) \tag{3.9}$$

By using the Maxwell equation:

$$\vec{\nabla}\times\vec{E} = \frac{\partial\vec{B}}{\partial t} \tag{3.10}$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}) = -\frac{\partial\vec{B}}{\partial t}$$
$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{j} + \left(\frac{\partial E'_y}{\partial x} - \frac{\partial E'_x}{\partial y}\right)\hat{k} = -\frac{\partial\vec{B}}{\partial t} \tag{3.11}$$

Now from Eq.(3.9), we get:

$$
\begin{aligned}
E_x &= E_o \cos w(\sqrt{\epsilon\mu} z - t) \\
E_y &= E_o \sin w(\sqrt{\epsilon\mu} z - t) \\
E_z &= 0
\end{aligned}
$$

Using the values in Eq.(3.11):

$$
\begin{aligned}
\left(-\frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z}\right)\hat{j} &= -\frac{\partial \vec{B}}{\partial t} \\
w\sqrt{\epsilon\mu}E_o \cos w(\sqrt{\epsilon\mu}z - t)\hat{i} + w\sqrt{\epsilon\mu}E_o \sin w(\sqrt{\epsilon\mu}z - t)\hat{j} &= -\frac{\partial \vec{B}}{\partial t} \\
w\sqrt{\epsilon\mu}E_o \cos w(\sqrt{\epsilon\mu}z - t)\hat{i} + w\sqrt{\epsilon\mu}E_o \sin w(\sqrt{\epsilon\mu}z - t)\hat{j} &= \frac{\partial \vec{B}}{\partial t}
\end{aligned}
$$

$$
\begin{aligned}
\left(-\frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z}\right)\hat{j} &= -w\sqrt{\epsilon\mu}E_o \int \cos w(\sqrt{\epsilon\mu}z - t)\hat{i} - \hat{j}E_o \sin w(\sqrt{\epsilon\mu}z - t)dt \\
\left(-\frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z}\right)\hat{j} &= \sqrt{\epsilon\mu}E_o \sin w(\sqrt{\epsilon\mu}z - t)\hat{r} + \sqrt{E\mu}E_o \cos w(\sqrt{\epsilon\mu}z - t
\end{aligned}
$$

This is required E_q for magnetic field.

$$
\begin{aligned}
\vec{B} &= \mu\vec{H} \\
\vec{H} &= \frac{1}{\mu}\vec{B} \\
\vec{H} &= -\sqrt{\frac{\epsilon}{\mu}}E_o \sin w(\sqrt{\epsilon\mu}z - t)\hat{i} + \sqrt{\frac{\epsilon}{\mu}}E_o \cos w\sqrt{\frac{\epsilon}{\mu}}z - t\hat{j}
\end{aligned}
$$

The Poynting vector $\vec{S}$ is given by:

$$
\begin{aligned}
\vec{S} &= \vec{E} \times \vec{H} \\
\vec{S} &= \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
E_o \cos w(\sqrt{\epsilon\mu}z - t) & E_o \sin w(\sqrt{\epsilon\mu}z - t) & 0 \\
-\frac{\epsilon}{\mu}E_o \sin w(\sqrt{\epsilon\mu}z - t) & \frac{E}{\mu}E_o \cos w(\sqrt{\epsilon\mu}z - t) & 0
\end{vmatrix}
\end{aligned}
$$

$$\vec{S} = \hat{k}\left[\frac{E}{\mu}E_o^2\cos^2 w(\sqrt{\epsilon\mu}z - t) + \frac{E}{\mu}E_o^2\sin^2 w(\sqrt{\epsilon\mu}z - t)\right]$$
$$\vec{S} = \frac{E}{\mu}E_o^2\left[\cos^2 w(\sqrt{\epsilon\mu}z - t) + \sin^2 w(\sqrt{\epsilon\mu}z - t)\right]\hat{k}$$
$$\vec{S} = \frac{E}{\mu}E_o^2\hat{k}$$

This is the required equation poynting vector.

Chapter 4

Applications of Maxwell's Equations

SOLVED PROBLEMS

Problem: 4.1- Write down the equation for right waves and left circularly polarized plane waves

Solution

By taking the z axis along the direction of the propagation of wave. The electric vector polarized light can be represented by the real parts of:

$$E_p(z,t) = (E_o e_x + E_o e^{-i\frac{\pi}{2}} e_y) e^{-iwt+ike+z}$$

And that the left circularly light by real points of:

$$E_L(z,t) = (E_o e_x + E_o e^{-i\frac{\pi}{2}} e_y) e^{-iwt+ike-z}$$

Where:

$$k+ = \frac{w}{c} n+$$
$$k- = \frac{w}{c} n-$$

Problem: 4.2- Given a plane polarized electric wave

$$E = E_o \exp\left(iw\left[t - \frac{n}{c}(k.r)\right]\right)$$

Drive from the Maxwell equations the relations between, E, K and H field. Obtain an expression for the index of refraction n in terms of w_i, t_i, μ, σ (the conductivity).

Solution

Maxwell's equations for a charge free ohmic conducting medium are:

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial D}{\partial t} \tag{4.1}$$

$$\overrightarrow{\nabla} \times \overrightarrow{H} = J + \frac{\partial D}{\partial t} \tag{4.2}$$

$$\overrightarrow{\nabla}.\overrightarrow{D} = 0 \tag{4.3}$$

$$\overrightarrow{\nabla}.\overrightarrow{B} = 0 \tag{4.4}$$

$$D = eE$$
$$B = \mu H$$
$$J = \sigma E$$

For the given type of wave we have $\frac{\partial D}{\partial t} \rightarrow iw$, $\nabla \rightarrow -i\frac{nw}{c}K$ Eq.(4.3) and Eq.(4.4) then given:

$$E.K = B.K = 0$$

And gives Eq.(4.1):

$$i\frac{nw}{c}K \times E = iw\mu H$$
$$H = \frac{nw}{\mu c}K \times E$$

Taking curl of both sides of Eq.(4.1) and using Eq.(4.2) and Eq.(4.3) we have:

$$\nabla^2 E = \mu H\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \left(\mu\epsilon - i\frac{\mu\sigma}{w}\right)\frac{\partial^2 E}{\partial t^2} = 0$$

Which is the equation for a wave propagating with phase velocity v given by:

$$v^2 = \left(\mu\epsilon - i\frac{\mu\sigma}{w}\right)^{-1} v^2 = \qquad \frac{1}{\mu\epsilon}\left(1 - i\frac{\mu\sigma}{wc}\right)^{-1}$$

Hence the index of the refraction of the medium is:

$$n = \frac{c}{v} = \left[\frac{\mu\epsilon}{\mu_o\epsilon o}\left(1 - i\frac{\mu\sigma}{wc}\right)\right]^{\frac{3}{2}}$$

Writing $n = \sqrt{\frac{\mu\epsilon}{\mu_o\epsilon o}}(\beta - i\alpha)$, we have:

$$\beta^2 - \alpha^2 = 1$$

$$\alpha\beta = \frac{\alpha}{2w\epsilon}$$

Solving for α and β, we have:

$$n = \sqrt{\frac{\mu\epsilon}{2\mu_o\epsilon o}}\left[\sqrt{\left(1 + \frac{\sigma^2}{\epsilon^2 w^2}\right)^{\frac{1}{2}}} + 1 - i\sqrt{\left(\frac{\sigma^2}{\epsilon^2 w^2}\right)^{\frac{1}{2}-1}}\right]$$

Problem: 4.3- What is the attenuation distance for a plane wave propagation in a good conductor ?. Express your answer in terms of conductivity σ, per mobility μ and frequency w.

Solution

For a ohmic conducting medium of permittivity ϵ, permeability μ and conductivity σ, the general wave equation to be used is:

$$\nabla^2 E - \mu\epsilon\overline{E} = 0$$

For please electromagnetic waves of angular frequency w, $E(r,t) = E_o(r)e^{-iwt}$, the above becomes:

$$\nabla^2 E_o + \mu\epsilon w^2 \left(1 + \frac{\sigma}{\epsilon w}\right) E_o \ = 0$$

Comparing this with the wave equation foe a dielectric, we see that for the conductor we have to replace:

$$\mu\epsilon \to \mu\epsilon \left(1 + \frac{\sigma}{\epsilon w}\right) E_o$$

If, we wish to use the results for a dielectric.
Consider the plane wave an incident on the conducting along the inward normal, whose direction is taken to be the z axis. Then in the conductor the electromagnetic wave can be represented:

$$E \ = E_o e^{i(kx - wt)}$$

The wave vector has magnitude:

$$k \ = \frac{w}{v} = w\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{\epsilon w}\right)^{\frac{1}{2}}$$

Let $k = \beta = i\alpha$, we have:

$$\beta^2 - \alpha^2 \ = w^2\mu\epsilon \ \ , \ \ \alpha\beta = \frac{1}{2}w\mu\sigma$$

For a good conductor, i.e. for $\frac{\sigma}{iw} >> 1$, we have the solution:

$$\alpha = \beta \ = \pm\sqrt{\frac{w\epsilon\sigma}{2}}$$

In the conductor, we have the:

$$E \ = E_o e^{-ax} e^{i(\beta z - wt)}$$

By the definition of the wave vector, β has to take the positive sign. As the wave can not amplify in the conductor, α has also to take the positive sign. The attenuation length δ is the wave travels for its amplitude to reduce to e^{-1} of its initial value. Thus:

$$\delta = \frac{1}{\alpha}\sqrt{\frac{2}{w\mu\sigma}}$$

Problem: 4.4- A plane wave of the angular frequency w and wave vector number $|K|$ propagates in a neutral, homogenous anisotropic,non conducting medium with $\mu = 1$. Show that $H$is orthogonal to E, D and K and also that D and H are transverse but $E$is not ?

Solution

Use the gaussian system of units.

(A). Maxwell's equations for the given medium are:

$$\begin{aligned}
\nabla \times E &= -\frac{1}{c}\frac{\partial B}{\partial t} \\
\nabla \times B &= \frac{1}{c}\frac{\partial D}{\partial t} \\
\nabla . B &= 0 \\
\nabla . D &= 0
\end{aligned}$$

The plane wave can be represented by $e^{i(k.x-wt)}$, so that:

$$\nabla\times \equiv iKx, \quad \nabla. \equiv iK; \quad \frac{\partial}{\partial t} = -iw$$

and the above equations reduces to:

$$\begin{aligned}
K \times E &= -\frac{w}{c}H \\
K \times B &= -\frac{w}{c}D \\
K.B &= K.D = 0
\end{aligned}$$

As, $\mu = 1$, from these we have:

$$\begin{aligned}
D.H &= -\frac{w}{c}(K \times H).H \equiv 0 \\
K.H &= -\frac{w}{c}(K.K) \times E \equiv 0
\end{aligned}$$

Hence K, D and H are mutually perpendicular i.e. D and H are transverse to K. However, as

$$
\begin{aligned}
K \times (K \times E) &= \frac{w}{c} K \times H \\
K.E &= \frac{1}{K}(\frac{w}{c} K \times H + K^2 E) \\
K.E &= \frac{1}{K}(-(\frac{w}{c})^2 D + K^2 E) \neq 0
\end{aligned}
$$

Unless, $K^2 = (\frac{w}{c})^2, E$ need not to be transverse to K.

An electromagnetic wavewith electric field given by:

$$
E_y = E_o e^{i(k_z - wt)} \quad , \quad E_o = E_y = 0
$$

Problem: 4.5- Find the magnetic field and wavelength of the electromagnetic wave for a given(allowable) w. Neglect the magnetic force on the electrodes.

Solution

(c) Using Maxwell's equation.

$$
\begin{aligned}
\nabla \times E &= -\frac{\partial B}{\partial t} \\
\nabla \times E &= ike_x + E_y e_y = -iKE_z e_z \\
\frac{\partial B}{\partial t} &= -iwB \\
B &= -\frac{K}{w} E_x e^{i(K_z - wt)e_z}
\end{aligned}
$$

Chapter 5

Optical Dispersion in Materials

SOLVED PROBLEMS

Problem: 5.1- Suppose you embedded some free charge in a piece of glass. About how long would it take for the charge to flow to the surface?

Solution

For Glass,

$$\begin{aligned}
\sigma =& \frac{1}{\rho} = 10^{-12} \Omega^{-1} m^{-1} \\
\tau =& \frac{\epsilon}{\sigma} = \frac{\epsilon_\circ \epsilon_r}{\sigma} \\
\tau =& \frac{\epsilon_\circ n^2}{\sigma} \qquad \because \epsilon_r \approxeq n^2 \\
\tau =& \frac{(8.85 \times 10^{-12})(1.5)^2}{10^{-12}} \\
\tau =& 205
\end{aligned}$$

Problem: 5.2- Silver is an excellent conductor, but it is expensive. Suppose you were designing a microwave experiment to operate at frequency of $10^{10}\,Hz$. How thick would you make the silver coating?

Solution

For Silver,

$$\rho = 1.59 \times 10^{-8}\,\Omega m$$
$$\sigma = \frac{1}{\rho} = 6.25 \times 10^{7}$$

If $\epsilon \approxeq \epsilon_\circ$ So,

$$\omega\epsilon = 2\pi f \epsilon_\circ$$
$$= 2(3.14) \times 10^{10} \times 8.85 \times 10^{-12}$$
$$\omega\epsilon = 0.56$$

Since $\sigma \gg \omega\epsilon$, then

$$d = \frac{1}{K} = \sqrt{\frac{2}{\omega\sigma\mu}}$$
$$d = \sqrt{\frac{2}{2(3.14) \times 10^{10} \times 6.25 \times 10^{7} \times 4\pi \times 10^{-7}}}$$
$$d = 6.4 \times 10^{-7} m \quad or \quad 6.4 \times 10^{-4} mm$$

Problem: 5.3- Show that skin depth in poor conductor $\sigma << \omega\epsilon$ is $(\frac{2}{\sigma})\sqrt{\frac{\epsilon}{\mu}}$. Find the skin depth for pure water.

Solution

$$K \approxeq \omega\sqrt{\frac{\epsilon\mu}{2}}[1 + \frac{1}{2}(\frac{\sigma}{\epsilon\omega}) - 1]^{\frac{1}{2}}$$
$$= \omega\sqrt{\frac{\epsilon\mu}{2}}\frac{1}{\sqrt{2}}\frac{\sigma}{\epsilon\omega}$$
$$= \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

For pure water; $\epsilon_r = 80.1$

$$\begin{aligned}
\mu =& \mu_\circ(1 + x_m) \\
=& \mu_\circ(1 - 9.0 \times 10^{-6}) \approxeq \mu_\circ \\
\sigma =& \frac{1}{\rho} = \frac{1}{2.5 \times 10^5} \\
d =& (2)(2.5 \times 10^5)\sqrt{\frac{(80.1)(8.85 \times 10^{-12})}{4\pi \times 10^{-7}}} \\
d =& 1.19 \times 10^4 m
\end{aligned}$$

Problem: 5.4- Calculate the reflection coefficient of light at an air ω silver interface. $\mu_1 - \mu_2 = \mu_\circ$, $\epsilon_1 = \epsilon_\circ$, $\sigma = 6 \times 10^7 \Omega^{-1} m^{-1}$ at optical frequency $\omega = 4 \times 10^{15} Hz$

Solution

$$R = |\frac{E_{\circ R}}{E_{\circ t}}|^2 = |\frac{1 - \beta}{1 + \beta}|^2$$

Where,

$$\beta = \frac{\mu_1 V_1}{\mu_2 \omega}\tilde{K}_2$$

Since silver is good conductor so, $\sigma \gg \epsilon\omega$

$$\begin{aligned}
\gamma =& C\sqrt{\frac{\sigma\mu_\circ}{2\omega}} \\
=& 3 \times 10^8 \sqrt{\frac{6 \times 10^7 \times 4\pi \times 10^{-7}}{2 \times 4 \times 10^{15}}} \\
\gamma =& 29
\end{aligned}$$

Then,

$$R = \frac{(1 - \gamma^2)^2 + \gamma^2}{(1 + \gamma^2)^2 + \gamma^2} = 0.93$$

Evidently 93% of the light is reflected.

Problem: 5.5- Find the radiation resistance of the wire joining two ends of the dipole. This resistance gives same average power loss to heat, as the oscillating dipole infact puts. Show that $R = 790(\frac{d}{\lambda})^2\Omega$

Solution

$$\begin{aligned} P =& I^2R \\ I =& q_\circ\omega\sin(\omega t) \\ P =& q_\circ^2\omega^2\sin^2(\omega t)R \end{aligned}$$

So,

$$< P > = \frac{1}{2}q_\circ^2\omega^2R$$

Equate this equation to

$$\frac{1}{2}q_\circ^2\omega^2R = \frac{\mu_\circ q_\circ d^2\omega^4}{12\pi C}$$

Hence,

$$R = \frac{\mu_\circ d^2\omega^2}{6\pi C}$$

Since $\omega = \frac{2\pi C}{\lambda}$

$$\begin{aligned} R =& \frac{\mu_\circ d^2}{6\pi C}\frac{4\pi^2C^2}{\lambda^2} \\ R =& \frac{2}{3}\pi\mu_\circ C(\frac{d}{\lambda})^2 \\ =& \frac{2}{3}(3.14)(4\pi\times10^{-7})(3\times10^8)(\frac{d}{\lambda})^2 \\ R =& 789.6(\frac{d}{\lambda})^2\Omega \end{aligned}$$

Chapter 6

Electrodynamics and Relativity

SOLVED PROBLEMS

Problem: 6.1- As the outlaws escape in their getaway car, which goes $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$. Does the bullet reach its target,

(a)-According to Galileo?

(b)-According to Einstein?

Solution

$$\text{Velocity of getaway car} = \frac{3}{4}c = 0.75c$$

$$\text{Velocity of bullet w.r.t. gun} = \frac{1}{3}c$$

$$\text{Velocity of pursuit car} = \frac{1}{2}c$$

(a)

According to Galileo,

$$\text{Velocity of bullet with respect to ground}$$

$$= \frac{1}{2}c + \frac{1}{3}c$$

$$= \frac{5}{6}c = 0.833c$$

Result

As the velocity of bullet with respect to ground is 0.833c and is greater then velocity of getaway car. Hence it will reach the target.

(a)

According to Einstein,

$$\text{Velocity of bullet with respect to ground}$$
$$= \frac{\frac{1}{2}c + \frac{1}{3}c}{1 + \frac{1}{2} \cdot \frac{1}{3}}$$
$$= \frac{5}{7}c = 0.714c$$

So According to Einstein, velocity of bullet is less than velocity of getaway car. Hence it will not reach the target.

Problem: 6.2- In a laboratory experiment, a muon is observed to travel 800m before disintegrating. A graduate student looks up the lifetime of muon $(2 \times 10^{-6}s)$ and conclude its speed was $4 \times 10^{8}ms^{-1}$, faster than light! Identify the student's error. Find the actual speed of muon.

Solution

First, taken into account, time dilation of the muon's "internal clock".

In laboratory, the muon lasts $T'^{=} = \frac{t_\circ}{\sqrt{1-\frac{v^2}{c^2}}}$

Where $t_\circ$ is proper time $2 \times 10^{-6}s$. Thus,

$$V = \frac{d}{t'} = \frac{d}{\frac{t_\circ}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{d}{t_\circ}\sqrt{1 - \frac{v^2}{c^2}}$$

Taking square on both sides and rearranging the equation

$$\begin{aligned}
v^2 =& \frac{d^2}{t^2}(1-\frac{v^2}{c^2}) \\
\frac{t_\circ^2 v^2}{t^2} =& 1-\frac{v^2}{c^2} \\
v^2(\frac{t_\circ^2}{d^2}+\frac{1}{c^2}) =& 1 \\
v^2 =& \frac{1}{\frac{t_\circ^2}{d^2}+\frac{1}{c^2}} \\
v^2 =& \frac{1}{\frac{1}{c^2}[\frac{c^2t_\circ^2}{d^2}+1]} \\
v^2 =& \frac{c^2}{(\frac{ct_\circ}{d})^2+1} \\
\frac{v^2}{c^2} =& \frac{1}{(\frac{ct_\circ}{d})^2+1} \\
\Rightarrow \frac{t_\circ c}{d} =& \frac{(2\times10^{-6}s)(3\times10^8 ms^{-1})}{800m} \\
=& \frac{3}{4}
\end{aligned}$$

Use it in above equation

$$\begin{aligned}
\frac{v^2}{c^2} =& \frac{1}{(\frac{3}{4})^2+1} = \frac{16}{25} \\
\frac{v}{c} =& \frac{4}{5} \\
v =& \frac{4}{5}c \\
v =& 0.8c = 2.4\times10^8 ms^{-1}
\end{aligned}$$

Hence velocity of muon is less than speed of light.

Problem: 6.3- Proper velocity (η) defined as $\vec{\eta} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\vec{u}$ in terms of ordinary velocity (u). Invert that equation to get formula for u in terms of η.

Solution

$$\eta^2 = \frac{1}{1-\frac{u^2}{c^2}}u^2$$
$$(1-\frac{u^2}{c^2})\eta^2 = u^2$$

Replace the term u with η and η with u, we get

$$(1-\frac{u^2}{c^2})u^2 = \eta^2$$
$$\vec{u} = \frac{1}{\sqrt{1-\frac{\eta^2}{c^2}}}\vec{\eta}$$

Problem: 6.4- A particle of mass (m) whose total energy is twice of its rest energy collides with an identical partial at rest. If they stick together, what is the mass of resulting composite particle? What is its velocity?

Solution

Initial momentum:

$$E^2 - P^2c^2 = m^2c^4$$
$$E^2 - m^2c^4 = P^2c^2$$
$$(2mc^2)^2 - m^2c^4 = P^2c^2$$
$$3m^2c^4 = P^2c^2$$
$$P = \sqrt{3\,m^2c^2}$$
$$P = \sqrt{3}mc$$

Initial energy of total system before collision

$$2mc^2 + mc^2 = 3mc^2$$

As we know in elastic collision both momentum and energy will remain conserved. So,

$$E^2 - P^2c^2 = M^2c^4$$

Where M is the mass of composite particle

$$\begin{aligned}(3mc^2)^2 - (\sqrt{3}mc)^2c^2 &= M^2c^4\\ 9m^2c^4 - 3m^2c^4 &= M^2c^4\\ 6m^2c^4 &= M^2c^4\\ M &= \sqrt{6}m = 2.5m\end{aligned}$$

In this process some kinetic energy was converted into rest energy so, $M > 2m$. Now,

$$\begin{aligned}\text{velocity} = v &= \frac{Pc^2}{E}\\ v &= \frac{\sqrt{3}mcc^2}{3mc^2} = \frac{c}{\sqrt{3}}\\ v &= 0.577c\end{aligned}$$

Problem: 6.5- A particle of mass (m) is subjected to a constant force (F). If it starts from rest the origin, at time $t = 0$, find its position (x) as a function of time.

Solution

$$\begin{aligned}\frac{dP}{dt} &= F\\ dP &= Fdt\end{aligned}$$

Integrating both side

$$P = Ft + \text{constant}$$

Since P=0 at t=0 constant must be zero. Therefore,

$$P = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} = Ft$$

Solving for U we obtain

$$u = \frac{(\frac{F}{m})t}{\sqrt{1+(\frac{Ft}{mc})^2}}$$

The numerator, of course, is the classical answer. Its approximately right, If $(\frac{F}{m})t \ll c$. But the relativistic denominator ensures that u never exceeds c; in fact, as $t \to \infty, u \to c$. Again integrating above equation, we get

$$x(t) = \frac{F}{m}\int_0^t \frac{t'}{\sqrt{1+(\frac{Ft'}{mc})^2}}dt'$$

$$x(t) = \frac{mc^2}{F}|\sqrt{1+(\frac{Ft'}{mc})^2}|_0^t$$

$$x(t) = \frac{mc^2}{F}[\sqrt{1+(\frac{Ft}{mc})^2}-1]$$

Result

In place of classical parabola, $x(t) = \frac{F}{2m}t^2$, The graph of above equation is hyperbola; For the reason, motion under a constant force is often called hyperbolic motion.


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