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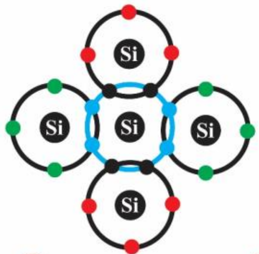

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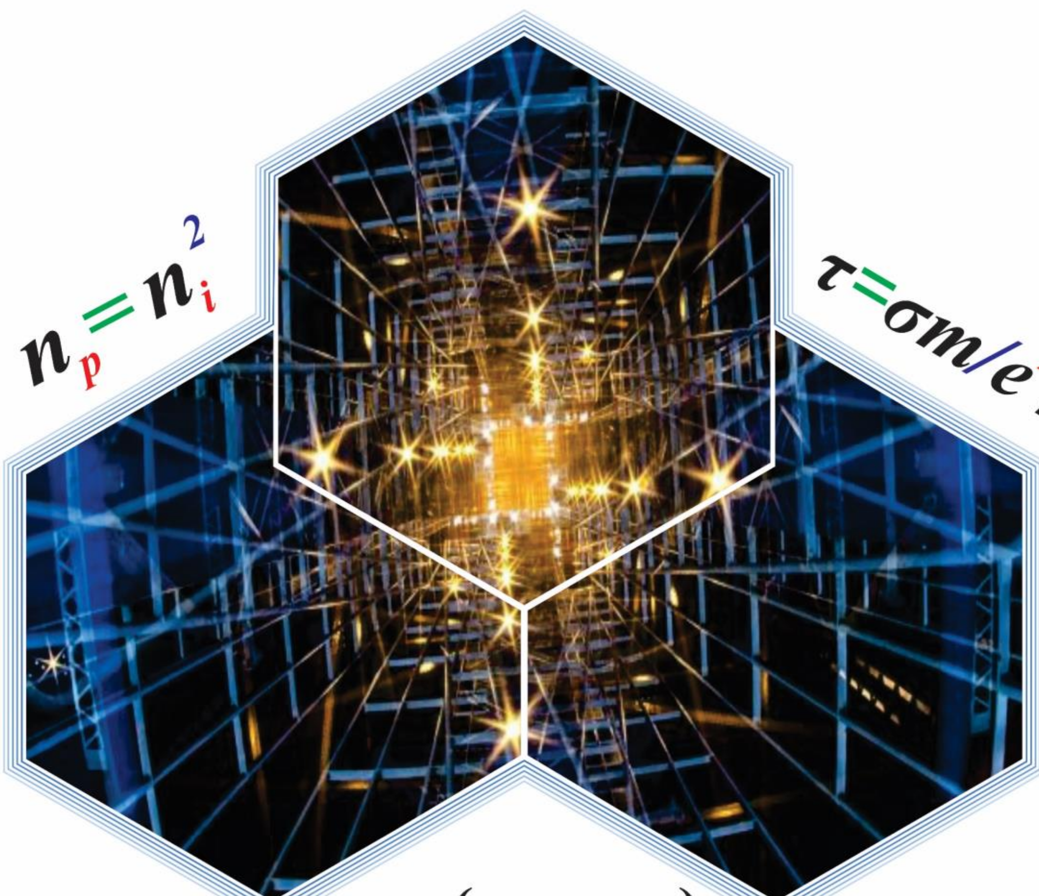
SOLID STATE PHYSICS - II

For BS/M.Sc Physics Programme

1st Edition



$n_p = n_i^2$

$\tau = \sigma m / e^2 n$

$\sigma = e(n\mu_n - p\mu_p)$

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TEACH YOURSELF

SOLID STATE PHYSICS - II

1st Edition

For **BS/M.Sc Physics** students of all Pakistani Universities/Colleges

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Chapter 1

Free Electron Fermi Gas

SOLVED PROBLEMS

Problem: 1.1- Calculate the number of energy states available for the electrons in a cubical box of side 0.05 cm lying below an energy of 1 eV.

Solution

$$\begin{aligned} m &= 9.1 \times 10^{-31}\ kg \\ h &= 6.63 \times 10^{-34}\ Js \\ V &= 0.05 \times 0.05 \times 0.05\ cm^3 \\ V &= 1.25 \times 10^{-10}\ m^3 \\ E &= 1\ eV\ =\ 1 \times 1.6 \times 10^{-19}\ J \\ \text{Number of energy states} &= ? \end{aligned}$$

Since, we know that

$$Z(E)dE\ =\ 4\pi V\left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}dE$$

Also, the number of energy states below 1 eV is

$$\begin{aligned}
\int_0^E Z(E)dE &= 4\pi V\left(\frac{2m}{h^2}\right)^{3/2}\int_0^E E^{1/2}dE \\
\int_0^E Z(E)dE &= 4\pi V\left(\frac{2m}{h^2}\right)^{3/2}\left[\frac{2}{3}E^{3/2}\right]_0^1 \\
\int_0^E Z(E)dE &= 4\pi V\left(\frac{2m}{h^2}\right)^{3/2}\left[\frac{2}{3}E^{3/2}\right] \\
\int_0^E Z(E)dE &= 4\times 3.14\times 1.25\times 10^{-10}\left(\frac{2\times 9.1\times 10^{-31}}{(6.63\times 10^{-34})^2}\right)^{3/2}\times\frac{2}{3}\left[(1.6\times 10^{-19})\right]^{3/2} \\
\int_0^E Z(E)dE &= 4\times 3.14\times 1.25\times 10^{-10}\times 8.463\times 10^{54}\times\frac{2}{3}\times 6.4\times 10^{-29} \\
\int_0^E Z(E)dE &= 5.669\times 10^{17}
\end{aligned}$$

Problem: 1.2- Evaluate the temperature at which there is one percent probability that a state with an energy 0.4 eV above the Fermi energy, will be occupied by an electron.

Solution

$$\begin{aligned}
k_B &= 1.38\times 10^{-23}\ J/K \\
E &= E_F + 0.4\ eV \\
E - E_F &= 0.4\ eV \\
F(E) &= 1\% = \frac{1}{100} \\
T &= ?
\end{aligned}$$

Since, we have to know that

$$F(E) = \frac{1}{1+\exp\left[\frac{E-E_F}{k_BT}\right]}$$

$$\frac{1}{100} = \frac{1}{1+\exp\left[\frac{E-E_F}{k_BT}\right]}$$

$$\text{or} \quad 100 = 1+\exp\left[\frac{E-E_F}{k_BT}\right]$$

$$100 = 1+\exp\left[\frac{0.4}{k_BT}\right]$$

$$\text{or} \quad \exp\left[\frac{0.4}{k_BT}\right] = 100-1$$

$$\exp\left[\frac{0.4}{k_BT}\right] = 99$$

$$\text{or} \quad \frac{0.4}{k_BT} = \log[99]$$

$$\frac{0.4}{k_BT} = 2.303\times\log_{10}99$$

$$k_BT = \frac{0.4}{2.303\times\log_{10}99}$$

$$k_BT = 0.087\ eV$$

$$\text{or} \quad T = \frac{0.087}{k_B}$$

$$T = \frac{0.087}{1.38\times10^{-23}}$$

$$T = 1008.69\ K$$

Problem: 1.3- A sample of SI is doped with 10^{17} phosphorous atoms per cm^3. What is its resistivity? What is the expected Hall voltage in a sample of 200 μm thickness if the current density is 1 A/cm^2 and magnetic field of 1×10^{-5} Wb/cm^2 is applied perpendicular to the direction of current flow. The mobility is given as 600 $cm^2/volt-sec$.

Solution

$$n = 10^{17}\ \text{electrons}/cm^3$$

$$\mu = 600\ cm^2/volt - sec$$
$$B_z = 1 \times 10^{-5}\ Wb/cm^2$$
$$e = 1.6 \times 10^{-19}\ C$$
$$d = 200\ \mu m$$
$$J_x = 1\ A/cm^2$$
$$\sigma = ?$$
$$\rho = ?$$
$$R_H = ?$$
$$V_H = ?$$

Since, we know that the conductivity is defined as

$$\sigma = \frac{\mu}{R_H}$$
$$\sigma = \frac{\mu}{\frac{1}{ne}} \qquad \because R_H = \frac{1}{ne}$$
$$\sigma = \mu ne$$
$$\sigma = 600 \times 10^{17} \times 1.6 \times 10^{-19}$$
$$\sigma = 9.6\ \Omega - cm$$

Now, the resistivity is defined as:

$$\rho = \frac{1}{\sigma}$$
$$\rho = \frac{1}{9.6}$$
$$\rho = 0.104\ \Omega - cm$$

Hall coefficient can be defined as:

$$R_H = -\frac{1}{ne}$$
$$R_H = -\frac{1}{10^{17} \times 1.6 \times 10^{-19}}$$
$$R_H = -62.5\ cm^3/C$$

And, Hall voltage is given as:

$$\begin{aligned} V_H &= E_H d \\ V_H &= (J_x B_z R_H) d \qquad \because E_H = J_x B_z R_H \\ V_H &= 1 \times 1 \times 10^{-5} \times (-62.5) \times (2 \times 10^{-2}) \\ V_H &= 12.5 \times 10^{-6}\ V \\ V_H &= 12.5\ \mu V \end{aligned}$$

Problem: 1.4- In a Hall effect experiment on Zinc, a potential of 4.5 μV is developed across a foil of thickness 0.02 mm when a current of 1.5 A is passed in a direction perpendicular to a magnetic field of 2.0 T. Calculate the Hall coefficient and the electron density.

Solution

$$\begin{aligned} V_H &= 4.5\ \mu m \\ V_H &= 4.5 \times 10^{-6}\ V \\ e &= 1.6 \times 10^{-19}\ C \\ d &= 0.02\ mm \\ d &= 0.02 \times 10^{-3} = 2 \times 10^{-5}\ m \\ I &= 1.5\ A \\ B &= 2T \\ R_H &= ? \\ n &= ? \end{aligned}$$

Since, the Hall coefficient is defined as

$$\begin{aligned} R_H &= \frac{V_H d}{BI} \\ R_H &= \frac{4.5 \times 10^{-6} \times 2 \times 10^{-5}}{2 \times 1.5} \\ R_H &= 0.3 \times 10^{-10}\ m^3 C^{-1} \end{aligned}$$

Also, the electron density is defined as:

$$\begin{aligned} R_H &= \frac{1}{ne} \\ \text{or} \qquad n &= \frac{1}{eR_H} \\ n &= \frac{1}{1.6 \times 10^{-19} \times 0.3 \times 10^{-10}} \\ n &= 2.08 \times 10^{29}\ m^{-3} \end{aligned}$$

Problem: 1.5- Derive pressure versus volume relationship for a free electron gas at $0K$.

Solution For, thermodynamics, we have

$$P = -\frac{\partial E}{\partial V}$$

where E is the internal energy of a system of particles occupying a volume V at pressure P. For a free electron gas containing N electrons with average kinetic energy $\bar{E}_o$ at $0K$, the energy E may be replaced by $N\bar{E}_o$. Therefore, we have

$$\begin{aligned} P &= -N\frac{\partial \bar{E}_o}{\partial V} \\ \text{or} \qquad P &= -N\frac{\partial\left(\frac{3}{5}E_{F_o}\right)}{\partial V} \qquad \because \bar{E}_o = \frac{3}{5}E_{F_o} \\ \text{or} \qquad P &= -\frac{3}{5}N\frac{\partial\left(E_{F_o}\right)}{\partial V} \end{aligned}$$

Since, we know that, $E_{F_o} = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$. Now, we get from the above equation:

$$\begin{aligned} P &= -\frac{3}{5}N\frac{\partial\left(\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}\right)}{\partial V} \\ P &= -\frac{3}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}\frac{\partial\left(\frac{1}{V}\right)^{2/3}}{\partial V} \\ P &= -\frac{3}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}\frac{\partial}{\partial V}\left(\frac{1}{V}\right)^{2/3} \end{aligned}$$

$$P = -\frac{3}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}\frac{\partial}{\partial V}(V)^{-2/3}$$
$$P = -\frac{3}{5}\left(-\frac{2}{3}\right)N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}(V)^{-2/3-1}$$
$$P = \frac{2}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}(V)^{-2/3-1}$$
$$P = \frac{2}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}(V)^{-5/3}$$

$$\text{or} \quad P = \frac{2}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}\left(\frac{1}{V}\right)^{5/3}$$
$$\text{or} \quad P = \frac{2}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}\left(\frac{1}{V}\right)^{2/3+1}$$
$$\text{or} \quad P = \frac{2}{5}N\frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}\left(\frac{1}{V}\right)^{2/3}\left(\frac{1}{V}\right)$$
$$\text{or} \quad P = \frac{2}{5}N\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}\left(\frac{1}{V}\right)$$
$$\text{or} \quad P = \frac{2}{5}\frac{NE_{F_o}}{V}$$

This is the pressure versus volume relationship for a free electron gas at $0K$.

Chapter 2

Band Theory of Solids

SOLVED PROBLEMS

Problem: 2.1- Using the Kronig-Penny model, show that for $P << 1$, the energy of the lowest energy band is

$$E = \frac{\hbar^2 P}{ma^2}$$

Solution

Since, the energy of the lowest band corresponds to $k = \pm\pi/a$, i.e., when

$$P\left[\frac{\sin\alpha a}{\alpha a}\right] + \cos\alpha a = \pm 1$$

Considering only the magnitude on the right hand side, we obtain

$$\begin{aligned} P\left[\frac{\sin\alpha a}{\alpha a}\right] &= 1 - \cos\alpha a \\ \text{or} \qquad \frac{P}{\alpha a}\left[\sin\alpha a\right] &= 1 - \cos\alpha a \\ \text{or} \qquad \frac{2P}{\alpha a}\sin\left[\frac{\alpha a}{2}\right]\cos\left[\frac{\alpha a}{2}\right] &= 2\sin^2\left[\frac{\alpha a}{2}\right] \end{aligned}$$

For $P << 1$, we can write as:

$$\begin{aligned} \tan\left[\frac{\alpha a}{2}\right] &= \frac{P}{\alpha a} = \tan\left[\frac{P}{\alpha a}\right] \\ \text{or} \quad \frac{\alpha a}{2} &= \frac{P}{\alpha a} \\ \text{or} \quad \alpha^2 a^2 &= 2P \\ \text{or} \quad \alpha^2 &= \frac{2P}{a^2} \end{aligned}$$

Also, we know that:

$$\alpha^2 = \frac{2mE}{\hbar^2}$$

Now, on comparing, we get

$$\begin{aligned} \frac{2mE}{\hbar^2} &= \frac{2P}{a^2} \\ \frac{mE}{\hbar^2} &= \frac{P}{a^2} \\ E &= \frac{\hbar^2 P}{ma^2} \end{aligned}$$

Hence, this is the energy of the lowest energy band.

Problem: 2.2- The energy near the valence band edge of a crystal is given by

$$E = -Ak^2$$

where $A = 10^{-39}\ Jm^2$. An electron with wave vector $\vec{k} = 10^{10}\vec{k}_x\ m^{-1}$ is removed from an orbital in the completely filled valence band. Determine the effective mass, velocity, momentum and energy of the hole.

Solution

$$\begin{aligned} E &= -Ak^2 \\ A &= 10^{-39}\ Jm^2 \\ \vec{k}_e &= 10^{10}\ \hat{k}_x m^{-1} \\ \Rightarrow \quad \vec{k}_h &= -10^{10}\ \hat{k}_x m^{-1} \end{aligned}$$

$$\hbar = 1.05 \times 10^{-34}\ Js$$
$$m_{\text{eff.}} = ?$$
$$\vec{P}_h = ?$$
$$\vec{v}_h = ?$$
$$E_h = ?$$

Since, we have to know that

$$E = -Ak^2$$
$$\text{or} \quad \frac{dE}{dk} = -2Ak$$
$$\text{or} \quad \frac{d^2E}{dk^2} = -2(1)A$$
$$\frac{d^2E}{dk^2} = -2A$$
$$\frac{d^2E}{dk^2} = -2 \times 10^{-39}$$

Since, the effective mass of an electron is given as:

$$m_{\text{eff.}} = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$$
$$m_{\text{eff.}} = \frac{\left(\frac{h}{2\pi}\right)^2}{(-2 \times 10^{-39})}$$
$$m_{\text{eff.}} = -\frac{(1.053 \times 10^{-34})^2}{2 \times 10^{-39}}$$
$$m_{\text{eff.}} = -5.5 \times 10^{-30}\ kg$$

Since, the effective mass of a hole is opposite to that of an electron at the same location in the energy band, the effective mass of hole is

$$m_h^* = -m_e^*$$
$$\text{or} \quad m_h^* = -(-5.5 \times 10^{-30})$$
$$m_h^* = 5.5 \times 10^{-30}\ kg$$

The momentum of the hole is calculated as:

$$
\begin{aligned}
\vec{P}_h &= \hbar \vec{K}_h \\
\vec{P}_h &= 1.053 \times 10^{-34} \times (-10^{10}\ \hat{k}_x) \\
\vec{P}_h &= -\ 1.053 \times 10^{-34} \times 10^{10}\ \hat{k}_x \\
\vec{P}_h &= -\ 1.053 \times 10^{-24}\ \hat{k}_x\ Js\ m^{-1}
\end{aligned}
$$

Now, the velocity of the hole is:

$$
\begin{aligned}
\vec{v}_h &= \frac{\vec{P}_h}{m_h^*} \\
\vec{v}_h &= \frac{-1.053 \times 10^{-24}}{5.5 \times 10^{-30}} \\
\vec{v}_h &= -\ 1.9 \times 10^{5}\ \hat{k}_x\ ms^{-1}
\end{aligned}
$$

Since the energy of the electron with wave vector $\vec{k}_e$ is

$$
\begin{aligned}
E_e &= -\ Ak^2 \\
E_e &= -\ (10^{-39}) \times (10^{10}\ \hat{k}_x)^2 \\
E_e &= -\ 10^{-39} \times 10^{20}\ \hat{k}_x \\
E_e &= -\ 10^{-19}\ J
\end{aligned}
$$

Therefore, the energy of the hole referred to zero at the top of the valence band is;

$$
\begin{aligned}
E_h &= -\ E_e \\
E_h &= -\ (-10^{-19}) \\
E_h &= 10^{-19}\ J
\end{aligned}
$$

Problem: 2.3- A sample of SI is doped with 10^{17} phosphorous atoms per cm^3. What is its resistivity? What is the expected Hall voltage in a sample of 200 μm thickness if the current density is 1 A/cm^2 and magnetic field of 1×10^{-5} Wb/cm^2 is applied perpendicular to the direction of current flow. The mobility is given as 600 $cm^2/volt - sec$.

Solution

$$
\begin{aligned}
n &= 10^{17} \text{ electrons}/cm^3 \\
\mu &= 600\ cm^2/volt - sec \\
B_z &= 1 \times 10^{-5}\ Wb/cm^2 \\
e &= 1.6 \times 10^{-19}\ C \\
d &= 200\ \mu m \\
J_x &= 1\ A/cm^2 \\
\sigma &= ? \\
\rho &= ? \\
R_H &= ? \\
V_H &= ?
\end{aligned}
$$

Since, we know that the conductivity is defined as

$$
\begin{aligned}
\sigma &= \frac{\mu}{R_H} \\
\sigma &= \frac{\mu}{\frac{1}{ne}} \quad \because R_H = \frac{1}{ne} \\
\sigma &= \mu ne \\
\sigma &= 600 \times 10^{17} \times 1.6 \times 10^{-19} \\
\sigma &= 9.6\ \Omega - cm
\end{aligned}
$$

Now, the resistivity is defined as:

$$\rho = \frac{1}{\sigma}$$
$$\rho = \frac{1}{9.6}$$
$$\rho = 0.104\ \Omega - cm$$

Hall coefficient can be defined as:

$$R_H = -\frac{1}{ne}$$
$$R_H = -\frac{1}{10^{17} \times 1.6 \times 10^{-19}}$$
$$R_H = -62.5\ cm^3/C$$

And, Hall voltage is given as:

$$V_H = E_H d$$
$$V_H = (J_x B_z R_H) d \quad \because E_H = J_x B_z R_H$$
$$V_H = 1 \times 1 \times 10^{-5} \times (-62.5) \times (2 \times 10^{-2})$$
$$V_H = 12.5 \times 10^{-6}\ V$$
$$V_H = 12.5\ \mu V$$

Problem: 2.4- In a Hall effect experiment on Zinc, a potential of 4.5 μV is developed across a foil of thickness 0.02 mm when a current of 1.5 A is passed in a direction perpendicular to a magnetic field of 2.0 T. Calculate the Hall coefficient and the electron density.

Solution

$$V_H = 4.5\ \mu m$$
$$V_H = 4.5 \times 10^{-6}\ V$$
$$e = 1.6 \times 10^{-19}\ C$$
$$d = 0.02\ mm$$

$$d = 0.02 \times 10^{-3} = 2 \times 10^{-5}\ m$$
$$I = 1.5\ A$$
$$B = 2T$$
$$R_H = ?$$
$$n = ?$$

Since, the Hall coefficient is defined as

$$R_H = \frac{V_H d}{BI}$$
$$R_H = \frac{4.5 \times 10^{-6} \times 2 \times 10^{-5}}{2 \times 1.5}$$
$$R_H = 0.3 \times 10^{-10}\ m^3C^{-1}$$

Also, the electron density is defined as:

$$R_H = \frac{1}{ne}$$
$$\text{or} \quad n = \frac{1}{eR_H}$$
$$n = \frac{1}{1.6 \times 10^{-19} \times 0.3 \times 10^{-10}}$$
$$n = 2.08 \times 10^{29}\ m^{-3}$$

Problem: 2.5- The energy near the valence band edge of the crystal is given by $E = -Ak^3$, where $A = 10^{-36}\ Jm^2$. Calculate the effective mass of an electron with wave vector having magnitude of $10^9\ m^{-1}$.

Solution

$$E = -Ak^3$$
$$A = 10^{-36}\ Jm^2$$
$$k = 10^9\ m^{-1}$$
$$m_{\text{eff.}} = ?$$

Since, we have to know that

$$
\begin{aligned}
E &= -\,Ak^3 \\
\text{or} \quad \frac{dE}{dk} &= -\,3Ak^2 \\
\text{or} \quad \frac{d^2E}{dk^2} &= -\,3(2)Ak \\
\frac{d^2E}{dk^2} &= -\,6Ak \\
\frac{d^2E}{dk^2} &= -\,6 \times 10^{-36} \times 10^9
\end{aligned}
$$

Since, the effective mass of an electron is given as:

$$
\begin{aligned}
m_{\text{eff.}} &= \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \\
m_{\text{eff.}} &= \frac{\left(\frac{h}{2\pi}\right)^2}{(-6 \times 10^{-36} \times 10^9)} \\
m_{\text{eff.}} &= -\,\frac{(1.053 \times 10^{-34})^2}{6 \times 10^{-45}} \\
m_{\text{eff.}} &= -\,0.184 \times 10^{-23}\ kg
\end{aligned}
$$

Chapter 3

Semiconductors

SOLVED PROBLEMS

Problem: 3.1- An insulator has an optical absorption which occurs for all wavelengths lesser than 1400A°. Find the width of the forbidden energy band for the insulator.

Solution

$$\begin{aligned} \lambda &= 1400A^\circ \\ \lambda &= 1400 \times 10^{-10}\ m \\ c &= 3 \times 10^8\ m/s \\ h &= 6.63 \times 10^{-34}\ Js \\ E_g &= ? \end{aligned}$$

Since, the corresponding frequency is defined as:

$$\begin{aligned} f\lambda &= c \\ \text{or} \qquad f &= \frac{c}{\lambda} \\ f &= \frac{3 \times 10^8}{1400 \times 10^{-10}} \\ f &= 0.00214 \times 10^{18} \\ f &= 2.14 \times 10^{15}\ Hz \end{aligned}$$

Hence, the energy gap is given as:

$$
\begin{aligned}
E_g &= hf \\
E_g &= 6.63 \times 10^{-34} \times 2.14 \times 10^{15} \\
E_g &= 14.182 \times 10^{-19}\ J \\
\text{or} \quad E_g &= 1.41 \times 10^{-18}\ J \\
\text{or} \quad E_g &= \frac{1.41 \times 10^{-18}}{.6 \times 10^{-19}}\ eV\ =\ 8.81\ eV
\end{aligned}
$$

Problem: 3.2- Determine the concentration of conduction electrons per meter cube in intrinsic semiconductor whose conductivity is $3 \times 10^4\ \Omega - m^{-1}$. Electron and hole mobilities are 0.14 and 0.06 m^2/Vs, respectively.

Solution

$$
\begin{aligned}
\sigma_e &= 3 \times 10^4\ \Omega - m^{-1} \\
\mu_e &= 0.14\ m^2/Vs \\
\mu_h &= 0.06\ m^2/Vs \\
e &= 1.6 \times 10^{-19}\ C \\
n_e &= ?
\end{aligned}
$$

Since, we have to know that

$$\sigma_e\ =\ n_e e \mu_e + n_p e \mu_h$$

For an intrinsic semiconductor, $n_p = n_e$, we get

$$
\begin{aligned}
\sigma_e &= n_e e \mu_e + n_e e \mu_h \\
\sigma_e &= n_e e(\mu_e + \mu_h) \\
\text{or} \quad n_e &= \frac{\sigma_e}{e(\mu_e + \mu_h)} \\
n_e &= \frac{3 \times 10^4}{1.6 \times 10^{-19}(0.14 + 0.06)}
\end{aligned}
$$

$$n_e = \frac{3 \times 10^4}{1.6 \times 10^{-19}(0.20)}$$
$$n_e = 9.37 \times 10^{23}$$

Problem: 3.3- The Fermi level in certain semi-conducting material is 1.75 eV at a particular temperature. Calculate the number of free electrons per unit volume in the semiconductor at the same temperature. Given the lattice parameter $a = \frac{\pi}{3}$.

Solution

$$E_F = 1.75\ eV$$
$$E_F = 1.75 \times 1.6 \times 10^{-19}\ J$$
$$E_F = 2.8 \times 10^{-19}\ J$$
$$a = \frac{\pi}{3}$$
$$m = 9.1 \times 10^{-31}\ kg$$
$$h = 6.63 \times 10^{-34}\ Js$$
$$n = ?$$

Since, we have to know that:

$$E_F = \frac{n^2\pi^2\hbar^2}{2ma^2}$$
$$E_F = \frac{n^2\pi^2\left[\frac{h}{2\pi}\right]^2}{2ma^2}$$
$$E_F = \frac{n^2\pi^2\frac{h^2}{4\pi^2}}{2ma^2}$$
$$E_F = \frac{n^2\pi^2h^2}{2ma^2 \times 4\pi^2}$$
$$E_F = \frac{n^2\pi^2h^2}{8ma^2\pi^2}$$
$$E_F = \frac{n^2h^2}{8ma^2}$$

$$n^2 = \frac{8ma^2E_F}{h^2}$$

$$\sqrt{n^2} = \sqrt{\frac{8ma^2E_F}{h^2}}$$

$$n = \sqrt{\frac{8mE_F}{h^2}}a$$

$$n = \sqrt{\frac{8 \times 9.1 \times 10^{-31} \times 2.8 \times 10^{-19}}{(6.63 \times 10^{-34})^2}} \times \frac{\pi}{3}$$

$$n = 2.15 \times 10^{18} \times \frac{\pi}{3}$$

$$n = 2.25 \times 10^{18} \text{ electron per } m^3$$

Problem: 3.4- Calculate the concentration of electrons and holes in N-type semiconductor if the donor density is 10^{22} atoms per meter cube and the intrinsic carrier concentration is 1.5×10^{20} per meter cube at room temperature.

Solution

$$\text{The number density of donors } N_d = 10^{22} \; atoms/m^3$$

$$\text{The number of intrinsic carriers } n_i = 1.5 \times 10^{20} \; /m^3$$

$$n_p = ?$$

As, we know that

$$n_e n_p = n_i^2$$

$$N_d n_p = n_i^2$$

$$n_p = \frac{n_i^2}{N_d}$$

$$n_p = \frac{(1.5 \times 10^{20})^2}{10^{22}}$$

$$n_p = 2.25 \times 10^{18} \text{ atoms}/m^3$$

Problem: 3.5- The electron and hole mobilities in a Si sample are 0.135 and 0.048 m^2/Vs, respectively. Determine the conductivity of intrinsic Si at 300 K if the intrinsic carrier concentration is 1.5×10^{16} atoms per meter cube. The sample is then doped with 10^{23} phosphorus atoms per meter cube. Determine the equilibrium hole concentration, conductivity and position of the Fermi level relative to the intrinsic level.

Solution

$$\begin{aligned}
\mu_n &= 0.135\ m^2/Vs \\
\mu_p &= 0.048\ m^2/Vs \\
n_i &= 1.5 \times 10^{16}\ m^{-3} \\
e &= 1.6 \times 10^{-19}\ C \\
n &= N_d^+ = 10^{23}\ atoms/m^3 \\
\sigma_1 &= ? \\
p &= ? \\
\sigma_2 &= ?
\end{aligned}$$

In case of intrinsic semiconductors, $n = p = n_i$. Therefore, the conductivity is given by:

$$\begin{aligned}
\sigma_1 &= en_i(\mu_n + \mu_p) \\
\sigma_1 &= 1.6 \times 10^{-19} \times 1.5 \times 10^{16}(0.135 + 0.048) \\
\sigma_1 &= 4.39 \times 10^{-4}\ (\Omega - m)^{-1}
\end{aligned}$$

In the extrinsic case, since $N_d >> n_i$, and assuming all the donors to be ionized. Therefore, the equilibrium hole concentration is

$$\begin{aligned}
np &= n_i^2 \\
p &= \frac{n_i^2}{n} \\
p &= \frac{(1.5 \times 10^{16})^2}{10^{23}} \\
p &= 2.25 \times 10^9\ m^{-3}
\end{aligned}$$

Now, the conductivity is given as:

$$\sigma_2 = en\mu_n$$
$$\sigma_2 = 1.6 \times 10^{-19} \times 10^{23} \times 0.135$$
$$\sigma_2 = 2.16 \times 10^{2}\ (\Omega - m)^{-1}$$

Also, we have to know that

$$E_F - E_i = kT \ln \left[\frac{n}{n_i}\right]$$
$$E_F - E_i = 8.62 \times 10^{-5} \times 300 \ln \left[\frac{10^{23}}{1.5 \times 10^{16}}\right]$$
$$E_F - E_i = 0.406\ eV$$

Chapter 4

Magnetism in Solids

SOLVED PROBLEMS

Problem: 4.1- A bar magnet of length 10 cm has pole strength of 10 NT^{-1}. Calculate its magnetic dipole moment.

Solution

$$
\begin{aligned}
2l &= 10\ cm \\
2l &= 10\ cm \\
\frac{10}{100}&\ m \\
2l &= 0.1\ m \\
m &= 10\ N/T \\
\mu &= ?
\end{aligned}
$$

Since, the magnetic dipole moment is given as:

$$
\begin{aligned}
\mu &= 2ml \\
\text{or} \quad \mu &= m(2l) \\
\mu &= 10 \times 0.1 \\
\mu &= 10 \times \frac{1}{10} \\
\mu &= 1\ Am^2
\end{aligned}
$$

Problem: 4.2- Calculate magnetic susceptibility of a material assuming one electron and taking $m = 9.1\times10^{-31}\ kg$, $R = 0.1\ nm$, $N = 5\times10^{28}\ m^{-3}$ and $e = 1.6\times10^{-19}\ C$.

Solution

$$
\begin{aligned}
m &= 9.1\times10^{-31}\ kg\\
R &= 0.1\ nm\\
R &= 0.1\times10^{-9}\ m\\
N &= 5\times10^{28}\ m^{-3}\\
e &= 1.6\times10^{-19}\ C\\
Z &= 1 \quad \text{for electron}\\
\mu_o &= 4\pi\times10^{-7}\ Wb/A-m\\
\chi &= ?
\end{aligned}
$$

Since, we have to know that

$$
\begin{aligned}
\chi &= -\frac{\mu_o N Z e^2}{6m}\langle R^2\rangle\\
\chi &= -3\times10^{-32}
\end{aligned}
$$

Problem: 4.3- A paramagnetic substance has 10^{28} atoms/m^3. The magnetic moment of each atom is $1.79\times10^{-23}\ Am^2$. Calculate the paramagnetic susceptibility of the material at temperature 320 K. What would be the dipole moment of the rod of this material 0.1 m long and 1 cm^2 cross-section placed in a field of $7\times10^4\ A/m$?

Solution

$$
\begin{aligned}
N &= 10^{28}\ \text{atoms}/m^3\\
\mu &= 1.79\times10^{-23}/Am^2\\
\mu_o &= 4\pi\times10^{-7}\ Wb/A-m\\
k &= 1.38\times10^{-38}\ J/K\\
T &= 320\ K
\end{aligned}
$$

$$H = 7 \times 10^4 \ A/m$$
$$V = 0.1 \ m \times 1 \ cm^2$$
$$V = 10^{-5} \ m^3$$
$$\chi = ?$$
$$M = ?$$
$$\mu = ?$$

The susceptibility of paramagnetic material is given by

$$\chi = \frac{N\mu^2\mu_o}{kT}$$
$$\chi = \frac{10^{28} \times (1.79 \times 10^{-23})^2 \times 4\pi \times 10^{-7}}{1.38 \times 10^{-38} \times 320T}$$
$$\chi = 9.11 \times 10^{-4}$$

Now, the magnetization is given as:

$$M = \chi H$$
$$M = 9.11 \times 10^{-4} \times 7 \times 10^4$$
$$M = 63.77 \ Am^{-1}$$

The magnetization is given as net dipole moment per unit volume, therefore, magnetic dipole moment is

$$\mu = M \times V$$
$$\mu = 63.77 \times 10^{-5} \ Am^2$$

Problem: 4.4- Calculate the diamagnetic susceptibility of atomic hydrogen in the ground state at S.T.P. using the wave function

$$\psi(r) = \frac{1}{(\pi a_o^3)^{1/2}} \exp\left(-\frac{r}{a_o}\right)$$

where $a_o = 0.46 \ \mathring{A}$ is the atomic radius.

Solution

The wave function for the ground state of hydrogen atom is

$$\psi(r) = \frac{1}{(\pi a_o^3)^{1/2}} \exp\left(-\frac{r}{a_o}\right)$$

The mean square distance of electronic charge distribution from the nucleus is calculated as:

$$\langle r^2 \rangle = \int \psi^* r^2 \psi dr$$

$$\langle r^2 \rangle = 4\pi \int_0^\infty \psi^* r^2 \psi r^2 dr$$

$$\langle r^2 \rangle = \frac{4\pi}{\pi a_o^3} \int_0^\infty r^4 \exp\left(-\frac{2r}{a_o}\right) dr$$

Put $-\frac{2r}{a_o} = t$, therefore,

$$r = -\frac{a_o}{2} t$$

$$\text{or} \quad dr = -\frac{a_o}{2} dt$$

Now,

$$\langle r^2 \rangle = \frac{4}{a_o^3}\left(\frac{a_o^4}{16}\right)\left(-\frac{a_o}{2}\right)\int_0^\infty t^4 e^{-t} dt$$

$$\langle r^2 \rangle = \frac{4}{a_o^3}\left(\frac{a_o^4}{16}\right)\left(-\frac{a_o}{2}\right) \times 24$$

$$\langle r^2 \rangle = 3a_o^2$$

Because $\because \int_0^\infty t^4 e^{-t} dt = 24$. Since, we know that

$$\chi_{\text{dia.}} = -\frac{N\mu_o Z e^2}{6m} \langle r^2 \rangle$$

$$\chi_{\text{dia.}} = -\frac{N\mu_o Z e^2}{6m} 3a_o^2$$

$$\chi_{\text{dia.}} = -\frac{N\mu_o Z e^2}{2m} a_o^2$$

Here,

$$N = \frac{6.02 \times 10^{26}}{2.24 \times 10^{-2}}$$
$$N = 2.69 \times 10^{28}\ m^{-3}$$
$$Z = 1$$
$$a_o = 0.46 \times 10^{-10}\ m$$
$$e = 1.6 \times 10^{-19}\ C$$
$$m = 9.1 \times 10^{-31}\ kg$$
$$\mu_o = 4\pi \times 10^{-7}\ Wb/Am$$

Therefore, by putting values, we get

$$\chi_{\text{dia.}} = -1.01 \times 10^{-6}$$

Problem: 4.5- An iron rod of 0.5 cm^2 area of cross section is subjected to a magnetizing field of 1200 Am^{-1}. If susceptibility of iron is 599, then calculate (I)-μ, (II)-B and (III)-ϕ magnetic flux produced.

Solution

$$A = 0.5\ cm^2$$
$$A = 0.5 \times 10^{-4}\ m^2$$
$$H = 12200\ Am^{-1}$$
$$\chi_m = 599$$
$$\mu_o = 4\pi \times 10^{-7}\ Wb/Am$$
$$\mu = ?$$
$$B = ?$$
$$\phi = ?$$

As, we know that

$$\mu_r = 1 + \chi_m$$
$$\mu_r = 1 + 599$$
$$\mu_r = 600 \; Wb/Am$$

Also, (I)-

$$\mu_r = \frac{\mu}{\mu_o}$$
$$\text{or} \quad \mu = \mu_o \mu_r$$
$$\mu = 4\pi \times 10^{-7} \times 600$$
$$\mu = 4 \times 3.14 \times 10^{-7} \times 600$$
$$\mu = 7.54 \times 10^{-4} \; Wb/Am$$

(II)-
Now, we have the relation for B, as

$$B = \mu H$$
$$B = 7.54 \times 10^{-4} \times 1200$$
$$B = 0.905 \; T$$

(III)-
Also, we know that

$$\phi = BA$$
$$\phi = 0.905 \times 0.5 \times 10^{-4}$$
$$\phi = 4.525 \times 10^{-5} \; Wb$$

Chapter 5

Introduction to Superconductor

SOLVED PROBLEMS

Problem: 5.1- Calculate the superconducting electron density of mercury at 3.5 K. Given transition temperature of mercury is 4.22 K.

Solution

The normal current density in mercury can be found in terms of molecular weight and density. Therefore,

$$\begin{aligned} n_o &= \frac{N_\rho}{M} \\ n_o &= \frac{6.02 \times 10^{26} \times 13.55 \times 10^3}{200.6} \\ n_o &= 4.06 \times 10^{28} \ /m^3 \end{aligned}$$

Since, we know that

$$\frac{n_o}{n_s} = \frac{1}{\left[1 - \left(\frac{T}{T_c}\right)^4\right]}$$

$$\text{or} \quad \frac{n_s}{n_o} = \left[1 - \left(\frac{T}{T_c}\right)^4\right]$$

$$\text{or} \quad n_s = n_o \left[1 - \left(\frac{T}{T_c}\right)^4\right]$$

$$n_s = 4.06 \times 10^{28} \left[1 - \left(\frac{3.5}{4.22}\right)^4\right]$$

$$n_s = 2.138 \times 10^{28} \ /m^3$$

Problem: 5.2- The critical temperature T_c for H_g with isotopic mass 199.5 amu is 4.185 K. Calculate its critical temperature, when isotopic mass changes to 203.4 amu.

Solution

$$T_{c_1} = 4.185 \ K$$

$$M_1 = 199.5 \ amu$$

$$M_2 = 203.4 \ amu$$

$$T_{c_2} = ?$$

Since, we have to know that

$$T_c\sqrt{M} = \text{constant}$$

$$\therefore \quad \frac{T_{c_1}}{T_{c_2}} = \sqrt{\frac{M_2}{M_1}}$$

$$\text{or} \quad \frac{T_{c_2}}{T_{c_1}} = \sqrt{\frac{M_1}{M_2}}$$

$$T_{c_2} = T_{c_1}\sqrt{\frac{M_1}{M_2}}$$

$$T_{c_2} = 4.185 \times \sqrt{\frac{199.5}{203.4}}$$

$$T_{c_2} = 4.14 \ K$$

Problem: 5.3- Calculate the critical current, which can flow through a long thin superconducting wire of diameter $10^{-4}\ m$. The critical field of aluminium is $7.9 \times 10^3\ A/m$.

Solution

$$\begin{aligned} d &= 10^{-4}\ m \\ \Rightarrow \quad r &= \frac{d}{2} \\ r &= \frac{10^{-4}}{2} \\ r &= 0.5 \times 10^{-4}\ m \\ H_c &= 7.9 \times 10^3\ A/m \\ I_c &= ? \end{aligned}$$

Since, we know that

$$\begin{aligned} H_c &= \frac{I_c}{2\pi r} \\ \text{or} \quad I_c &= 2\pi r H_c \\ I_c &= 2 \times 3.14 \times 0.5 \times 10^{-4} \times 7.9 \times 10^3 \\ I_c &= 24.81 \times 10^{-1}\ A \\ I_c &= 2.48\ A \end{aligned}$$

Problem: 5.4- The transition temperature of lead is $7.2\ K$. However, it loses the superconducting property if subjected to a magnetic field of $3.3 \times 10^4\ A/m$ at $5\ K$. Find the value of $H_c(0)$ which will allow the metal to retain its superconductivity at $0\ K$.

Solution

$$\begin{aligned} T_c &= 7.2\ K \\ T &= 5\ K \\ H_c &= 3.3 \times 10^4\ A/m \\ H_c(0) &= ? \end{aligned}$$

Since, we know that

$$
\begin{aligned}
H_c &= H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right] \\
\Rightarrow \quad H_c(0) &= \frac{H_c}{\left[1-\left(\frac{T}{T_c}\right)^2\right]} \\
H_c(0) &= \frac{3.3\times 10^4}{\left[1-\left(\frac{5}{7.2}\right)^2\right]} \\
H_c(0) &= \frac{3.3\times 10^4}{\left[1-\left(\frac{25}{51.28}\right)\right]} \\
H_c(0) &= \frac{3.3\times 10^4}{[1-0.487]}
\end{aligned}
$$

$$
\begin{aligned}
H_c(0) &= \frac{3.3\times 10^4}{0.513} \\
H_c(0) &= 6.43\times 10^4 \ A/m
\end{aligned}
$$

Problem: 5.5- The transition temperature of mercury with an average atomic mass of 200.59 amu is 4.153 K. Determine the transition temperature of one of its isotopes, $_{80}Hg^{204}$.

Solution

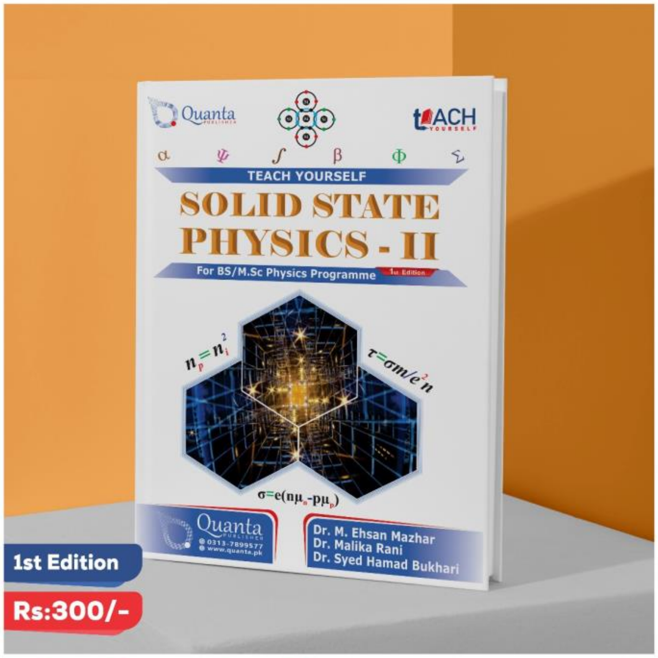
$$
\begin{aligned}
T_{c_1} &= 4.153\ K \\
M_1 &= 200.59\ amu \\
M_2 &= 204\ amu \\
T_{c_2} &= ?
\end{aligned}
$$

The transition temperature of a superconductor is related to its isotopic mass as

$$T_c \propto \frac{1}{\sqrt{M}}$$

Which gives,

$$\frac{T_{c_2}}{T_{c_1}} = \sqrt{\frac{M_1}{M_2}}$$
$$T_{c_2} = T_{c_1}\sqrt{\frac{M_1}{M_2}}$$
$$T_{c_2} = 4.153\sqrt{\frac{200.59}{204}}$$
$$T_{c_2} = 4.118\ K$$



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