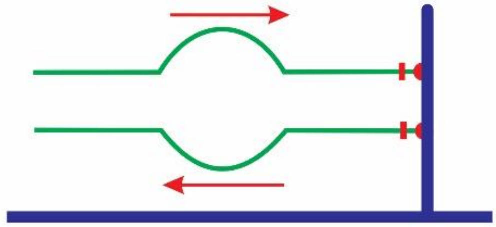

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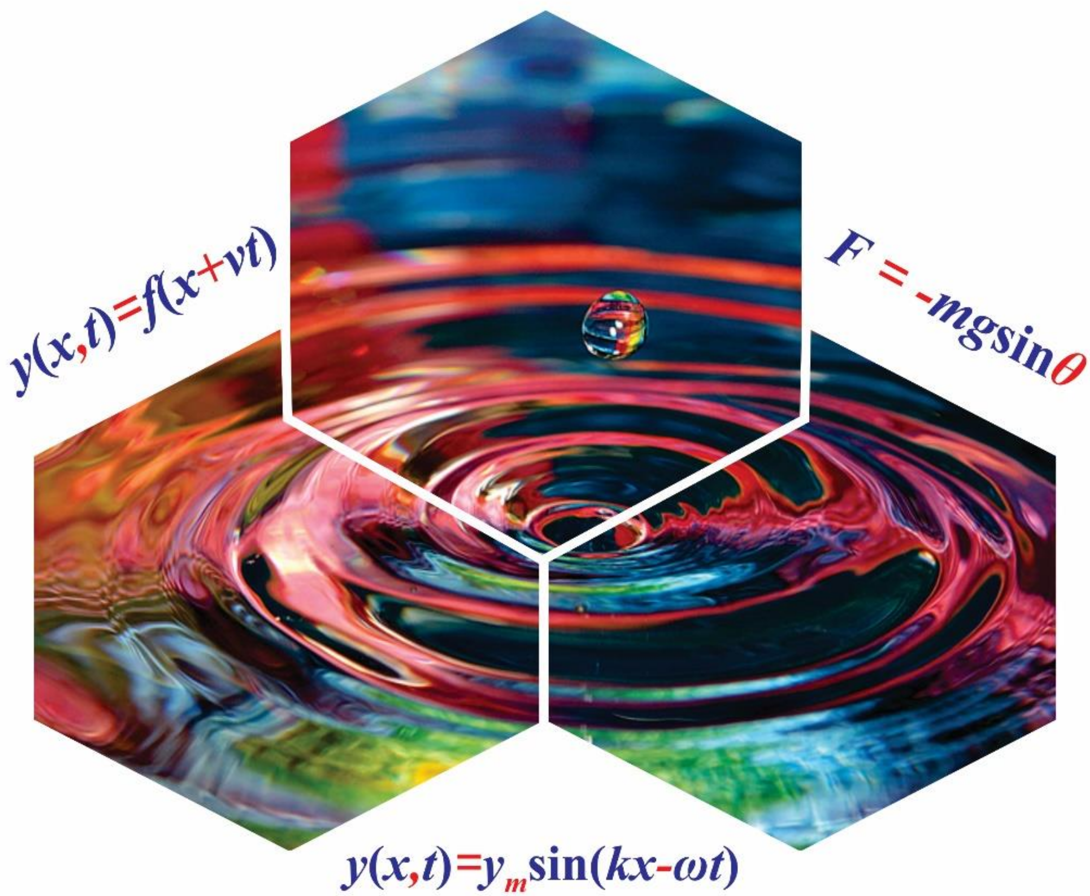
TEACH YOURSELF

WAVES AND OSCILLATIONS



Asim Javed siddique

Dr. Syed Hamad Bukhari Hammad Abbas

TEACH YOURSELF

WAVES & OSCILLATIONS

2nd Edition

For **BS Physics** students of all Pakistani Universities/Colleges

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Asim Javed Saddique
Department of Physics
Bahauddin Zakariya University, Multan

&

Dr. Syed Hamad Bukhari
Department of Physics
G.C. University Faisalabad, Sub-Campus, Layyah

&

Hammad Abbas
Department of Physics
Emerson College, Multan

•

Assisted by
Fahmeeda Shaheen
Department of Physics
G.C. University Faisalabad, Sub-Campus, Layyah

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Contents

Chapter 1

Harmonic Motion

SOLVED PROBLEMS

Problem: 1.1- The period of a disk of radius 10.2 cm executing small oscillations about a pivot at its rim is measured to be 0.784 s. Find the value of g, the acceleration duet to gravity at that location.

Solution

$$
\begin{aligned}
R &= 10.2\ cm = \frac{10.2}{100}\ m \\
R &= 0.102\ m \\
T &= 0.784\ s \\
g &= ?
\end{aligned}
$$

Since, we know that

$$
\begin{aligned}
T &= 2\pi\sqrt{\frac{3R}{2g}} \\
T^2 &= 4\pi^2\frac{3R}{2g}
\end{aligned}
$$

$$
\begin{aligned}
g &= \frac{6\pi^2 R}{T^2} \\
g &= \frac{6 \times (3.14)^2 \times 0.102}{(0.784)^2} \\
g &= \frac{6 \times 9.8596 \times 0.102}{0.6147} \\
g &= \frac{6.0337}{0.6147} \\
g &= 9.815\ ms^{-2}
\end{aligned}
$$

Problem: 1.2- An oscillation block-spring system has a mechanical energy of 1.18 J, amplitude of 9.84 cm, and a maximum speed of 1.22 ms^{-1}. Find the force constant of the spring, mass of the block and the frequency of oscillation.

Solution

$$
\begin{aligned}
E &= 1.18\ J \\
x_m &= 9.84\ cm\ = \frac{9.84}{100}\ m \\
x_m &= 0.0984\ m \\
v_{\text{max.}} &= 11.22\ ms^{-1} \\
k &= ? \\
m &= ? \\
f &= ?
\end{aligned}
$$

Since, the total mechanical energy of the mass-spring system is

$$
\begin{aligned}
E &= \frac{1}{2} k x_m^2 \\
k &= \frac{2E}{x_m^2} \\
k &= \frac{2 \times 1.18}{(0.0984)^2} \\
k &= \frac{2.36}{0.009682} \\
k &= 243.75\ Nm^{-1}
\end{aligned}
$$

The maximum velocity is given by

$$\begin{aligned}
v_{\text{max.}} &= \sqrt{\frac{k}{m}} x_m \\
v_{\text{max.}}^2 &= \frac{k}{m} x_m^2 \\
m &= \frac{k}{v_{\text{max.}}^2} x_m^2 \\
m &= \frac{243.75}{(1.22)^2} (0.0984)^2 \\
m &= 1.61\ kg
\end{aligned}$$

The frequency can be determine from:

$$\begin{aligned}
f &= \frac{1}{2\pi} \sqrt{\frac{k}{m}} \\
f &= \frac{1}{2 \times 3.14} \sqrt{\frac{243.75}{1.6}} \\
f &= 1.96\ Hz
\end{aligned}$$

Problem: 1.3- A 0.05 kg mass is attached to the bottom of a vertical spring and set vibration. If maximum speed of mass is 0.15 ms^{-1} and the period is 0.5 s. Find spring constant of spring, amplitude of motion and frequency of oscillation.

Solution

$$\begin{aligned}
m &= 0.05\ kg \\
T &= 0.5\ s \\
v_{\text{max.}} &= 0.15\ ms^{-1} \\
k &= ? \;\; ; \;\; x_m = ? \;\; ; \;\; f = ?
\end{aligned}$$

Using the relation

$$\begin{aligned}
T &= 2\pi\sqrt{\frac{m}{k}} \\
T^2 &= 4\pi^2\frac{m}{k} \\
k &= \frac{4\pi^2 m}{T^2} \\
k &= \frac{4\times(3.14)^2\times 0.05}{(0.5)^2} \\
k &= 7.9\ Nm^{-1}
\end{aligned}$$

Also,

$$\begin{aligned}
v_{\text{max.}} &= \omega x_m \\
x_m &= \frac{v_{\text{max.}}}{\omega} \\
x_m &= \frac{v_{\text{max.}}}{\frac{2\pi}{T}} \\
x_m &= \frac{v_{\text{max.}}\times T}{2\pi} \\
x_m &= \frac{0.15\times 0.5}{2\times 3.14} \\
x_m &= 0.012\ m
\end{aligned}$$

Also, frequency is the reciprocal of time period, so

$$f = \frac{1}{T} = \frac{1}{0.5} = 2\ Hz$$

Problem: 1.4- A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through the stick a distance x from the 0.5 m mark. The period of oscillation is obtained to be 2.5 s. Find x.

Solution

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$T = 2\pi\sqrt{\frac{\frac{mL^2}{12} + mx^2}{mgx}}$$

$$T^2 = 4\pi^2\frac{\frac{mL^2}{12} + mx^2}{mgx}$$

$$mgx \times T^2 = 4\pi^2\frac{mL^2}{12} + 4\pi^2 mx^2$$

$$gx \times T^2 = \frac{\pi^2 L^2}{3} + 4\pi^2 x^2$$

$$4\pi^2 x^2 - g \times T^2 x + \frac{\pi^2 L^2}{3} = 0$$

Now, using the quadratic formula, which is $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get

$$x = \frac{-(-gT^2) \pm \sqrt{(gT^2)^2 - 4(4\pi^2) \times \frac{\pi^2 L^2}{3}}}{2 \times 4\pi^2}$$

$$x = \frac{gT^2}{8\pi^2} - \frac{\sqrt{gT^4 - \frac{16\pi^4 L^2}{3}}}{8\pi^2}$$

$$x = \frac{9.8 \times (2.5)^2}{8(3.14)^2} - \frac{\sqrt{9.8 \times (2.5)^4 - \frac{16\times(3.14)^4(1)^2}{3}}}{8 \times (3.14)^2}$$

$$x = \frac{61.25 - \sqrt{135.66}}{78.88}$$

$$x = \frac{49.6}{78.88} = 0.628\ m$$

Problem: 1.5- A linear oscillator consisting of mass m fastened to a spring of spring constant k oscillates on a frictionless surface at $t = 0$ its displacement $x_\circ$ is $-8.50\ cm$ and velocity $v_\circ$ is $-0.92\ ms^{-1}$ its acceleration $a_\circ$ is $47.0\ ms^{-2}$. What is the angular frequency ω and frequency f of the system. What is the phase constant.

Solution

$$x_\circ = -8.50\ cm = -0.085\ m$$

$$v_\circ = -0.92\ ms^{-1}$$

$$
\begin{aligned}
a_\circ &= 47.0\ ms^{-2} \\
\omega &= ? \\
f &= ? \\
\phi &= ?
\end{aligned}
$$

Since, we know that

$$
\begin{aligned}
a_\circ &= -x_\circ\omega^2 \\
\omega^2 &= -\frac{a_\circ}{x_\circ} \\
\omega &= \sqrt{-\frac{a_\circ}{x_\circ}} \\
\omega &= \sqrt{\frac{-47}{-0.085}} \\
\omega &= 23.5 \text{ rad } s^{-1}
\end{aligned}
$$

Also, we know that

$$
\begin{aligned}
\omega &= 2\pi f \\
f &= \frac{\omega}{2\pi} \\
f &= \frac{23.5}{2\times 3.14} \\
f &= 3.74\ Hz
\end{aligned}
$$

And,

$$
\begin{aligned}
\tan\phi &= \frac{v_\circ}{\omega x_\circ} \\
\phi &= \tan^{-1}\left(\frac{v_\circ}{\omega x_\circ}\right) \\
\phi &= \tan^{-1}\left(\frac{-0.92}{233.5\times(-0.085)}\right) \\
\phi &= \tan^{-1}(0.461) \\
\phi &= 180 - 25 = 155^\circ
\end{aligned}
$$

Chapter 2

Wave in Physical Media

SOLVED PROBLEMS

Problem: 2.1- A string 2.7 m long has a mass of 260 g. The tension in the string is 36 N. What must be the frequency of traveling waves of amplitude 7.7 mm in order that the average transmitted power be 85 W.

Solution

$$\begin{aligned}
\text{Length of a string } &= L = 2.7\ m \\
\text{Mass of a string } &= m = 260\ g = \frac{260}{1000}\ Kg \\
\text{Mass of a string } &= m = 0.26\ kg \\
\text{Tension in the string } &= F = 36\ N \\
\text{Average power } &= P = 85\ W \\
\text{Max. displacement } &= y_m = 7.7\ mm = 7.7 \times 10^{-3}\ m \\
\text{Frequency of traveling wave } &= f = ?
\end{aligned}$$

Since, we know that

$$P = 2\pi^2 \sqrt{F\mu} f^2 y_m^2$$

$$
\begin{aligned}
f^2 &= \frac{P}{2\pi^2\sqrt{F\mu}y_m^2} \\
f^2 &= \frac{P}{2\pi^2\sqrt{F\left(\frac{m}{L}\right)}y_m^2} \\
f &= \sqrt{\frac{P}{2\pi^2\sqrt{F\left(\frac{m}{L}\right)}y_m^2}} \\
f &= \sqrt{\frac{85}{2(3.14)^2\sqrt{36\left(\frac{0.26}{2.7}\right)}\left(7.7\times10^{-3}\right)^2}} \\
f &= \sqrt{0.0438\times10^6} \\
f &= 0.209\times10^3\ Hz \\
f &= 209\ Hz
\end{aligned}
$$

Problem: 2.2- The speed of a wave on a string is 172 ms^{-1} when the tension is 123 N. To what value must the tension be increased in order to raise the wave speed to 180 ms^{-1}.

Solution

First, we have to determine the linear mass density, so we write the speed of a wave in terms of tension and linear mass density as

$$
\begin{aligned}
v &= \sqrt{\frac{F}{\mu}} \\
\mu &= \frac{F}{v^2} \\
\mu &= \frac{123}{(172)^2} \\
\mu &= 4.16\times10^{-3}\ kgm^{-1}
\end{aligned}
$$

Now, we are given that to determine the tension in the string and we are given the speed and we calculate μ above, so we use

$$
\begin{aligned}
v &= \sqrt{\frac{F}{\mu}} \\
v^2 &= \frac{F}{\mu} \\
F &= \mu v^2 \\
F &= 4.16 \times 10^{-3} \times (180)^2 \\
F &= 135\ N
\end{aligned}
$$

Problem: 2.3- The equation of a transverse wave on a string is

$$y = 2.0\ mm \sin\left(20\ m^{-1}x - (600\ s^{-1})\right) t$$

The tension in the string is 15 N. Find the linear density of this string in $g\ m^{-1}$.

Solution

Given equation is

$$y = 2.0\ mm \sin\left(20\ m^{-1}x - (600\ s^{-1})\right) t$$

and the general equation is

$$y = y_m \sin(kx - \omega t)$$

By comparing the above two equations, we get

$$
\begin{aligned}
y_m &= 2.0\ mm = 2 \times 10^{-3}\ m \\
k &= 20\ m^{-1} \\
\omega &= 600\ s^{-1}
\end{aligned}
$$

$$\text{Since,} \quad v = \frac{\omega}{k} \quad \Rightarrow \quad v = \frac{600}{20} \quad \Rightarrow \quad v = 30\ ms^{-1}$$

Also,

$$
\begin{aligned}
v &= \sqrt{\frac{F}{\mu}} \\
v^2 &= \frac{F}{\mu} \\
\mu &= \frac{F}{v^2} \\
\mu &= \frac{15}{(30)^2} = \frac{15}{900} \\
\mu &= \frac{1}{60} = 0.016\ kgm^{-1} \\
\mu &= 0.016 \times 1000\ gm^{-1} \\
\mu &= 16\ gm^{-1}
\end{aligned}
$$

Problem: 2.4- A wave of frequency of 493 Hz has a speed of 353 ms^{-1}. How far apart are the two points differing in phase by 35°. Find the difference in phase between two displacement at the same point but at times differing by 1.12 ms.

Solution

$$
\begin{aligned}
f &= 493\ Hz \\
v &= 353\ ms^{-1} \\
\theta &= 35^\circ \\
\lambda &= ? \quad \text{and} \quad \phi = ?
\end{aligned}
$$

Since, we know that

$$
\begin{aligned}
v &= f\lambda \\
\lambda &= \frac{v}{f} \\
\lambda &= \frac{353}{493} = 0.72\ m
\end{aligned}
$$

Also,

$$
d = \frac{\lambda}{2\pi}\theta
$$

$$d = \frac{0.72}{2 \times 3.14} \times 35^\circ$$
$$d = 0.07\ m\ =\ 7\ cm$$

And, the time period and frequency are reciprocal of each other so,

$$T = \frac{1}{f}$$
$$T = \frac{1}{493}$$
$$T = 2.03\ ms$$
$$\phi = \frac{0.72}{2.03} \times 1.12$$
$$\phi = 0.34\ m$$

Problem: 2.5- What are the three lowest frequencies for standing waves on a wire 10 m long having a mass of 100 g, which is stretched under a tension of 250 N.

Solution

$$\text{Length of a string} = L\ =\ 10\ m$$
$$\text{Mass of a string} = m\ =\ 100\ g\ =\ \frac{100}{1000}\ Kg$$
$$\text{Mass of a string} = m\ =\ 0.1\ kg$$
$$\text{Tension in the string} = F\ =\ 250\ N$$
$$f_1 = ?$$
$$f_2 = ?$$
$$f_3 = ?$$

Since, we know that

$$f_n = n\frac{v}{2L}$$

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$$
$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\frac{m}{L}}}$$
$$f_n = \frac{n}{2L}\sqrt{\frac{F \times L}{m}}$$

Apply, $v = f_1\lambda_1 \Longrightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$. So,

$$f_1 = \frac{1}{2L}\sqrt{\frac{F \times L}{m}}$$
$$f_1 = \frac{1}{2 \times 10}\sqrt{\frac{250 \times 10}{0.1}}$$
$$f_1 = 8\ Hz$$

For, $f_2 = 2f_1$. So,

$$f_2 = 2\frac{1}{2L}\sqrt{\frac{F \times L}{m}}$$
$$f_2 = \frac{1}{L}\sqrt{\frac{F \times L}{m}}$$
$$f_2 = \frac{1}{10}\sqrt{\frac{250 \times 10}{0.1}}$$
$$f_2 = 116\ Hz$$

For, $f_3 = 3f_1$.

$$f_3 = 3f_1$$
$$f_3 = 3 \times 8$$
$$f_3 = 24\ Hz$$

Chapter 3

Beats and Polarization

SOLVED PROBLEMS

Problem: 3.1- An ambulance emitting a whistle at 1600 Hz overtakes and passes a cyclist pedaling a bike at 340 ms^{-1}. After being passed the cyclist hears a frequency of 1590 Hz. How fast the ambulance moving.

Solution

$$\begin{aligned} f &= 1600\ Hz \\ f' &= 1590\ Hz \\ v_s &= ? \end{aligned}$$

As, source moving away from the listener, therefore frequency of the source decrease. So, by using formula

$$\begin{aligned} f' &= \left(\frac{v}{v+v_s}\right) f \\ 1590 &= \left(\frac{340}{340+v_s}\right) 1600 \\ \frac{1590}{1600} &= \frac{340}{340+v_s} \end{aligned}$$

$$\begin{aligned}
\frac{1600}{1590} &= \frac{340 + v_s}{340} \\
1.006 \times 340 &= v_s + 340 \\
342.13 &= v_s + 340 \\
v_s &= 342.13 - 340 \\
v_s &= 2.14\ ms^{-1}
\end{aligned}$$

Problem: 3.2- The 15.8 kHz whine of the turbines in the jet engines of an aircraft moving with speed 193 ms^{-1} is heard at what frequency by the pilot of a second craft trying to overtake the first at a speed of 246 ms^{-1}

Solution

$$\begin{aligned}
f' &= 15.8\ kHz\ =\ 15.8 \times 10^3\ Hz \\
v &= 343\ ms^{-1} \\
v_\circ &= 246\ ms^{-1} \\
v_s &= 193\ ms^{-1} \\
f &= ?
\end{aligned}$$

If both the source and observer move through the transmitting medium, then we use the relation to determine the frequency heard by the observer

$$f\ =\ \left(\frac{v \pm v_\circ}{v \mp v_s}\right) f'$$

Here the observer is moving towards the source (with a speed $v_\circ$) and the source is moving away from the observer (with a speed v_s), so we use the positive sign with the speed of observer (because the observer is moving towards the source) and also a positive sign with the speed of source (because the source is moving away from the observer), that is

$$f = \left(\frac{v + v_\circ}{v + v_s}\right) f'$$
$$f = \left(\frac{343 + 246}{343 + 193}\right) 15.8 \times 10^3$$
$$f = 17.4 \times 10^3$$
$$f = 17.4\ kHz$$

So, the pilot will hear a frequency of 17.4 kHz.

Problem: 3.3- Diagnostic ultrasound of frequency 4.5 MHz is used to examine tumors in soft tissues. What is the wavelength in air of such sound wave. If the speed of sound in tissue is 1500 ms^{-1}, what is the wavelength of this wave is tissue.

Solution

$$f = 4.5\ MHz = 4.5 \times 10^6\ Hz$$
$$\lambda = ?$$
$$v = 340\ ms^{-1}$$
$$v' = 1500\ ms^{-1}$$
$$\lambda' = ?$$

Since, we know that

$$\lambda = \frac{v}{f}$$
$$\lambda = \frac{340}{4.5 \times 10^6}$$
$$\lambda = 75.56 \times 10^{-6}\ m = 75.56\ \mu m$$

Also,

$$\lambda' = \frac{v'}{f}$$
$$\lambda' = \frac{1500}{4.5 \times 10^6}$$
$$\lambda' = 3.33 \times 10^{-4}\ m$$

Problem: 3.4- An acoustic burglar alarm consists of a source emitting waves of frequency 280 kHz. What will be the beat frequency of waves reflected from an intruder walking at an average speed of 0.950 ms^{-1} directly away from the alarm.

Solution

$$
\begin{aligned}
f &= 280\ kHz\ =\ 28000\ Hz \\
v &= 340\ ms^{-1} \\
v_s &= 0.950\ ms^{-1} \\
f' &= ?
\end{aligned}
$$

Since, we know that

$$
\begin{aligned}
f' &= \left(\frac{v}{v+v_s}\right) f \\
f' &= \left(\frac{340}{340+0.950}\right) \times 28000 \\
f' &= 27921.98\ Hz
\end{aligned}
$$

Problem: 3.5- A tuning fork of unknown frequency makes three beats per second with a standard of frequency 384 Hz. The beat frequency decreases when a small piece of wax is put on a prong of the first fork. What is the frequency of this fork?

Solution

$$
\begin{aligned}
\text{Number of beats per second} &= 3 \\
f_{\text{beat}} &= \frac{3}{s}\ =\ 3\ Hz \\
\text{Fork frequency} &= f_B\ =\ 384\ Hz \\
f_A &= ?
\end{aligned}
$$

We have to find f_B, by using the relation

$$
|f_A - f_B| = f_{\text{beat}}
$$

$$
\begin{aligned}
|f_A - 384| &= 3 \\
f_A - 384 &= \pm\ 3 \\
f_A &= 3 + 384 \\
f_A &= 387\ Hz
\end{aligned}
$$

And,

$$
\begin{aligned}
f_A &= -3 + 384 \\
f_A &= 381\ Hz
\end{aligned}
$$

Chapter 4

Coupled Oscillators and Normal Modes

SOLVED PROBLEMS

Problem: 4.1- Consider two discs with a weight less spring of force constant $10\frac{dynes}{cm}$. Calculate the frequency of oscillation of the spring when the system is resting on a table.

Solution

When the system is resting in the table, the equation of motion of the spring is given by

$$m_1\frac{d^2y}{dt^2} = -ky$$

Where y is the displacement of mass m_1 at any instant t. Here $m_1 = 100gm$ and $K = 10^5\frac{dynes}{cm}$. Hence frequency of oscillation

$$\begin{aligned} m_1 =& \frac{1}{2\pi}\sqrt{\frac{K}{m_1}} \\ =& \frac{1}{2\pi}\sqrt{\frac{10^5}{100}} \ = \ 5 \ s^{-1} \end{aligned}$$

Problem: 4.2- Consider two discs are shown in figure with a weight less spring of force constant $10\frac{dynes}{cm}$. Calculate when the table is removed and the system is falling freely. Also deduce the amplitude of oscillation of the lower disc.

Solution

When the table is removed and the system is falls freely under the gravity, the equation of motion of the spring, when observed about the centre of motion of the system, is given by

$$\mu\frac{d^2y}{dt^2} = -ky$$

Where μ is the reduced mass of the system and is equal to

$$\frac{m_1m_2}{m_1+m_2} = \frac{100\times 200}{100+200} = \frac{200}{3}$$

Hence the frequency of oscillation is given by

$$n_1 = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{10^5\times 3}{200}}$$

The force exerted by the lower disc on the spring is equal to the weight of the disc and in the maximum displacement position from the normal the weight is balanced by the restoring force produced on the spring. Thus $200\times 980 = A\times 10^5$.
Hence amplitude

$$A = \frac{200\times 980}{10^5} = 1.95\ cm$$

Problem: 4.3- From the spectroscopic measurement the fundamental vibrational angular frequency of the HF molecule is given by $7.55 \times 10^{14} \frac{\text{rad}}{S}$. Deduce the force constant for HF molecule.

Solution

Reduced mass

$$\mu = \frac{1 \times 19}{1 + 19} \times 1.66 \times 10^{-24}$$

Angular frequency

$$\begin{aligned} \omega =& \frac{1}{2\pi}\sqrt{\frac{K}{\mu}} \\ K =& \mu\omega^2 \\ =& \frac{19 \times 1.66 \times 10^{-24} \times (7.55)^2 \times 10^{26}}{20} \\ \simeq 9.0 \times 10^5 \frac{dynes}{cm} \end{aligned}$$

Problem: 4.4- From the spectroscopic measurement the fundamental vibrational angular frequency of the HF molecule is given by $7.55 \times 10^{14} \frac{\text{rad}}{S}$. Deduce work done in stretching the atoms by $0.5\mathring{A}$.

Solution

Work

$$\begin{aligned} W =& \frac{1}{2}K(r - r_0)^2 \\ =& \frac{1}{2} \times 9 \times 10^5 \times (0.5 \times 10^{-8})^2 \\ \simeq& 1 \times^{-11} egs \\ \simeq& 6\ eV \end{aligned}$$

Problem: 4.5- In HCl molecules the force required to alter the distance between the atoms from equilibrium is $5.4 \times 10^{-5} \frac{dynes}{cm}$. Calculate the fundamental frequency of vibration of the HCl molecules.

Given

$$
\begin{aligned}
m_H =& 1.66 \times 10^{-24} gm \\
m_{Cl} =& 35.5 \times 1.66 \times 10^{-24} gm
\end{aligned}
$$

Solution

The reduced mass of the molecule is

$$
\begin{aligned}
\mu =& \frac{m_H \times m_{Cl}}{m_H + m_{Cl}} \\
\mu =& \frac{35.5 \times (1.66 \times 10^{-24})^2}{(1 + 35.5)(1.66 \times 10^{24})} \\
\mu =& 1.6 \times 10^{-24} \; gm
\end{aligned}
$$

The frequency of HCl molecule

$$
\begin{aligned}
\nu =& \frac{1}{2\pi} \sqrt{\frac{K}{\mu}} \\
\nu =& \frac{1}{2 \times 3.14} \sqrt{\frac{5.4 \times 10^5}{1.6 \times 10^{-24}}} \\
\nu =& 9.2 \times 10^{12} \; s^{-1}
\end{aligned}
$$

Chapter 5

Diffraction and Interference

SOLVED PROBLEMS

Problem: 5.1- An astronaut in a satellite claims, she can just barely resolve two point sources on the earth $1.6 \times 10^5\ m$ below. Calculate their angular and linear separation. Take $\lambda = 540 \times 10^{-9}\ m$ and the pupil diameter of astronaut's eye to be $5 \times 10^{-3}\ m$.

Solution

$$\begin{aligned} D &= 1.6 \times 10^5\ m \\ d &= 5 \times 10^{-3}\ m \\ \lambda &= 540 \times 10^{-9}\ m \\ \theta_R &= ? \\ l &= ? \end{aligned}$$

Since, we know that

$$\begin{aligned} \theta_R &= 1.22\frac{\lambda}{d} \\ \theta_R &= 1.22\frac{540 \times 10^{-9}}{5 \times 10^{-3}} \\ \theta_R &= 1.3 \times 10^{-4}\ \text{rad.} \end{aligned}$$

Also, the linear separation is given as

$$\begin{aligned} l &= D\theta_R \\ l &= 1.6 \times 10^5 \times 1.3 \times 10^{-4} \\ l &= 2.08 \times 10 \\ l &= 20.8\ m \end{aligned}$$

Problem: 5.2- A double slit arrangement produces interference fringes from sodium light $\lambda = 589 \times 10^{-9}\ m$ that are 0.2° apart. What is angular fringe separation, if entire arrangement is in water.

Solution

$$\begin{aligned} \lambda &= 589 \times 10^{-9}\ m \\ \theta &= 0.2^\circ \\ n &= 1.33 \\ \theta' &= ? \end{aligned}$$

Since, we know that

$$\begin{aligned} d\sin\theta &= m\lambda \\ d\theta &= \lambda \\ \theta &= \frac{\lambda}{d} \\ \theta &= \frac{\frac{v}{f}}{d} \end{aligned}$$

Since, $n = \frac{c}{v} \Longrightarrow v = \frac{c}{n}$. Then,

$$\begin{aligned} \theta' &= \frac{\frac{c}{nf}}{d} \\ \theta' &= \frac{\frac{c}{f}}{nd} \\ \theta' &= \frac{\lambda}{nd} \end{aligned}$$

$$\theta' = \frac{\theta}{n} \qquad \because \theta = \frac{\lambda}{d}$$
$$\theta' = \frac{0.2}{1.33}$$
$$\theta' = 0.15^\circ$$

Problem: 5.3- A double slit arrangement produces interference fringes for sodium light $\lambda = 589\ nm$, $d = 0.10\ mm$ and $D = 20\ cm$. On a viewing screen, what is the distance between the fifth maximum and seventh minimum from the central maximum?

Solution

Given data:

$$\lambda = 589\ nm = 589 \times 10^{-9}\ m$$
$$d = 0.10\ mm = 0.1 \times 10^{-3}\ m$$
$$D = 20\ cm = 0.2\ m$$
$$y_7 - y_5 = ?$$

For maxima,

$$y_m = m\frac{\lambda D}{d}$$

For, fifth maxima,

$$y_5 = 5\frac{\lambda D}{d}$$

For, minima

$$y_m = \left(m + \frac{1}{2}\right)\frac{\lambda D}{d}$$

For seventh minima,

$$y_7 = \left(6 + \frac{1}{2}\right)\frac{\lambda D}{d}$$
$$y_7 = \frac{13}{2}\frac{\lambda D}{d}$$
$$y_7 - y_5 = \frac{13}{2}\frac{\lambda D}{d} - 5\frac{\lambda D}{d}$$

$$y_7 - y_5 = 3\frac{\lambda D}{2d}$$
$$y_7 - y_5 = 3\frac{546 \times 10^{-9} D \times 0.2}{2 \times 0.10 \times 10^{-3}}$$
$$y_7 - y_5 = 1.64 \times 10^{-3}\ m$$

Problem: 5.4- A diffraction grating has 1.26×10^4 rulings uniformly spaced over width $W = 25.4\ mm$. It is illuminated at normal incidence by light of wavelength 450 nm (blue) and 625 nm (red). At what angles occurs? Also find the line width for both blue and red lines in the 2nd order.

Solution

$$N = 1.26 \times 10^4$$
$$W = 25.4\ mm = 25.4 \times 10^{-3}\ m$$
$$d = \frac{W}{N} = \frac{25.4 \times 10^{-3}}{1.26 \times 10^4}$$
$$d = 2.0116 \times 10^{-6}\ m = 2016 \times 10^{-9}\ m$$
$$\lambda_1 = 450\ nm = 450 \times 10^{-9}\ m$$
$$\lambda_2 = 625\ nm = 625 \times 10^{-9}\ m$$
$$m = 2$$
$$\theta_1 = \theta_2 = ?$$
$$d_1 = d_2 = ?$$

Since, we know that

$$d \sin\theta = m\lambda$$

So,

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda_1}{d}\right)$$
$$\theta_1 = \sin^{-1}\left(\frac{2 \times 450 \times 10^{-9}}{2016 \times 10^{-9}}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{900}{2016}\right)$$
$$\theta_1 = \sin^{-1}(0.446)$$
$$\theta_1 = 26.5^\circ$$

And,

$$\theta_2 = \sin^{-1}\left(\frac{m\lambda_2}{d}\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{2 \times 625 \times 10^{-9}}{2016 \times 10^{-9}}\right)$$
$$\theta_2 = \sin^{-1}\left(\frac{1250}{2016}\right)$$
$$\theta_2 = \sin^{-1}(0.62)$$
$$\theta_2 = 38.3^\circ$$

Now,

$$d_1 = \frac{\lambda_1}{\sin\theta_1}$$
$$d_1 = \frac{450 \times 10^{-9}}{\sin(26.5)}$$
$$d_1 = \frac{450 \times 10^{-9}}{0.446}$$
$$d_1 = 1008.9 \times 10^{-9}$$
$$d_1 = 1008.9\ nm$$

And,

$$d_2 = \frac{\lambda_2}{\sin\theta_2}$$
$$d_2 = \frac{625 \times 10^{-9}}{\sin(38.3)}$$

$$
\begin{aligned}
d_2 &= \frac{625 \times 10^{-9}}{0.62} \\
d_2 &= 1008.0 \times 10^{-9} \\
d_2 &= 1008.0\ nm
\end{aligned}
$$

Problem: 5.5- Find the sum y of the following quantities $y_1 = 10.0 \sin \omega t$ and $y_2 = 8 \sin(\omega t + 30)$.

Solution

Horizontal component is

$$
\begin{aligned}
y_x &= y_{1x} + y_{2x} \\
y_x &= 10 + 8 \cos 30 \\
y_x &= 10 + 8(0.866) \\
y_x &= 10 + 6.93 \\
y_x &= 16.93
\end{aligned}
$$

Vertical component is

$$
\begin{aligned}
y_y &= y_{1y} + y_{2y} \\
y_y &= 0 + 8 \sin 30 \\
y_y &= 8(0.5) \\
y_y &= 4
\end{aligned}
$$

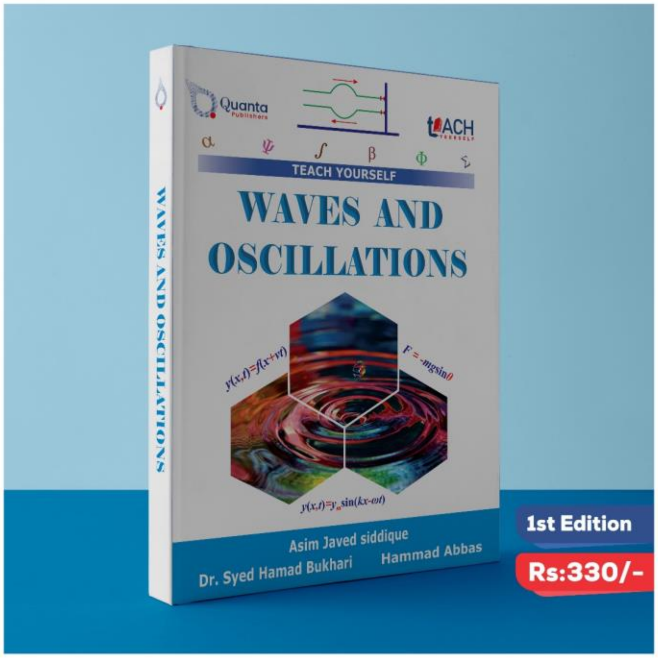
Now, the sum y is

$$
\begin{aligned}
y_m &= \sqrt{y_x^2 + y_y^2} \\
y_m &= \sqrt{(16.93)^2 + (4)^2} \\
y_m &= \sqrt{286.62 + 16} \\
y_m &= \sqrt{302.62} \\
y_m &= 17.4
\end{aligned}
$$

$$\tan\phi = \frac{8\sin 30^\circ}{10 + 8\cos 30^\circ}$$
$$\tan\phi = 0.238$$
$$\phi = \tan^{-1}(0.238)$$
$$\phi = 13^\circ$$

Now, the equation of resultant wave is

$$y = y_1 + y_2$$
$$y = y_m \sin(\omega t + \phi)$$
$$y = 17\sin(\omega t + 13^\circ)$$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