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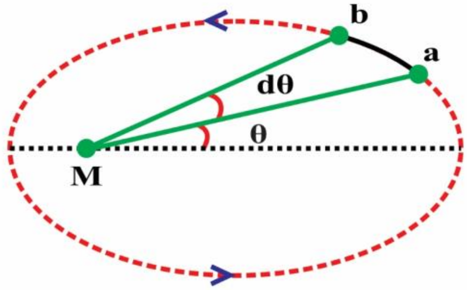

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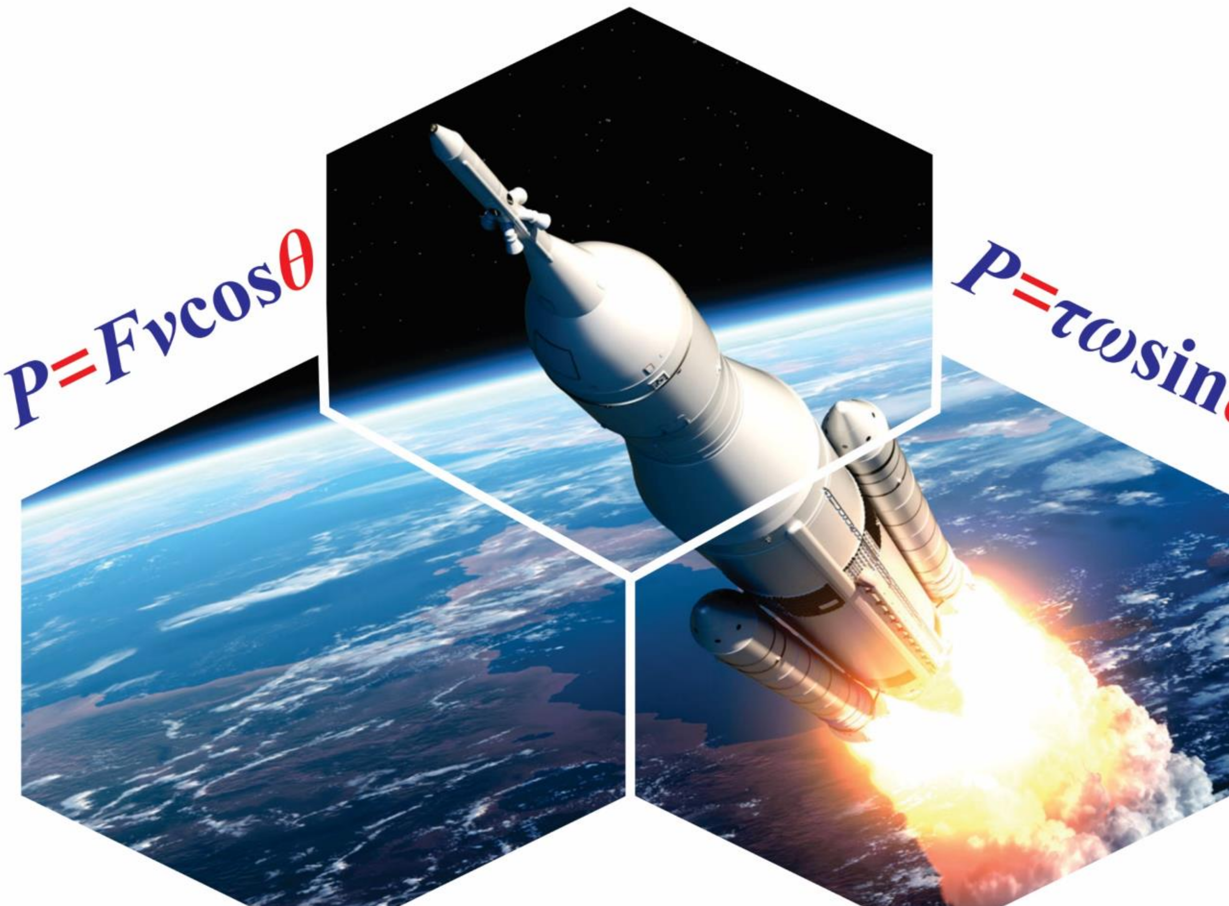
MECHANICS -II



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TEACH YOURSELF

Mechanics-II

1st Edition

For **BS Physics/Chemistry/Mathematics** students

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Contents

Chapter 1

Rotational Dynamics

SOLVED PROBLEMS

Problem: 1.1- Engine or a car develops 120 hp power when rotating at 1750 $rev\ m^{-1}$. How much torque does it deliver?

Solution

$$\text{Power} = P = 120\ hp = 120 \times 746\ watt \qquad \because 1hp = 746\ watt$$
$$\text{Power} = P = 8.95 \times 10^4\ W$$
$$\text{Angular velocity} = \omega = 1750\ rev.m^{-1} = \frac{1750 \times 2\pi}{60}\ rads^{-1}$$
$$\text{Angular velocity} = \omega = 183.17\ rads^{-1}$$
$$\text{Torque} = \tau = ?$$

Since,

$$P = \tau\omega$$
$$\Rightarrow \tau = \frac{P}{\omega}$$

$$\tau = \frac{8.95 \times 10^4}{183.17}$$
$$\tau = 488.73\ Nm$$

Problem: 1.2- Calculate the rotational inertia of solid sphere of mass 30 kg and diameter 20 cm about its any diameter and about tangential axis.

Solution

$$\begin{aligned}
\text{Mass of solid sphere} &= M = 30\ kg \\
\text{Diameter of solid sphere} &= d = 20\ cm = \frac{20}{100}\ m = 0.2\ m \\
\text{Radius of solid sphere} &= R = \frac{d}{2} = \frac{0.2}{2} = 0.1\ m \\
\text{Rotational inertia about any diameter} &= I = ? \\
\text{Rotational inertia about tangential axis} &= I_{\text{tan.}} = ?
\end{aligned}$$

Since we know that:

$$\begin{aligned}
I &= \frac{2}{5}MR^2 \\
I &= \frac{2}{5} \times 30 \times (0.1)^2 \\
I &= \frac{2}{5} \times 30 \times 0.01 \\
I &= 2 \times 6 \times 0.01 \\
I &= 0.12\ kgm^2
\end{aligned}$$

and, according to parallel axes theorem:

$$\begin{aligned}
I_{\text{tan.}} &= I + MR^2 \\
I_{\text{tan.}} &= \frac{2}{5}MR^2 + MR^2 \\
I_{\text{tan.}} &= \frac{7}{5}MR^2
\end{aligned}$$

$$\begin{aligned}
I_{\text{tan.}} &= \frac{7}{5} \times 30 \times (0.1)^2 \\
I_{\text{tan.}} &= 7 \times 6 \times 0.01 \\
I_{\text{tan.}} &= 0.42\ kgm^2
\end{aligned}$$

Problem: 1.3- Calculate the angular speed of a car rounding a circular turn of radius 110 m at 52.4 km/h.

Solution

$$\begin{aligned}
\text{Radius } &= r \;=\; 110\ m \\
\text{Speed } &= v \;=\; 52.4\ km/h \\
\text{Speed } &= v \;=\; \frac{52.4 \times 1000}{3600}\ m/s \\
\text{Speed } &= v \;=\; 14.6\ m/s \\
\text{Angular speed } &= \omega \;= ?
\end{aligned}$$

Since we know that the relation between linear and angular speed is:

$$\begin{aligned}
v &= r\omega \\
\Rightarrow\ \omega &= \frac{v}{r} \\
\omega &= \frac{14.6}{110} \\
\omega &= 0.133\ rad\ s^{-1}
\end{aligned}$$

Problem: 1.4- A fly wheel of mass 500 kg and radius 1 m makes 500 rev/min. Assuming the mass to be concentrated along the rim, find rotational kinetic energy of fly wheel.

Solution

$$\begin{aligned}
\text{Mass } &= m \;=\; 500\ kg \\
\text{Radius } &= r \;=\; 1\ m \\
\text{Angular speed } &= \omega \;=\; 500\ rev/min \;=\; \frac{2\pi \times 500}{60}\ rad/s \\
\text{Angular speed } &= \omega \;=\; 52.33\ rad/s \\
\text{Rotational kinetic energy } &= K.E_{\text{rot.}} \;= ?
\end{aligned}$$

Since the relation is:

$$
\begin{aligned}
K.E_{\text{rot.}} &= \frac{1}{2} I\omega^2 \\
K.E_{\text{rot.}} &= \frac{1}{2} m r^2 \omega^2 \qquad \because I = mr^2 \\
K.E_{\text{rot.}} &= \frac{1}{2} \times 500 \times (1)^2 \times (52.33)^2 \\
K.E_{\text{rot.}} &= 6.871 \times 10^5 \ J
\end{aligned}
$$

Problem: 1.5- A pulley wheel 8.14 cm in diameter has a 5.63 m long cord wrapped around its periphery. Starting from rest, the wheel is given an angular acceleration of 1.47 rad/s^2. Through what angle must the wheel turn for the cord to unwind?

Solution

$$
\begin{aligned}
s &= 5.63 \ m \\
r &= \frac{d}{2} = \frac{8.14}{2} \ cm \\
r &= 4.07 \ cm = 4.07 \times 10^{-2} \ m \\
\phi &= ?
\end{aligned}
$$

As we know that:

$$
\begin{aligned}
s &= r\phi \\
\Rightarrow \ \phi &= \frac{s}{r} \\
\phi &= \frac{5.63}{4.07 \times 10^{-2}} \\
\phi &= 1.38 \times 10^2 \ rad \\
\phi &= 138 \ rad
\end{aligned}
$$

Chapter 2

Angular Momentum

SOLVED PROBLEMS

Problem: 2.1- The angular momentum of a particle is given as $\vec{L} = 6t^4\hat{i} - 3t^2\hat{j} + 13t^3\hat{k}$, find torque at $t = 1$sec and at $t = 2$sec.

Solution

It is given that

$$\vec{L} = 6t^4\hat{i} - 3t^2\hat{j} + 13t^3\hat{k}$$
$$\vec{\tau} = ?$$
$$\because \ \vec{\tau} = \frac{d\vec{L}}{dt}$$
$$\vec{\tau} = \frac{d}{dt}(6t^4\hat{i} - 3t^2\hat{j} + 13t^3\hat{k})$$
$$\vec{\tau} = 24t^3\hat{i} - 6t\hat{j} + 39t^2\hat{k}$$

$$\vec{\tau} = 24\hat{i} - 6\hat{j} + 39\hat{k} \text{ units} \quad \text{at } t = 1s$$
$$\vec{\tau} = 192\hat{i} - 12\hat{j} + 156\hat{k} \text{ units} \quad \text{at } t = 2s$$

Problem: 2.2- Find the angular momentum of earth about its own axis.

Solution

Angular momentum of earth about its own axis is,

$$L = I\omega$$
$$L = \frac{2}{5}MR^2 \times \frac{2\pi}{T}$$

Since,

$$\text{Moment of inertia of earth } = I = \frac{2}{5}MR^2$$
$$\text{Angular frequency } = \omega = \frac{2\pi}{T}$$
$$\text{Mass of earth } = M = 6 \times 10^{24} kg$$
$$\text{Radius of earth } = R = 6.4 \times 10^6 m$$
$$\text{Time period } = T = 86400s$$

so,

$$L = \frac{2}{5} \times 6 \times 10^{24}(6.4 \times 10^6)^2 \times \frac{2\pi}{86400}$$
$$L = \frac{2}{5} \times 6 \times 10^{24} \times 40.96 \times 10^{12} \times \frac{6.28}{86400}$$
$$L = \frac{3086.74}{432000} \times 10^{36}$$
$$L = 0.0071 \times 10^{36}$$
$$L = 7.1 \times 10^{33} Js$$

Problem: 2.3- What is angular momentum of a $95kg$ man running with a speed of $5.1ms^{-1}$ on a circular track of radius $25m$?

Solution

$$\text{Mass } = m = 95kg$$
$$\text{Speed } = v = 5.1ms^{-1}$$
$$\text{Radius } = r = 25m$$

Angular momentum is defined as:

$$L = mvr$$
$$L = 95 \times 5.1 \times 25$$
$$L = 1.2 \times 10^4 kgm^2s^{-1}$$

Problem: 2.4- A star (considering uniform sphere) of radius $2.3 \times 10^8 m$ rotates with an angular speed $2.6 \times 10^{-6}\ rads^{-1}$. If this star collapses to radius of $20000m$, find its final angular speed.

Solution

$$R_1 = 2.3 \times 10^8 m$$
$$R_2 = 20000m$$
$$\omega_1 = 2.6 \times 10^{-6}\ rads^{-1}$$
$$\omega_2 = ?$$

According to law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$
$$\omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1$$
$$\omega_2 = \left(\frac{\frac{2}{5}MR_1^2}{\frac{2}{5}MR_2^2}\right)\omega_1$$
$$\omega_2 = \left(\frac{2.3 \times 10^8}{20000}\right)^2 \times 2.6 \times 10^{-6}$$
$$\omega_2 = \frac{12.696}{4 \times 10^8} \times 10^{10}$$
$$\omega_2 = 3.174 \times 10^2 = 317\ rads^{-1}$$

Problem: 2.5- In a light wind, a wind mill experiences a constant torque of 255 Nm. If windmill is initially at rest, what is its angular momentum after 2 s?

Solution

$$\tau = 255\ Nm$$
$$L_i = 0 \quad \text{[As windmill is at rest]}$$
$$dt = 2s$$
$$L_f = ?$$

Since the relation between the torque and angular momentum is given as:

$$\begin{aligned} \tau &= \frac{dL}{dt} \\ \Rightarrow\ dL &= \tau dt \\ \Rightarrow\ L_f - L_i &= \tau dt \\ L_f - 0 &= 255 \times 2 \\ L_f &= 510 kgm^2 s^{-1} \end{aligned}$$

Chapter 3

Gravitation

SOLVED PROBLEMS

Problem: 3.1- Calculate the potential energy of the moon-earth system relative to the potential energy at infinite separation.

Solution

As we know that:

$$\begin{aligned}
\text{Mass of earth} &= M = 6 \times 10^{24}\ kg \\
\text{Mass of moon} &= m = 7.36 \times 10^{22}\ kg \\
\text{Separation distance} &= r = 3.82 \times 10^{8}\ m \\
\text{Gravitational constant} &= G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}
\end{aligned}$$

Since,

$$\begin{aligned}
U(r) &= -\frac{GMm}{r} \\
U(r) &= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.36 \times 10^{22}}{3.82 \times 10^{8}} \\
U(r) &= -\frac{294.55 \times 10^{35}}{3.82 \times 10^{8}} \\
U(r) &= -77.10 \times 10^{27} \\
U(r) &= -7.71 \times 10^{28}\ J
\end{aligned}$$

Problem: 3.2- Calculate the gravitational force between two 7.3 kg bowling balls separated by 0.65 m between their centers.

Solution

$$\begin{aligned}
\text{Mass of each bowling ball} &= m_1 = m_2 = 7.3\ kg \\
\text{Distance between centers of balls} &= r = 0.65\ m \\
\text{Gravitational constant} &= G = 6.67 \times 10^{-11}\ Nm^2kg^{-2} \\
\text{Gravitational force} &= F = ?
\end{aligned}$$

Since we know that:

$$\begin{aligned}
F &= G\frac{m_1 m_2}{r^2} \\
F &= 6.67 \times 10^{-11}\frac{7.3 \times 7.3}{(0.65)^2} \\
F &= \frac{355.44 \times 10^{-11}}{0.4225} \\
F &= 841.28 \times 10^{-11} \\
F &= 8.41 \times 10^{-9}\ N
\end{aligned}$$

Problem: 3.3- A satellite orbits at a height of 230 km above the surface of earth. Calculate the period of satellite.

Solution

$$\begin{aligned}
\text{Mass of earth} &= M = 6 \times 10^{24}\ kg \\
\text{Height} &= h = 230\ km = 230 \times 10^3\ m \\
\text{Radius of satellite orbit} &= r = R + h = 6400 + 230 = 6630 \times 10^3\ m \\
\text{Gravitational constant} &= G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}
\end{aligned}$$

Now according to law of periods, we have

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$T = \sqrt{\frac{4 \times (3.14)^2 \times (6630 \times 10^3)^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}}}$$

$$T = \sqrt{\frac{39.44 \times 2.91 \times 10^{11} \times 10^9}{40.02 \times 10^{13}}}$$

$$T = \sqrt{\frac{114.77 \times 10^{20}}{40.02 \times 10^{13}}}$$

$$T = \sqrt{2.8678 \times 10^7}$$

$$T = \sqrt{28.678 \times 10^6}$$

$$T = 5.355 \times 10^3$$

$$T = 5355\ s$$

Problem: 3.4- A reconnaissance spacecraft circles the moon at very low altitude. Calculate its speed.

Solution

$$\text{Mass of the moon} = M = 7.36 \times 10^{22}\ kg$$

$$\text{Radius of orbit} = r = 1.74 \times 10^6\ m$$

$$\text{Gravitational constant} = G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$$

$$\text{Speed of spacecraft} = v = ?$$

Since the relation is:

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 7.36 \times 10^{22}}{1.74 \times 10^6}}$$

$$v = \sqrt{\frac{49.091 \times 10^{11}}{1.74 \times 10^6}}$$

$$v = \sqrt{28.213 \times 10^5}$$

$$v = \sqrt{282.13 \times 10^4}$$

$$v = 16.79 \times 10^2 \ ms^{-1}$$
$$v = 1.67 \times 10^3 \ ms^{-1}$$

Problem: 3.5- Find the mass of Mars having radius $3.39 \times 10^6 \ m$. Given that acceleration due to gravity on surface of Mars is $3.73 \ ms^{-2}$.

Solution

$$\text{Radius of Mars} = R = 3.39 \times 10^6 \ m$$
$$\text{Gravitational acceleration} = g = 3.73 \ ms^{-2}$$
$$\text{Gravitational constant} = G = 6.67 \times 10^{-11} \ Nm^2kg^{-2}$$
$$\text{Mass of Mars} = M = ?$$

As we know that:

$$g = \frac{GM}{R^2}$$
$$\Rightarrow M = \frac{gR^2}{G}$$
$$M = \frac{3.73 \times (3.39 \times 10^6)^2}{6.67 \times 10^{-11}}$$
$$M = 6.42 \times 10^{23} \ kg$$

Chapter 4

Bulk Properties of Matters

SOLVED PROBLEMS

Problem: 4.1- A cube of Al of side 10 cm is subjected to a shearing force of $100N$. The top surface of cube is displaced by 0.01 cm w.r.t. bottom. Calculate shearing stress, shearing strain and modulus of rigidity.

Solution

$$\text{Length of side of cube} = l = 0.1m$$
$$\text{Area of one side of cube} = A = 0.1 \times 0.1 = 0.01m^2$$
$$\text{Tangential force} = F = 100N$$

so the shearing stress is given as

$$\text{Shearing stress} = \frac{F}{A}$$
$$\text{Shearing stress} = \frac{100}{0.01} = 10^4 Nm^{-2}$$

also,

$$\text{Displacement} = \Delta x = 0.01cm = 0.0001m$$
$$\text{Thickness} = L = 0.1m$$

so,

$$\begin{aligned}
\text{Shearing strain } &= \frac{\Delta x}{L} \\
\text{Shearing strain } &= \frac{0.0001}{0.1} = 10^{-3} \\
\text{and } \text{Modulus of rigidity } &= \frac{\text{Shearing stress}}{\text{Shearing strain}} \\
\text{Modulus of rigidity } &= \frac{10^4}{10^{-3}} \\
\text{Modulus of rigidity } &= 10^7 Nm^{-2}
\end{aligned}$$

Problem: 4.2- Find the pressure in mega pascal 118 m below the surface of ocean. The density of sea water is 1.024 gm/cm^3 and atmospheric pressure at sea level is $1.013 \times 10^5\ Nm^{-2}$.

Solution

$$\begin{aligned}
\text{height or depth } = h &= 118\ m \\
\text{density } = \rho &= 1.024\ gm\ cm^{-3} = 1024\ kgm^{-3} \\
g &= 9.8\ ms^{-2} \\
\text{atmospheric pressure } = P_\circ &= 1.013 \times 10^5\ Nm^{-2} \\
P &= ?
\end{aligned}$$

since, we know that

$$\begin{aligned}
P &= P_\circ + \rho g h \\
P &= 1.013 \times 10^5 + 1024 \times 9.8 \times 118 \\
P &= 1.013 \times 10^5 + 1.184 \times 10^6 \\
P &= 1.285 \times 10^6\ Pa \\
P &= 1.285\ MPa \qquad \because 10^6 = M
\end{aligned}$$

Problem: 4.3- Human lungs operate against a pressure difference less than 0.05 atm. How far below the water level can a diver breathe through a long tube.

Solution

$$\text{Pressure difference} = \Delta P = 0.05\ atm$$
$$\text{Pressure difference} = \Delta P = 0.05 \times 1.01 \times 10^5\ Pa$$
$$\text{Pressure difference} = \Delta P = 5.1 \times 10^3\ Pa$$
$$\text{Water density} = \rho = 1000\ kgm^{-3}$$
$$\text{Gravitational acceleration} = g = 9.8\ ms^{-2}$$
$$\text{Height} = h = ?$$

Since we know that the pressure difference is:

$$P = P_0 + \rho g h$$
$$\Delta P = P - P_0 = \rho g h$$
$$\Delta P = \rho g h$$
$$\Rightarrow h = \frac{\Delta P}{\rho g}$$
$$h = \frac{5.1 \times 10^3}{1000 \times 9.8}$$
$$h = 0.52\ m$$

Problem: 4.4- A flat plate of area 10 cm^2 is separated from a large plate by a layer of glycerine 1 mm thick. If viscosity coefficient of glycerine is 20 $gm/cm\ sec$. What force is required to keep the plate moving with velocity of 1 cm/sec ?

Solution

$$\text{Area} = A = 10\ cm^2$$
$$\text{Distance} = d = 1\ mm = 0.1\ cm$$
$$\text{Coefficient of viscosity} = \eta = 2.0\ gm/cm\ sec$$

$$\text{Velocity } = v = 1\ cm/sec$$
$$\text{Force } = F = ?$$

Since we know that the coefficient of viscosity is:

$$\eta = \frac{Fd}{vA}$$
$$\Rightarrow \quad F = \frac{\eta v A}{d}$$
$$F = \frac{2.0 \times 1 \times 10}{0.1}$$
$$F = 200\ dynes = 200 \times 10^{-5} N$$
$$F = 2 \times 10^{-3} N \qquad \because 1 dyne = 10^{-5} N$$

Problem: 4.5- A structural steel rod has a radius of 9.5 mm and a length of 81 cm. A force of $6.2 \times 10^4\ N$ stretches it axially. What is the stress on the rod? What is the elongation of the rod under this load if young's modulus is $2.0 \times 10^{11}\ Nm^{-2}$?

Solution

$$\text{Radius } = r = 9.5\ mm = 9.5 \times 10^{-3}\ m$$
$$\text{Length } = L = 81\ cm = 0.81\ m$$
$$\text{Force } = F = 6.2 \times 10^4\ N$$
$$\text{Young's modulus } = Y = 2.0 \times 10^{11}\ Nm^{-2}$$
$$\text{Stress } = ?$$
$$\text{Change in length } = \Delta L = ?$$

Since we know that

$$\text{Tensile stress } = \frac{F}{A} = \frac{F}{\pi r^2}$$
$$\text{Tensile stress } = \frac{6.2 \times 10^4}{3.14 \times (9.5 \times 10^{-3})^2}$$
$$\text{Tensile stress } = 2.2 \times 10^8\ Nm^{-2}$$

Also, Young's modulus is defined as:

$$
\begin{aligned}
Y &= \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \\
\Rightarrow\ Y &= \frac{FL}{A\Delta L} \\
\Rightarrow\ \Delta L &= \frac{FL}{AY} \\
\Rightarrow\ \Delta L &= \frac{(F/A)L}{Y} \\
\Delta L &= \frac{2.2 \times 10^{8} \times 0.81}{2.0 \times 10^{11}} \\
\Delta L &= 8.9 \times 10^{-4}\ m \\
\Delta L &= 0.89\ mm
\end{aligned}
$$

Chapter 5

Special Theory of Relativity

SOLVED PROBLEMS

Problem: 5.1- Calculate the speed of a particle whose total energy is equal to twice its rest energy.

Solution

$$
\begin{aligned}
E &= 2E_0 \\
E &= 2m_\circ c^2 \\
mc^2 &= 2m_\circ c^2 \qquad \because E = mc^2 \\
\frac{m_\circ c^2}{\sqrt{1-\frac{v^2}{c^2}}} &= 2m_\circ c^2 \qquad \because m = \frac{m_\circ}{\sqrt{1-\frac{v^2}{c^2}}} \\
\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= 2
\end{aligned}
$$

Since $\beta^2 = \frac{v^2}{c^2}$, so

$$
\begin{aligned}
\frac{1}{\sqrt{1-\beta^2}} &= 2 \\
\text{or} \quad \sqrt{1-\beta^2} &= \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
1-\beta^2 &= \frac{1}{4} \qquad \because \text{squaring on both sides}\\
\beta^2 &= 1-\frac{1}{4} = \frac{3}{4}\\
\Rightarrow \frac{v^2}{c^2} &= \frac{3}{4}\\
\Rightarrow \frac{v}{c} &= \sqrt{\frac{3}{4}}\\
\Rightarrow v &= \frac{\sqrt{3}}{2}c
\end{aligned}
$$

Problem: 5.2- A space ship of rest length $130m$ drifts past a timing station at a speed of $0.740c$. Calculate the length of the space ship as measured by the timing station.

Solution

$$
\begin{aligned}
l_\circ &= 130m\\
v &= 0.740c\\
l &= ?
\end{aligned}
$$

Using the relation of relativity of length contraction

$$
\begin{aligned}
l &= l_\circ\sqrt{1-\frac{v^2}{c^2}}\\
l &= 130\times\sqrt{1-\frac{(0.740c)^2}{c^2}}\\
l &= 130\times\sqrt{1-\frac{(0.740)^2c^2}{c^2}}\\
l &= 130\times\sqrt{1-0.5476}\\
l &= 130\times\sqrt{0.4524}\\
l &= 130\times 0.6726\\
l &= 87m
\end{aligned}
$$

Problem: 5.3- Observer S reports that an event occurred on the x-axis at $x = 3.20 \times 10^8 m$ at a time $t = 2.50s$. Observer S' is moving in the direction of increasing x at a speed of $0.380c$. What coordinates would S' report for the event?

Solution

$$
\begin{aligned}
x &= 3.20 \times 10^8 m \\
v &= 0.380c \\
t &= 2.50s \\
c &= 3 \times 10^8 \\
x' &= ? \\
t' &= ?
\end{aligned}
$$

Using the relation:

$$
\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\
x' &= \frac{3.20 \times 10^8 - 0.380 \times 3 \times 10^8 \times 2.50}{\sqrt{1 - \frac{(0.380c)^2}{c^2}}} \\
x' &= \frac{3.20 \times 10^8 - 0.380 \times 2.50 \times 3 \times 10^8}{\sqrt{1 - \frac{(0.380)^2 c^2}{c^2}}} \\
x' &= \frac{0.35 \times 10^8}{0.9249} \\
x' &= 0.3784 \times 10^8 \\
x' &= 3.78 \times 10^7 m
\end{aligned}
$$

and

$$
t' = \frac{t - v\frac{x}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}
$$

$$t' = \frac{2.50 - 0.380c\frac{3.20\times10^8}{c^2}}{\sqrt{1-\frac{(0.380c)^2}{c^2}}}$$

$$t' = \frac{2.50 - 0.380\frac{3.20\times10^8}{c}}{\sqrt{1-(0.380)^2}}$$

$$t' = \frac{2.50 - 0.380\frac{3.20\times10^8}{3\times10^8}}{0.9249}$$

$$t' = \frac{2.50 - 0.380\times 1.067}{0.9249}$$

$$t' = \frac{2.50 - 0.32}{0.9249}$$

$$t' = \frac{2.18}{0.9249}$$

$$t' = 2.36s$$

Problem: 5.4- The mean life of muons stopped in a lead black is measured $2.20\mu s$ and mean life of cosmic ray muons observed from earth is found to be $1.6\mu s$. Find the speed of cosmic ray muons.

Solution

$$t_\circ = 1.6\mu\ s = 1.6\times10^{-6}\ s$$

$$t = 2.20\mu\ s = 2.2\times10^{-6}\ s$$

$$c = 3\times10^8 ms^{-1}$$

$$v = ?$$

Since the relation for relativity of time is given as

$$t = \frac{t_\circ}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\text{or } \frac{1}{t} = \frac{\sqrt{1-\frac{v^2}{c^2}}}{t_\circ}$$

$$\frac{t_\circ}{t} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\begin{aligned}
\text{or } \frac{t_\circ^2}{t^2} &= 1 - \frac{v^2}{c^2} \\
\frac{v^2}{c^2} &= 1 - \frac{t_\circ^2}{t^2} \\
v^2 &= c^2\left(1 - \frac{t_\circ^2}{t^2}\right) \\
v &= c\sqrt{1 - \frac{t_\circ^2}{t^2}} \\
v &= 3\times 10^8\sqrt{1 - \frac{(1.6\times 10^{-6})^2}{(2.2\times 10^{-6})^2}} \\
v &= 3\times 10^8\sqrt{1 - \frac{2.56}{4.84}} \\
v &= 0.6862\times 3\times 10^8 \\
v &= 2.06\times 10^8 ms^{-1}
\end{aligned}$$

Problem: 5.5- What is momentum of proton moving with a speed of $v = 0.86c$.

Solution

Since the relation for relativistic momentum is given as

$$\begin{aligned}
P &= \gamma mv \\
P &= \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}
\end{aligned}$$

Also,

$$\begin{aligned}
\text{Mass of proton } &= m = \; 1.67\times 10^{-27} kg \\
\text{Speed of light } &= c = \; 3\times 10^8 ms^{-1} \\
v &= 0.86c
\end{aligned}$$

so,

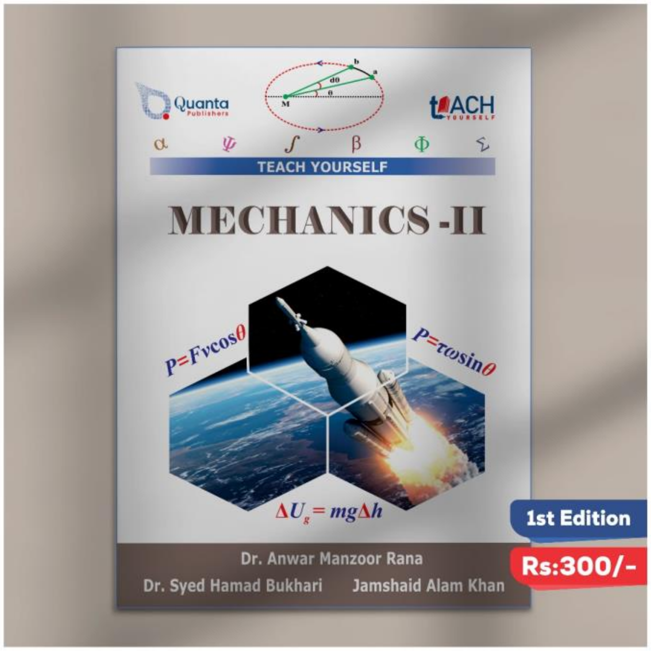
$$P = \frac{1.67\times 10^{-27}\times 0.86\times 3\times 10^8}{\sqrt{1-\frac{(0.86c)^2}{c^2}}}$$

$$P = \frac{1.67 \times 10^{-27} \times 0.86 \times 3 \times 10^{8}}{\sqrt{1 - \frac{(0.86)^2 c^2}{c^2}}}$$

$$P = \frac{1.67 \times 3 \times 0.86 \times 10^{-19}}{\sqrt{1 - 0.7396}}$$

$$P = \frac{4.3086 \times 10^{-19}}{0.5103}$$

$$P = 8.44 \times 10^{-19} kgms^{-1}$$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