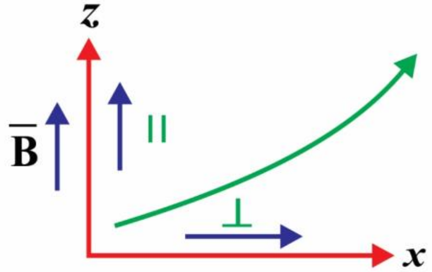
Quanta Publishers



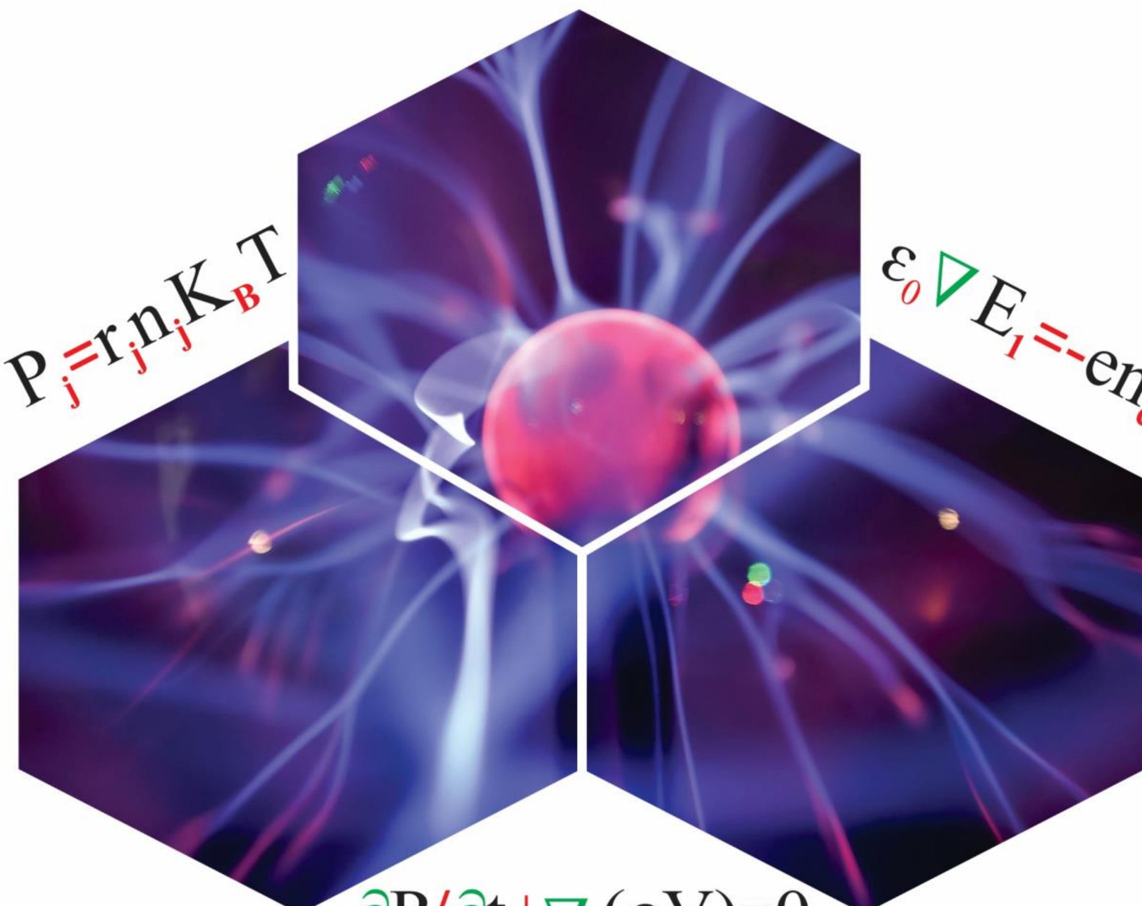
α Ψ ∫ β Φ Σ

TEACH YOURSELF

PLASMA PHYSICS

99% OF THE UNIVERSE IS IN PLASMA STATE

$P_j = r_j n_j K_B T$



$\varepsilon_0 \nabla E_1 = -en_{e1}$

$\partial P/\partial t + \nabla.(\rho V) = 0$

Shafiq Ahmad
Associate Professor
Department of Physics
G.D.C.G. Thana Malakand,
KPK.

Dr. Syed Hamad Bukhari
Assistant Professor
Department of Physics
G.C. University Faisalabad
Sub-Campus, Layyah.

TEACH YOURSELF

Plasma Physics

1st Edition

For **BS/M.Sc Physics** students of all Pakistani Universities

•

Shafiq Ahmad
Associate Professor
Department of Physics
Govt. Degree College, Thana, Malakand, KPK

&

Dr. Syed Hamad Bukhari
Assistant Professor
Department of Physics
G.C. University Faisalabad, Sub-Campus, Layyah

•

Assisted by
Qudsia Shamas
Department of Physics
G.C. University Faisalabad,
Sub-Campus, Layyah

Quanta Publisher, 2660/6C Raza Abad, Shah Shamas, Multan.

Contents

Chapter 1

Introduction to Plasma

SOLVED PROBLEMS

Problem: 1.1- Compute λ_D and N_D for the following cases: (a) A glow discharge with $n = 10^{16}\ m^{-3}$, $kT_e = 2eV$. (b) The earth ionosphere with $n = 10^{12}\ m^{-3}$, $kT_e = 0.1eV$. (c) A θ-pinch with $n = 10^{23}\ m^{-3}$, $kT_e = 800eV$.

Solution

(a)

$$\begin{aligned} n &= 10^{16}\ m^{-3} \\ kT_e &= 2eV \\ \lambda_D &= ? \\ N_D &= ? \end{aligned}$$

As, we know that

$$\begin{aligned} \lambda_D &= 7430\sqrt{\frac{kT_e}{n}} \\ \lambda_D &= 7430\sqrt{\frac{2}{10^{16}}} \\ \lambda_D &= 7430 \times 1.41 \times 10^{-8} \\ \lambda_D &= 1.04 \times 10^{-4}\ m \end{aligned}$$

and also, we know that

$$N_D = n\frac{4}{3}\pi\lambda_D^3$$

$$N_D = 10^{16}\frac{4}{3} \times 3.14 \times (1.04 \times 10^{-4})^3 \quad \Rightarrow \quad N_D = 4.7 \times 10^4$$

(b)

$$\lambda_D = 7430\sqrt{\frac{kT_e}{n}}$$

$$\lambda_D = 7430\sqrt{\frac{0.1}{10^{12}}}$$

$$\lambda_D = 7430 \times 0.32 \times 10^{-6}$$

$$\lambda_D = 2377.6 \times 10^{-6}$$

$$\lambda_D = 2.4 \times 10^{-3}\ m$$

and also, we know that

$$N_D = n\frac{4}{3}\pi\lambda_D^3$$

$$N_D = 10^{12}\frac{4}{3} \times 3.14 \times (2.4 \times 10^{-3})^3$$

$$N_D = 5.8 \times 10^4$$

(c)

$$\lambda_D = 7430\sqrt{\frac{kT_e}{n}}$$

$$\lambda_D = 7430\sqrt{\frac{800}{10^{23}}}$$

$$\lambda_D = 7430 \times 8.9 \times 10^{-11}$$

$$\lambda_D = 66127 \times 10^{-11}$$

$$\lambda_D = 6.6 \times 10^{-7}\ m$$

and also, we know that

$$N_D = n\frac{4}{3}\pi\lambda_D^3$$
$$N_D = 10^{23}\frac{4}{3} \times 3.14 \times (6.6 \times 10^{-7})^3$$
$$N_D = 1.2 \times 10^5$$

Problem: 1.2- Derive the constant A for normalized one-dimensional Maxwellion distribution

$$\hat{f}(u) = Ae^{(-\frac{mu^2}{2kT})}$$

such that $\int\limits_{-\infty}^{\infty} \hat{f}(u)du = 1$.

Solution

The one-dimensional Max-Well distribution is given by

$$f(u) = A\exp\left(\frac{-\frac{1}{2}mu^2}{kT}\right)$$

By calculating the average kinetic energy must know the value of velocity and number of particles or density n

$$n = \int_{-\infty}^{\infty} f(u)\, du$$

Now putting the value of $f(u)$, we get

$$n = \int_{-\infty}^{\infty} A\exp\left(\frac{-\frac{1}{2}mu^2}{kT}\right)du$$

$$n^2 = \left|\int_{-\infty}^{\infty} f(u)\, du\right|^2$$

$$n^2 = A^2\left|\int_{-\infty}^{\infty} e^{\frac{-\frac{1}{2}mu^2}{kT}}du\right|^2 \tag{1.1}$$

Let,

$$x = \sqrt{\frac{m}{2kT}}u$$
$$dx = \sqrt{\frac{m}{2kT}}du$$
$$du = \sqrt{\frac{2kT}{m}}dx$$

and

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$$

Now from Eq.(1.1), we get

$$n^2 = A^2 \int_{-\infty}^{\infty} \left| e^{-x^2}\sqrt{\frac{2kT}{m}}dx \right|^2$$
$$n^2 = A^2 \left(\frac{2kT}{m}\right)^{\frac{1}{2}\times 2} \int_{-\infty}^{\infty} \left| e^{-x^2}dx \right|^2$$
$$n^2 = A^2 \left(\frac{2kT}{m}\right)\left(\sqrt{\pi}\right)^2$$
$$n^2 = A^2 \left(\frac{2kT}{m}\right)\pi$$
$$A^2 = n^2 \left(\frac{m}{2\pi kT}\right)$$

$$\sqrt{A^2} = \sqrt{n^2\left(\frac{m}{2\pi kT}\right)}$$
$$A = n\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}$$

This is the normalization constant and the equation shows that particles are in one dimension.

Problem: 1.3- A distant galaxy contains a cloud of protons and antiprotons, each with density $n = 10^6\ m^{-3}$ and temperature $100\ K$. What is the Debye length.

Solution

$$n = 10^6\ m^{-3}$$
$$T = 100\ K$$
$$\lambda_D = ?$$

As, we know that

$$\lambda_D = \sqrt{\frac{8.85 \times 10^{-12} \times 1.38 \times 10^{-23} \times 100}{10^6 \times (1.6 \times 10^{-19})^2}}$$
$$\lambda_D = \sqrt{\frac{\varepsilon_\circ kT}{n_\circ e^2}}$$
$$\lambda_D = \sqrt{\frac{12.213 \times 10^{-33}}{2.56 \times 10^{-32}}}$$
$$\lambda_D = 0.69\ m$$

Problem: 1.4- In laser fusion, the core of a small pellet of DT is compressed to a density of $10^{33} m^{-3}$ at a temperature of 50000000 o^K. Estimate the number of particles in a Deybe sphere in this Plasma.

Solution

$$\lambda_D = ?$$
$$N_D = ?$$
$$T = 5 \times 10^7$$
$$n = 10^{33} m^{-3}$$
$$\lambda_D = 69(\frac{T}{n})^{\frac{1}{2}} \quad (T \text{ in the unit of } K)$$
$$\lambda_D = 69(5 \times \frac{10^7}{10^{33}})^{\frac{1}{2}}$$

$$\boxed{\lambda_D = 1.54 \times 10^{-11} m} \text{ Deybe length}$$

Naturally, the number of particles contained in a Deybe sphere is

$$N_D = \frac{4}{3}\pi\lambda_D^3 \times n$$
$$N_D = \frac{4}{3} \times (1.54)^3$$
$$N_D \approx 15$$

Problem: 1.5- Compute the pressure, in atmosphere and in Ions/ft^2 exerted by a thermonuclear Plasma on its container. Assume

$$kT_e = kT_i = 20keV$$
$$n = 10^{21} m^{-m}$$
$$p = nkT$$
$$\text{Where} \quad T = T_i + T_e$$

Solution

$$n = 10^{21} m^{-3}$$
$$P = nkT$$

$$\text{Assume} \quad kT_e = kT_i$$
$$T = T_i + T_e$$

This is just unit conversion.

$$1keV = 1\cdot 6\times 10^{-19}\text{J},\quad \text{So}$$

$$P = 10^{21}\times(20k_eV + 20k_eV) = 4\times 10^{22}m^{-3}keV$$

$$P = 4\times 10^{3}m^{-m}\text{J}$$

$$P = 4\times 10^{3}\frac{N}{m^2}$$

But $$1atm = 10^{5}\frac{N}{m^2} = 1\frac{ton}{ft^2}$$

$$P = 0.04atm = 0.04\frac{tan}{ft^2}$$

Chapter 2

Single Particle Motion

SOLVED PROBLEMS

Problem: 2.1- In the TFTR (Tokamak Fusion Test Reactor) at princetion, the plasma was heated by injection of 200 keV neutral deuterium atoms, which after entering the magnetic field, are converted to 200keV deuterium ions having $A = 2$ by charge exchange. These ions are confined only if $r_L << a$, where $a = 0.6\ m$ is the minor radius of the toroidal plasma. Compute the maximum Larmor radius in a 5 T field to see if this is satisfied.

Solution

For deuterium;

$$\begin{aligned}
\text{Atomic mass} &= A = 2 \\
\text{Mass} &= m = 2m_P \\
\text{Mass} &= m = 2 \times 1.6 \times 10^{-27} = 3.34 \times 10^{-27}\ kg \\
\text{Charge} &= q = 1.6 \times 10^{-19}\ C \\
\text{Magnetic field} &= B = 5\ T \\
\text{Energy} &= E = 200\ keV = 200 \times 10^3\ eV
\end{aligned}$$

$$\text{Energy } = E = 2 \times 10^5 \times 1.6 \times 10^{-19} \ J$$
$$\text{Larmor radius } = r_L = ?$$

Since, we know that

$$V_\perp = \left(\frac{2 \times 10^5 \times 1.6 \times 10^{-19}}{m} \right)^{1/2}$$
$$V_\perp = \left(\frac{2E}{3.34 \times 10^{-27}} \right)^{1/2}$$
$$V_\perp = \left(1.916 \times 10^{13}\right)^{1/2}$$
$$V_\perp = 0.437 \times 10^7 \ m/s$$

As, we know that

$$r_L = \frac{mV_\perp}{qB}$$
$$r_L = \frac{3.34 \times 10^{-27} \times 0.437 \times 10^7}{1.6 \times 10^{-19} \times 5}$$
$$r_L = 0.181 \times 10^{-1}$$
$$r_L = 0.018 \ m$$

As $r_L = 0.018 \ m << a = 0.6 \ m$, so the Larmor radius satisfies the confined ion condition.

Problem: 2.2- An ion engine has a 1 T magnetic field, and a hydrogen plasma is to be shot out at an $E \times B$ velocity of 1000 km/s. How much internal electric field must be present in the plasma?

Solution

Given that:

$$\text{Magnetic field } = B = 1 \ T$$
$$\text{Velocity } = v = 1000 \ km/s = 10^6 \ m/s$$
$$\text{Internal electric field } = E = ?$$

As, we know that

$$F = qvB$$

and

$$F = qE$$

On comparing the above two equations, we get

$$\begin{aligned} Eq &= qvB \\ E &= vB \\ E &= 10^6 \times 1 \\ E &= 10^6 \ V/m \end{aligned}$$

Problem: 2.3- A hydrogen plasma is heated by applying a radio-frequency wave with E perpendicular to B and with angular frequency $\omega = 10^9 \ rad/s$. The confining magnetic field is $1 \ T$. Is the motion of (a) The electrons and (b) The ions in response to this wave adiabatic?

Solution

Given data:

$$\begin{aligned} \omega &= 10^9 \ rad/s \\ B &= 1 \ T \\ \omega_e &= ? \\ \omega_i &= ? \end{aligned}$$

(a)

As, the Lamor frequency of electron is

$$\begin{aligned} \omega_e &= \frac{eB}{m_e} \\ \omega_e &= \frac{1.6 \times 10^{-19} \times 1}{9.11 \times 10^{-31}} \\ \omega_e &= 1.76 \times 10^{11} \ rad/s \end{aligned}$$

As, $\omega_e >> \omega$, so the motion of electron is adiabatic.

(b)

As, the Lamor frequency of ion is

$$\omega_i = \frac{eB}{m_i}$$
$$\omega_i = \frac{1.6 \times 10^{-19} \times 1}{1.67 \times 10^{-27}}$$
$$\omega_i = 9.8 \times 10^{7}\ rad/s$$

As, $\omega >> \omega_i$, so the motion of ion is not adiabatic.

Problem: 2.4- Derive the result $2|V_m|$ directly by using the invariance of J. (a) Let $\int V_{\|} ds \simeq V_{\|} L$ and differentiate with respect to time. (b) From this, get an expression for T in terms of $\frac{dL}{dt}$. Set $\frac{dL}{dt} = -2V_m$ to obtain the answer.

Solution

(a)

Given that

$$\int V_{\|} ds = V_{\|} L = \text{constant}$$

Now,

$$\frac{d}{dt}(V_{\|} L) = \frac{d}{dt}(constant)$$
$$V_{\|}\frac{dL}{dt} + L\frac{dV_{\|}}{dt} = 0$$
$$V_{\|}L' + LV_{\|}' = 0$$

(b)

Since,

$$V_{\|}L' + LV_{\|}' = 0$$
$$LV_{\|}' = -V_{\|}L'$$
$$\frac{V_{\|}'}{V_{\|}} = -\frac{L'}{L}$$

$$
\begin{aligned}
V_\parallel' &= -V_\parallel \frac{L'}{L} \\
\frac{\Delta V_\parallel}{T} &= \frac{V_\parallel}{L}(-L') \qquad \because V_\parallel' = \frac{dV_\parallel}{t} \\
T &= \frac{\Delta V_\parallel}{V_\parallel} \cdot \frac{L}{-L'} \\
T &= \frac{\Delta V_\parallel}{V_\parallel} \cdot \frac{L}{-\frac{dL}{dt}} \\
T &= \frac{\Delta V_\parallel}{V_\parallel} \cdot \frac{L}{-(-2V_m)} \\
T &= \frac{\Delta V_\parallel}{V_\parallel} \cdot \frac{L}{2V_m} \\
T &= \frac{2V_{\perp i} - V_{\perp i}}{\frac{1}{2}(2V_{\perp i} + V_{\perp i})} \cdot \frac{L}{2V_m} \\
T &= \frac{V_{\perp i}}{\frac{1}{2}3V_{\perp i}} \cdot \frac{L}{2V_m} \\
T &= \frac{2}{3}\frac{L}{2V_m}
\end{aligned}
$$

Since, $V_m = 10\ km/s = 10^4\ m/s, L = 10^{10}\ km = 10^{13}\ m$, so we get

$$
\begin{aligned}
T &= \frac{2}{3}\frac{10^{13}}{2 \times 10^4} \\
T &= 0.33 \times 10^9\ s \\
T &= 3.3 \times 10^8\ s
\end{aligned}
$$

Problem: 2.5- Compute r_L for the following cases if $V_\parallel$ is negligible: (a) A 10 keV electron in the earth's magnetic field of $5 \times 10^{-5}\ T$. (b) A solar wind proton with streaming velocity 300 km/s and field B is $5 \times 110^{-9}\ T$.

Solution

(a)

Given data:

$$\text{Energy of electron} = E = 10\ keV = 10 \times 10^3\ eV$$

$$
\begin{aligned}
\text{Energy of electron } &= E \ = \ 10^4 \times 1.6 \times 10^{-19} \ J \\
\text{Earth's magnetic field } &= B \ = \ 5 \times 10^{-5} \ T \\
\text{Charge on electron } &= e \ = \ 1.6 \times 10^{-19} \ C \\
\text{Mass of electron } &= m \ = \ 9.11 \times 10^{-31} \ kg \\
\text{Larmor radius } &= r_L \ = ?
\end{aligned}
$$

We know that,

$$
\begin{aligned}
r_L &= \frac{V_\perp}{\omega_c} \\
r_L &= \frac{mV_\perp}{qB} \qquad (2.1)
\end{aligned}
$$

Also,

$$
\begin{aligned}
E &= \frac{1}{2} m V_\perp^2 \\
V_\perp^2 &= \frac{2E}{m} \\
V_\perp &= \left(\frac{2E}{m}\right)^{1/2} \\
V_\perp &= \left(\frac{2 \times 10^4 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}\right)^{1/2} \\
V_\perp &= (0.3512 \times 10^{16})^{1/2} \\
V_\perp &= 0.593 \times 10^8 \\
V_\perp &= 5.93 \times 10^7 \ m/s
\end{aligned}
$$

Now, from Eq.(2.1), we get

$$
\begin{aligned}
r_L &= \frac{9.11 \times 10^{-31} \times 5.93 \times 10^7}{1.6 \times 10^{-19} \times 5 \times 10^{-5}} \\
r_L &= \frac{6.75 \times 10^{-24}}{10^{-24}} \\
r_L &= 6.75 \ m
\end{aligned}
$$

(b)

Given data:

$$V_\perp = 3300\ km/s = 300 \times 1000\ m/s$$
$$V_\perp = 3 \times 10^5\ m/s$$
$$B = 5 \times 10^{-9}\ T$$
$$m = 1.67 \times 10^{-27}\ kg$$
$$r_L = ?$$

As, we know that

$$r_L = \frac{mV_\perp}{qB}$$
$$r_L = \frac{1.67 \times 10^{-27} \times 3 \times 10^5}{1.6 \times 10^{-19} \times 5 \times 10^{-9}}$$
$$r_L = \frac{0.626 \times 10^{-22}}{10^{-28}}$$
$$r_L = 0.626 \times 10^{-22+28} = 0.626 \times 10^6$$
$$r_L = 6.26 \times 10^5\ m$$

Chapter 3

Plasma as Fluid

SOLVED PROBLEMS

Problem: 3.1- If the ion cyclotron frequency is denoted by Ω_C and the ion plasma frequency is defined by

$$\Omega_P = \left(\frac{ne^2}{\varepsilon_\circ M}\right)^{\frac{1}{2}}$$

where M is the ion mass, under what circumstances is the dielectric constant ε approximately equal to $\frac{\Omega_P^2}{\Omega_C^2}$?

Solution

As we know that

$$\varepsilon \cong 1 + \frac{nM}{\varepsilon_\circ B^2} \cong \frac{\Omega_P^2}{\Omega_C^2} \tag{3.1}$$

As,

$$\Omega_P = \left(\frac{ne^2}{\varepsilon_\circ M}\right)^{\frac{1}{2}}$$

$$\Omega_P^2 = \frac{ne^2}{\varepsilon_\circ M} \tag{3.2}$$

Also,

$$\begin{aligned}\Omega_C &= \frac{eB}{M}\\ \frac{1}{\Omega_C} &= \frac{M}{eB}\\ \frac{1}{\Omega_C^2} &= \frac{M^2}{e^2B^2}\end{aligned}\tag{3.3}$$

Put Eqs.(3.2) and (3.3) into Eq.(3.1), we get

$$\begin{aligned}\varepsilon &= \frac{ne^2}{\varepsilon_\circ M}\cdot\frac{M^2}{e^2B^2}\\ \varepsilon &= \frac{nM}{\varepsilon_\circ B^2}\end{aligned}$$

The dielectric constant ε is approximately equal to $\frac{\Omega_P^2}{\Omega_C^2}$ if $\varepsilon >> 1$.

Problem: 3.2- show that the expression for J_D on the right hand side of the equation

$$J_D = (kT_i + kT_e)\frac{B\times\nabla n}{B^2}$$

has the dimensions of current density.

Solution

$$\begin{aligned}J_D &= (kT_i + kT_e)\frac{B\times\nabla n}{B^2}\\ J_D &= (kT_i + kT_e)\frac{B\times\nabla n}{B^2}\propto\frac{kT}{e}\frac{ne}{BL}\end{aligned}\tag{3.4}$$

Since,

$$kT\propto e\phi \qquad \text{and} \qquad E\propto -\frac{\phi}{L}\Longrightarrow \phi = -EL$$

So, we get

$$\begin{aligned}kT &\propto e(-EL)\\ \frac{kT}{eL} &\propto E\end{aligned}\tag{3.5}$$

Putting the value of Eq.(3.5) into Eq.(3.4), we get

$$J_D \propto \frac{kT}{eL}\frac{ne}{B}$$
$$J_D \propto E\frac{ne}{B}$$
$$J_D \propto \frac{neE}{B}$$

As, $V_E = \frac{E}{B}$, so we get

$$J_D \propto neV_E$$

Problem: 3.3- Evaluate diamagnetic current density J_D in A/m^2 for $B = 0.4\ T$, $n_\circ = 10^{16}\ m^{-3}$, $kT_e = kT_i = 0.25\ eV, r = r_\circ = 1\ cm$.

Solution

Given data:

$$B = 0.4\ T$$
$$n_\circ = 10^{16}\ m^{-3}$$
$$kT_e = kT_i = 0.25\ eV$$
$$r = r_\circ = 1\ cm = 10^{-2}\ m$$
$$J_D = ?$$

We know that

$$J_D = n_e\left(|V_{De}| + |V_{Di}|\right) \tag{3.6}$$

where,

$$|V_{De}| = |V_{Di}| = \frac{(kT)_{eV}}{B}\frac{2r}{r_\circ^2}$$

Putting values, we get

$$|V_{De}| = |V_{Di}| = \frac{0.25 \times 2r}{0.4 \times r_\circ^2}$$
$$|V_{De}| = |V_{Di}| = 1.25 \times \frac{r}{r_\circ^2}\ ms^{-1} \tag{3.7}$$
$$|V_{De}| = |V_{Di}| = 1.25 \times \frac{r^2}{r}$$

$$|V_{De}| = |V_{Di}| = \frac{1.25}{r}$$
$$|V_{De}| = |V_{Di}| = \frac{1.25}{10^{-2}}$$
$$|V_{De}| = |V_{Di}| = 1.25 \times 10^2$$

Now, from Eq.(3.6), we get

$$J_D = n_e (2V_{De}) \varepsilon^{-1}$$
$$J_D = \frac{n_e (2V_{De})}{\varepsilon} \tag{3.8}$$

As,$n_\circ = 10^{16}, e = 1.6 \times 10^{-19}, V_{De} = 1.25 \times 10^2, \varepsilon = 2.718$, put in Eq.(3.8), we get

$$J_D = \frac{10^{16} \times 1.6 \times 10^{-19} (2 \times 1.25 \times 10^2)}{2.718}$$
$$J_D = 1.47 \times 10^{16-19+2}$$
$$J_D = 1.47 \times 10^{-1}$$
$$J_D = 0.147 \ A/m^2$$

Problem: 3.4- An isothermal plasma is confined between the planes $x = \pm a$ in a magnetic field $B = B_\circ \hat{Z}$. The density distribution is

$$n = n_\circ \left(1 - \frac{x^2}{a^2}\right)$$

Evaluate V_{De} at $x = \frac{a}{2}$, if $B = 0.2T, kT_e = 2eV$ and $a = 4cm$.

Solution

Given data:

$$B = 0.2T$$
$$kT_e = 2eV$$
$$a = 4cm = 4 \times 10^{-2} m$$
$$V_{De} = ?$$

As, we know that

$$
\begin{aligned}
V_{D_e} &= \frac{kT(eV)}{B(T)}\frac{1}{\Lambda}\ ms^{-1} \\
V_{D_e} &= \frac{2}{0.2}\frac{1}{\Lambda} \\
V_{D_e} &= 10\frac{1}{\Lambda} \\
V_{D_e} &= 10\Lambda^{-1} \qquad (3.9)
\end{aligned}
$$

As,

$$
\Lambda^{-1} = \left|\frac{n'}{n}\right| \qquad (3.10)
$$

But, given that

$$
\begin{aligned}
n &= n_\circ\left(1-\frac{x^2}{a^2}\right) \\
n &= n_\circ\left(1-\frac{\left(\frac{a}{2}\right)^2}{a^2}\right) \qquad \because x = \frac{a}{2} \\
n &= n_\circ\left(1-\frac{a^2}{4a^2}\right) \\
n &= n_\circ\left(1-\frac{1}{4}\right) \\
n &= n_\circ\left(\frac{3}{4}\right) \qquad (3.11)
\end{aligned}
$$

Also,

$$
\begin{aligned}
n &= n_\circ\left(1-\frac{x^2}{a^2}\right) \\
n &= n_\circ - n_\circ\frac{x^2}{a^2}
\end{aligned}
$$

Taking differentiation, we get

$$
\begin{aligned}
n' &= 0 - 2n_\circ\frac{x}{a^2} \\
n' &= -2n_\circ\frac{x}{a^2}
\end{aligned}
$$

$$n' = -2n_\circ \frac{\frac{a}{2}}{a^2} \qquad \because x = \frac{a}{2}$$
$$n' = -\frac{2n_\circ}{a^2}\frac{a}{2}$$
$$n' = -\frac{n_\circ}{a} \tag{3.12}$$

Put the value of Eqs.(3.11) and (3.12) in Eq.(3.10), we get

$$\Lambda^{-1} = \left|\frac{n'}{n}\right| = \left|\frac{-\frac{n_\circ}{a}}{n_\circ\left(\frac{3}{4}\right)}\right|$$
$$\Lambda^{-1} = \frac{\frac{1}{a}}{\frac{3}{4}}$$
$$\Lambda^{-1} = \frac{\frac{1}{0.04}}{\frac{3}{4}}$$
$$\Lambda^{-1} = \frac{25}{0.75}$$
$$\Lambda^{-1} = 33.3\ m^{-1}$$

Now, from Eq.(3.9), we get

$$V_D = 10 \times 33.3$$
$$V_D = 333\ ms^{-1}$$

Problem: 3.5- Calculate the plasma frequency when plasma density at 50 Km is $10^{18}\ m^{-3}$ and at 70 Km is $10^{17}\ m^{-3}$ and at 85 Km is $10^{14}\ m^{-3}$.

Solution

Given data:

Plasma density at 50 Km $= n_\circ = 10^{18}\ m^{-3}$

Plasma density at 70 Km $= n_\circ = 10^{17}\ m^{-3}$

Plasma density at 85 Km $= n_\circ = 10^{14}\ m^{-3}$

Mass of electron $= m_e = 9.11 \times 10^{-31}\ Kg$

Charge on electron $= e = 1.67 \times 10^{-27}\ C$

Plasma frequency $= f = ?$

As, we know that

$$f = \frac{\omega_P}{2\pi}$$

We also know that

$$\omega_P = \sqrt{\frac{n_\circ e^2}{m\varepsilon_\circ}}$$

$$\omega_P = e\sqrt{\frac{n_\circ}{m\varepsilon_\circ}}$$

So, we have

$$f = \frac{e\sqrt{\frac{n_\circ}{m\varepsilon_\circ}}}{2\pi}$$

$$f = \frac{e}{2\pi}\sqrt{\frac{n_\circ}{m\varepsilon_\circ}}$$

for $n_\circ = 10^{18}\ m^{-3}$, we have

$$f = \frac{1.67 \times 10^{-27}}{2 \times 3.14}\sqrt{\frac{10^{18}}{9.11 \times 10^{-31} \times 8.85 \times 10^{-12}}}$$

$$f = 8.97 \times 10^9\ Hz$$

$$f = 8.97\ GHz$$

for $n_\circ = 10^{17}\ m^{-3}$, we have

$$f = \frac{1.67 \times 10^{-27}}{2 \times 3.14}\sqrt{\frac{10^{17}}{9.11 \times 10^{-31} \times 8.85 \times 10^{-12}}}$$

$$f = 2.839 \times 10^9\ Hz$$

$$f = 2.839\ GHz$$

for $n_\circ = 10^{14}\ m^{-3}$, we have

$$f = \frac{1.67 \times 10^{-27}}{2 \times 3.14}\sqrt{\frac{10^{14}}{9.11 \times 10^{-31} \times 8.85 \times 10^{-12}}}$$

$$f = 89.79 \times 10^6\ Hz$$

$$f = 89.79\ MHz$$

Chapter 4

Plasma in Waves

SOLVED PROBLEMS

Problem: 4.1- Calculate the alfven speed in region of the magnetosphere where $B = 10^{-8}\ T, n = 10^{8}\ m^{-3}$ and $M = M_H = 1.67 \times 10^{27}\ kg$.

Solution

$$\begin{aligned} B &= 10^{-8} \\ n &= 10^{8}\ m^{-3} \\ M = M_H &= 1.67 \times 10^{-27}\ kg \\ V_A &= ? \end{aligned}$$

As

$$\begin{aligned} \rho &= n_{\circ} M \\ \rho &= 10^{8} \times 1.67 \times 10^{-27} \\ \rho &= 1.67 \times 10^{-19} \end{aligned}$$

We know that the Alfven speed is

$$V_A = \frac{B}{\sqrt{\mu_{\circ} \rho}}$$

$$
\begin{aligned}
V_A &= \frac{10^{-8}}{\sqrt{4\pi \times 10^{-7} \times 1.67 \times 10^{-19}}} \\
V_A &= \frac{10^{-8}}{\sqrt{20.98 \times 10^{-26}}} \\
V_A &= \frac{10^{-8}}{4.58 \times 10^{-13}} \\
V_A &= 0.218 \times 10^{-8+13} \\
V_A &= 0.218 \times 10^{5} \\
V_A &= 2.18 \times 10^{4}\ m/s
\end{aligned}
$$

Problem: 4.2- For electromagnetic waves, show that the index of refraction is equal to the square root of appropriate plasma dielectric constant ε.

Solution

As, we know that

$$\tilde{n} = \frac{ck}{\omega} \tag{4.1}$$

Also, from ordinary wave equation,

$$
\begin{aligned}
\omega^2 &= \omega_P^2 + c^2k^2 \\
\frac{\omega^2}{\omega^2} &= \frac{\omega_P^2}{\omega^2} + \frac{c^2k^2}{\omega^2} \\
1 &= \frac{\omega_P^2}{\omega^2} + \frac{c^2k^2}{\omega^2} \\
\frac{c^2k^2}{\omega^2} &= 1 - \frac{\omega_P^2}{\omega^2} \\
\sqrt{\frac{c^2k^2}{\omega^2}} &= \sqrt{1 - \frac{\omega_P^2}{\omega^2}} \\
\frac{ck}{\omega} &= \sqrt{1 - \frac{\omega_P^2}{\omega^2}}
\end{aligned}
$$

Now, from Eq.(4.1), we get

$$
\begin{aligned}
\tilde{n} &= \sqrt{1 - \frac{\omega_P^2}{\omega^2}} \\
\tilde{n} &= \sqrt{\varepsilon} \qquad \because \varepsilon = 1 - \frac{\omega_P^2}{\omega^2}
\end{aligned}
$$

Problem: 4.3- A hydrogen discharge in a 1 T field produces a density of $10^{16}\ m^{-3}$. (a) What is the Alfven speed V_A. (b) Suppose V_A had come out greater than c. Does this mean that Alfven waves travel faster than the speed of light?

Solution

(a)

$$\begin{aligned} B &= 1\ T \\ \rho &= 10^{16}\ m^{-3} \\ V_A &= ? \end{aligned}$$

As, we know that

$$\begin{aligned} V_A &= \frac{B}{\sqrt{\mu_\circ \rho}} \\ V_A &= \frac{1}{\sqrt{4\pi \times 10^{-7} \times 10^{16}}} \\ V_A &= 2.18 \times 10^{8}\ m/s \end{aligned}$$

(b)

The Alfven wave represents for phase velocity. And phase velocity did not carry information. So, it does not mean that wave can travel faster than light.

Problem: 4.4- Electron plasma waves are propagated in a uniform plasma with $kT_e = 100\ eV, n = 10^{16}\ m^{-3}$ and $B = 0$. If the frequency f is 1.1 GHz, what is the wavelength in cm?

Solution

$$\begin{aligned} kT_e &= 100\ eV \\ n &= 10^{16}\ m^{-3} \\ B &= 0 \\ f &= 1.1\ GHz = 1.1 \times 10^{9}\ Hz \\ \lambda &= ? \end{aligned}$$

From the dispersion relation of electron plasma wave, we have

$$\begin{aligned}
\omega^2 &= \omega_P^2 + \frac{3}{2}k^2 V_{\text{th}}^2 \\
\omega^2 &= \omega_P^2 + \frac{3}{2}k^2 \times \frac{2kTe}{m} \qquad \because V_{\text{th}}^2 = \frac{2kTe}{m} \\
\omega^2 &= \omega_P^2 + \frac{3kTe}{m}k^2 \\
\omega^2 - \omega_P^2 &= \frac{3kTe}{m}k^2 \\
k^2 &= \frac{\omega^2 - \omega_P^2}{\frac{3kTe}{m}} \qquad (4.2)
\end{aligned}$$

As, we know that

$$\begin{aligned}
\omega_P &= 2\pi\sqrt{n} \\
\omega_P &= 6.28 \times \sqrt{10^{16}} \\
\omega_P &= 6.28 \times 10^8 \ rad/sec
\end{aligned}$$

Also,

$$\begin{aligned}
\omega &= 2\pi f = 2 \times 3.14 \times 1.1 \times 10^9 \\
\omega &= 6.908 \times 10^9 \ rad/sec
\end{aligned}$$

Now,

$$\begin{aligned}
\frac{3kTe}{m} &= \frac{3 \times 100 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} \\
\frac{3kTe}{m} &= 52.68 \times 10^{12} \\
\frac{3kTe}{m} &= 5.27 \times 10^{13}
\end{aligned}$$

Putting the values in Eq.(4.2), we get

$$k^2 = \frac{(6.908 \times 10^9)^2 - (6.28 \times 10^8)^2}{5.27 \times 10^{13}}$$
$$k^2 = \frac{16.14}{5.27} \times 10^5$$
$$\sqrt{k^2} = \sqrt{\frac{16.14}{5.27} \times 10^5}$$
$$k = 553.17$$

As,

$$\lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{553.17}$$
$$\lambda = \frac{6.2831}{553.17}$$
$$\lambda = 1.13\ cm$$

Problem: 4.5- By writing the linearized Poisson equation used in the derivation of simple plasma oscillations in the form

$$\nabla \cdot (\varepsilon E) = 0$$

Derive an expression for the dielectric constant ε applicable to high frequency longitudinal motions.

Solution

From Gauss's law

$$\nabla E = \frac{\rho}{\varepsilon_\circ}$$

$$\nabla E_i = \frac{-en_1}{\varepsilon_\circ}$$
$$ikE_i = \frac{-en_1}{\varepsilon_\circ} \qquad \because \nabla = ik \tag{4.3}$$

From the equation of continuity, we get

$$\frac{\partial n}{\partial t} + \nabla n V = 0$$

$$\frac{\partial n_1}{\partial t} + \nabla n_\circ V_1 = 0$$

$$-i\omega n_1 + ikn_\circ V_1 = 0$$

$$-i\omega n_1 = -ikn_\circ V_1$$

$$n_1 = \frac{kn_\circ V_1}{\omega} \tag{4.4}$$

Now, from equation of motion, we get

$$mn\left(\frac{\partial V}{\partial t} + (V\cdot\nabla)V\right) = qn\left(E + V\times B\right) - \nabla\rho_e$$

$$mn\left(\frac{\partial V}{\partial t} + 0\right) = qn\left(E + 0\right) - 0$$

$$mn\frac{\partial V_1}{\partial t} = qnE_1$$

$$m\frac{\partial V_1}{\partial t} = qE_1$$

$$-i\omega m V_1 = -eE_1 \quad \because q = -e$$

$$V_1 = \frac{eE_1}{i\omega m}$$

$$V_1 = \frac{ieE_1}{i^2\omega m}$$

$$V_1 = -\frac{ieE_1}{\omega m} \tag{4.5}$$

Put the value of Eq.(4.5) in Eq.(4.4), we get

$$n_1 = \frac{kn_\circ\left(-\frac{ieE_1}{\omega m}\right)}{\omega}$$

$$n_1 = \frac{kn_\circ}{\omega}\cdot\frac{-ieE_1}{\omega m}$$

$$n_1 = \frac{-ekn_{\circ iE_1}}{m\omega^2} \tag{4.6}$$

Put the value of Eq.(4.6) in Eq.(4.3), we get

$$ikE_1 = \frac{-e}{\varepsilon_\circ} \cdot \frac{-ekn_{\circ iE_1}}{m\omega^2}$$

$$ikE_1 = \frac{e^2kn_{\circ iE_1}}{\varepsilon_\circ m\omega^2}$$

$$ikE_1 - \frac{e^2kn_{\circ iE_1}}{\varepsilon_\circ m\omega^2} = 0$$

$$ik\left(1 - \frac{n_\circ e^2}{\varepsilon_\circ m\omega^2}\right)E_1 = 0$$

$$ik\left(1 - \frac{\omega_P^2}{\omega^2}\right)E_1 = 0 \qquad \because \omega_P^2 = \frac{n_\circ e^2}{\varepsilon_\circ m}$$

$$\nabla \cdot (\varepsilon E) = 0 \qquad \because \varepsilon = 1 - \frac{\omega_P^2}{\omega^2}$$

Chapter 5

Plasma Confinement

SOLVED PROBLEMS

Problem: 5.1- Find the closest approach of a $2MeV$ proton to a gold nucleus. How does this distance compare with those for a deuteron and alpha particle of the same energy ?

Solution

The distance of closes approaches r is that distance from the nucleus at which the total energy of incident particle is potential and is given by

$$\frac{1}{2}Mv^2 = \frac{Zze^2}{4\pi\epsilon_0 r}$$

$$E = 2MeV = \ 2 \times 1.6 \times 10^{-13} joule$$

$$R = \frac{Zze^2}{4}\pi\epsilon_0 r$$

$$ER = 5.688 \times 10^{-14} meter$$

This distance is same for deuteron of the same energy as charge ze on the deuteron is same as that od proton. Since the charge on the alpha particle is double that of one the proton. Hence $r = 1.376 \times 10^{-14}m$.

Problem: 5.2- If the energy of alpha particle emitted by Am^{241} is $5.48MeV$, find the closest distance it can approach to a Au nucleus.

Solution

The distance of closest approach is given by $D = 2Ze^2/4\pi\in_{\circ}E$. As

$$\begin{aligned} E &= 5.48\text{MeV} \\ &= 5.48 \times 1.6 \times 10^{-13} \end{aligned}$$

$$D = \frac{9 \times 10^9 \times 2 \times 79 \times \left(1.6 \times 10^{-19}\right)^2}{5.48 \times 1.6 \times 10^{-13}}$$
$$D = 4.14 \times 10^{-14} m$$

Problem: 5.3- Consider a gas of atoms undergoing fusion. Calculate the temperature required to overcome the Coulomb barrier and the released if the gas consists of

1. 10B
2. 24Mg.

Solution

(1) Let us estimate the height of the Coulomb barrier. It is given by the relation

$$V_{coul} = \frac{Z_1 Z_2 e^2}{r}$$

Here, r is the separation between two nuclei at the point of closest approach. It is given by the sum of radii of two ^{10}B nuclei. The radius of each of the nucleus can be estimated using $r = 1.2A^{\frac{1}{3}}f$. Therefore,

$$r = 1.2 \times 10^{\frac{1}{3}} + 1.2 \times 10^{\frac{1}{3}} = 5.17f$$

Coulomb barrier can be written

$$\begin{aligned} V_{coul} &= \frac{Z_1 Z_2 \times \hbar c}{r} \times \frac{e^2}{\hbar c} \\ &= \frac{e^2}{\hbar c} = \frac{1}{137} = a \end{aligned}$$

Substituting various values, we get

$$V_{coul} = \frac{1}{137} \times \frac{5 \times 5 \times 197.5 MeV\ f}{5.17\ f}$$
$$= 6.97\ MeV$$
$$= 1.12 \times 10^{-12} J$$

In order to calculate the temperature required to overcome the Coulomb barrier, we equate this energy to thermal energy as

$$\frac{3}{2} KT = E = \ V_{coul}$$

where K is Boltzmann's constzant and T is absolute temperature. or

$$\frac{3}{2} \times 1.38 \times 10^{-23} T = 1.12 \times 10^{-12}$$
$$T = 5.4 \times 10^{10} K$$

Similar calculations are performed for the case of ^{24}Mg fusing with ^{24}Mg as under.

(2) Let us estimate the height of the Coulomb barrier. It is given by the relation

$$V_{coul} = \frac{Z_1 Z_2 e^2}{r}$$

Here, r is the separation between two nuclei at the point of closest approach. It is given by the sum of radii of two ^{24}Mg nuclei. The radius of each of the nucleus can be estimated using $r = 1.2 A^{\frac{1}{3}} f$. Therefore,

$$r = 1.2 \times 24 \frac{1}{3} + 1.2 \times 24^{\frac{1}{3}} = 6.92 f$$

Coulomb barrier can be written

$$V_{coul} = \frac{Z_1 Z_2 \times \hbar c}{r} \times \frac{e^2}{\hbar c}$$
$$= \frac{e^2}{\hbar c} = \ \frac{1}{137} = a$$

Substituting various values, we get

$$V_{coul} = \frac{1}{137} \times \frac{12 \times 12 \times 197.5 MeV\ f}{6.92\ f}$$
$$= 29.99\ MeV$$
$$= 4.80 \times 10^{-12} J$$

In order to calculate the temperature required to overcome the Coulomb barrier, we equate this energy to thermal energy as

$$\frac{3}{2} KT = E = V_{coul}$$

where K is Boltzmann's consztant and T is absolute temperature. or

$$\frac{3}{2} \times 1.38 \times 10^{-23} T = 4.80 \times 10^{-12}$$
$$T = 23.2 \times 10^{10} K$$

Problem: 5.4- Calculate the mass defect and Q-values for the fusion

$$d + d \longrightarrow {}^3He + n$$
$$d + d \longrightarrow {}^3H + p$$

Assuming these occur with the deuterons at rest, find the kinetic energies of the outgoing particles in each case. Given

$$m_p = 1.007825\ amu$$
$$m_n = 1.008665\ amu$$
$$m(^2H) = 2.014102\ amu$$
$$m(^3H) = 3.016049\ amu$$
$$m(^3He) = 3.016029\ amu$$

Solution

We have

$$d + d \longrightarrow {}^3He + n$$

Mass defect is given by the relation

$$Mass\ defect = 2 \times m_d - m(^3He) - m_n$$

Substituting various given masses, we get

$$\begin{aligned} Mass\ defect &= 2 \times 2.014102 - 3.016029 - 1.008665 \\ &= 0.00351\ amu \end{aligned}$$

and

$$\begin{aligned} Q - value &= mass\ defect(amu) \times 931.47\ Mev \\ &= 0.00351 \times 931.47\ MeV \\ &= 3.27\ MeV \end{aligned}$$

$$d + d \longrightarrow {}^3H + p$$

Mass defect is given by the relation

$$Mass\ defect = 2 \times m_d - m(^3H) - m_p$$

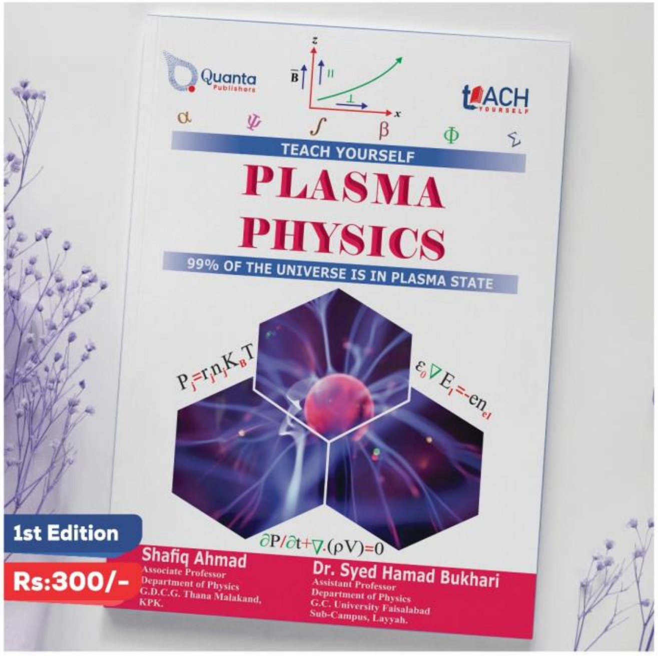
Substituting various given masses, we get

$$\begin{aligned} Mass\ defect &= 2 \times 2.014102 - 3.016049 - 1.007825 \\ &= 0.00433\ amu \end{aligned}$$

and

$$\begin{aligned} Q - value &= mass\ defect(amu) \times 931.47\ Mev \\ &= 0.00433 \times 931.47\ MeV \\ &= 4.03\ MeV \end{aligned}$$

4.03 Mev Assuming the initial state deuterons are essentially at rest then the final state kinetic energy is equal to Q. By applying the conservation of momentum it can be seen that the share of the kinetic energy that each particle has is inversely proportional to its mass. Thus, for these reactions, the heavier particle takes one quarter while the lighter particle takes three quarters of the total kinetic energy (Q)


Quanta Publishers
TEACH YOURSELF
TEACH YOURSELF
PLASMA PHYSICS
99% OF THE UNIVERSE IS IN PLASMA STATE
1st Edition
Rs:300/-
Shafiq Ahmad
Dr. Syed Hamad Bukhari