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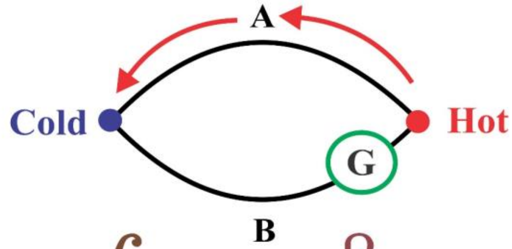

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TEACH YOURSELF

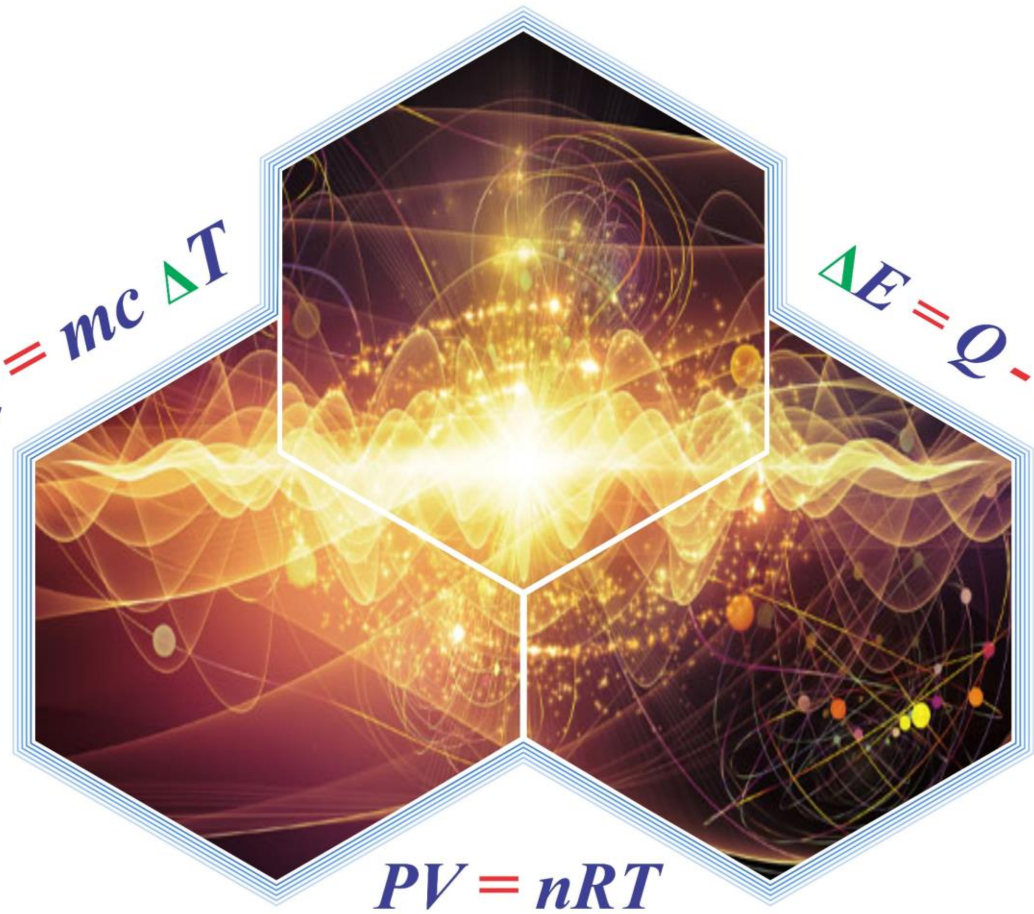
HEAT AND THERMODYNAMICS

For BS Physics Programme

2nd Edition

$Q = mc\,\Delta T$

$\Delta E = Q - W$



$PV = nRT$

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Dr. Syed Hamad Bukhari

TEACH YOURSELF

HEAT & THERMODYNAMICS

2nd Edition

For **BS Physics** students of all Pakistani Universities

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Contents

Chapter 1

Basic Concepts of Thermodynamics

NO SOLVED PROBLEMS ARE THERE FOR CHAPTER 1

Chapter 2

Heat and Temperature

SOLVED PROBLEMS

Problem: 2.1- A cylinder contains 3 *mol* of helium gas at a temperature of 300 K. If the gas is heated at constant volume, how much energy must be transferred by heat to the gas for its temperature to increase to 500 K. How much energy must be transferred by heat to the gas at constant pressure to raise the temperature to 500 K.

Solution

$$\begin{aligned}
C_v &= 12.5\ Jmol^{-1}K^{-1} \\
n &= 3\ mol \\
C_p &= 20.8\ Jmol^{-1}K^{-1} \\
T_1 &= 300\ K \\
T_2 &= 500\ K \\
\Delta T &= T_2 - T_1 \\
\Delta T &= 500 - 300\ =\ 200\ K
\end{aligned}$$

For the constant volume process,

$$Q = nC_v\Delta T$$

$$Q = 3 \times 12.5 \times 200$$
$$Q = 7500\ J$$

For the constant pressure process,

$$Q = nC_p \Delta T$$
$$Q = 3 \times 20.8 \times 200$$
$$Q = 12500\ J$$

Problem: 2.2- A fighter fires a silver bullet with a muzzle speed of 200 ms^{-1} into the pine wall of a saloon. Assume that all the internal energy generated by the impact remains with the bullet. Calculate the temperature change of the bullet.

Solution

$$\text{Velocity} = v = 200\ ms^{-1}$$
$$\text{Temperature difference} = \Delta T = ?$$

Since, we know that

$$Q = mc\Delta T$$
$$\Delta T = \frac{Q}{mc}$$

$$\Delta T = \frac{\frac{1}{2}mv^2}{mc} \qquad \because Q = K.E = \frac{1}{2}mv^2$$
$$\Delta T = \frac{v^2}{2c}$$
$$\Delta T = \frac{(200)^2}{234}$$
$$\Delta T = \frac{40000}{234}$$
$$\Delta T = 85.5°C$$

Problem: 2.3- A small electric immersion heater is used to boil 136 g of water for a cup of instant coffer. The heater is labeled 220 W. Calculate the time required to bring this water from 23.5°C to the boiling point, ignoring any heat losses.

Solution

$$\begin{aligned}
m &= 136\ g\ =\ \frac{136}{1000}\ kg \\
m &= 0.136\ kg \\
c &= 4190\ J\ kg^{-1}\ K^{-1} \\
T_2 &= 100°C \\
T_1 &= 23.5°C \\
\Delta T &= T_2 - T_1\ =\ (100 - 23.5)\ =\ 76.5°C \\
P &= 220\ W
\end{aligned}$$

The heat required to boil the water is

$$Q\ =\ mc\Delta T$$

But we have to determine the time required to boil the water, so replace Q by Pt. So, $P\ =\ \frac{Q}{t}$, we get

$$\begin{aligned}
Pt &= mc\Delta T \\
t &= \frac{mc\Delta T}{P} \\
t &= \frac{0.136 \times 4190 \times 76.5}{220} \\
t &= 198\ s
\end{aligned}$$

Problem: 2.4- How much water remains unfrozen after 50.4 kJ of heat have been extracted from 258 g of liquid water initially at $0°C$.

Solution

$$\text{Latent heat of fusion } = L_f = 333\ kJ\ kg^{-1}$$
$$\text{Heat } = Q = 50.4\ kJ$$
$$\text{Mass } = m = 258\ g = 0.258\ kg$$

If m_f is the frozen mass of water than it can be determine from the relation between mass m_f and heat Q is

$$Q = L_f m_f$$
$$m_f = \frac{Q}{L_f}$$
$$m_f = \frac{50.4}{333}$$
$$m_f = 0.151\ kg$$

Now, if m_f is the frozen mass and m_{uf} is the unfrozen mass, then

$$m = m_f + m_{uf}$$
$$m_{uf} = m - m_f$$
$$m_{uf} = 0.258 - 0.151$$
$$m_{uf} = 0.107\ kg$$
$$m_{uf} = 0.107 \times 1000\ g = 107\ g$$

Problem: 2.5- In a certain solar house, energy from the sum is stored in barrels filled with water. In a particular winter stretch of five cloudy days, 5.22 GJ are needed to maintain the inside of the house at 22 K. Assuming that the water in the barrels is at 50 K. Calculate the volume of water, which is required

Solution

$$\begin{aligned}
\text{Heat} &= Q = 5.22\ GJ \\
\text{Heat} &= Q = 5.22 \times 10^9\ J \\
\text{Density} &= \rho = 1000\ kgm^{-3} \\
\text{Specific heat of water} &= c = 4190\ Jkg^{-1}\ K^{-1} \\
\text{Initial temperature} &= T_1 = 22\ K \\
\text{Final temperature} &= T_2 = 50K \\
\text{Temperature difference} &= \Delta T = T_2 - T_1 \\
\text{Temperature difference} &= \Delta T = 50 - 22 = 28\ K
\end{aligned}$$

The heat required to boil the water is

$$\begin{aligned}
Q &= mc\Delta T \\
Q &= \rho V c\Delta T & \because \rho = \frac{m}{V} \\
V &= \frac{Q}{\rho c\Delta T} \\
V &= \frac{5.22 \times 10^9}{1000 \times 4190 \times 28} \\
V &= 44.5\ m^3
\end{aligned}$$

Chapter 3

Laws and Functions of Thermodynamics

SOLVED PROBLEMS

Problem: 3.1- The boiling point of water is $100°C$ at a pressure of $76cm$ of mercury. Find boiling point of water at a pressure of $75cm$ of mercury. The latent heat of steam at $100°C$ is 537 $calg^{-1}$ and specific volume is 1671 cm^3g^{-1}.

Solution

$$\begin{aligned}
\text{Molecular weight of } \mathrm{H_2O} &= 18g/mol = 18gmol^{-1} \\
\text{Temperature} &= T \ = \ 100°C \\
\text{Temperature} &= T \ = \ 100 + 273 \ K \ = \ 373 \ K \\
\text{Latent heat of steam} &= L \ = \ 537 \ calg^{-1} \\
\text{Latent heat of steam} &= L \ = \ 537 \times 18 \times 4.2 \times 10^7 \ erg \ g^{-1} \ mol^{-1} \\
\because \quad 1calg^{-1} &= 4.2 \times 10^7 ergg^{-1}. \\
dP &= 75 - 76 \ = \ -1 \ cm \text{ of } Hg \\
dP &= -1 \times 13.6 \times 981 \ dynes \ cm^{-2} \\
v_2 - v_1 &= (1671 - 1) \times 18 \ = \ 1670 \times 18 \ cm^3 mol^{-1} \\
dT &= ?
\end{aligned}$$

Now, using the relation,

$$\begin{aligned}
\frac{dP}{dT} &= \frac{L}{T\left(v_2 - v_1\right)} \\
\frac{dT}{dP} &= \frac{T\left(v_2 - v_1\right)}{L} \\
dT &= T\left(v_2 - v_1\right)\frac{dP}{L} \\
dT &= \frac{373 \times 1670 \times 18 \times (-1 \times 13.6 \times 981)}{537 \times 18 \times 4.2 \times 10^7} \\
dT &= -0.37^\circ C
\end{aligned}$$

Hence required boiling point:

$$\begin{aligned}
&= 100 - 0.37 \\
&= 99.63^\circ C
\end{aligned}$$

Problem: 3.2- Show that for a van der Waal gas,

$$C_P - C_V = R\left(1 + \frac{2a}{VRT}\right)$$

Solution

Van der Waal equation of state for one mole is

$$P + \frac{a}{V^2} = \frac{RT}{V - b}$$

Differentiating w.r.t. T at constant volume and pressure, we get

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V - b}$$

And,

$$-\frac{2a}{V^3}\left(\frac{\partial V}{\partial T}\right)_P = \frac{-RT}{(V - b)^2}\left(\frac{\partial V}{\partial T}\right)_P + \frac{R}{(V - b)}$$

$$\frac{RT}{(V-b)^2}\left(\frac{\partial V}{\partial T}\right)_P - \frac{2a}{V^3}\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{(V-b)}$$

$$\left(\frac{\partial V}{\partial T}\right)_P\left[\frac{RT}{(V-b)^2} - \frac{2a}{V^3}\right] = \frac{R}{(V-b)}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{\frac{R}{(V-b)}}{\left[\frac{RT}{(V-b)^2} - \frac{2a}{V^3}\right]}$$

Using above values in equation $C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_P$, we get

$$C_P - C_V = T\frac{R}{(V-b)}\frac{\frac{R}{(V-b)}}{\left[\frac{RT}{(V-b)^2} - \frac{2a}{V^3}\right]}$$

$$C_P - C_V = T\frac{\left(\frac{R}{V-b}\right)^2}{\left[\frac{RT}{(V-b)^2} - \frac{2a}{V^3}\right]}$$

$$C_P - C_V = \frac{R}{1 - \frac{2a}{V^3}\left(\frac{(V-b)^2}{RT}\right)}$$

Since b is small quantity, so $V - b = V$, we get

$$C_P - C_V = \frac{R}{1 - \frac{2a}{V^3}\left(\frac{(V-0)^2}{RT}\right)}$$

$$C_P - C_V = \frac{R}{1 - \frac{2a}{V^3}\left(\frac{(V)^2}{RT}\right)}$$

$$C_P - C_V = \frac{R}{1 - \frac{2a}{VRT}}$$

$$C_P - C_V = R\left(1 - \frac{2a}{VRT}\right)^{-1}$$

Expanding by binomial expansion and neglecting square and higher powers of $\frac{2a}{VRT}$, we get

$$C_P - C_V = R\left(1 + \frac{2a}{VRT}\right)$$

Problem: 3.3- Calculate pressure required to make ice freeze at 272 K.

Solution

$$\begin{aligned}
\text{Latent heat } &= L \;=\; 79.6 \times 4.2 \times 10^{7}\; erg\; g^{-1} \\
\text{Temperature difference } &= dT \;=\; 273 - 272 \;=\; 1\; K \\
\text{volume of 1 } g \text{ of water at } 273K &= v_1 \;=\; 1\; cc \\
\text{volume of 1 } g \text{ of ice at } 273K &= v_2 \;=\; 1.091\; cc \\
T &= 272\; K \\
dP &= ?
\end{aligned}$$

Now, using the relation,

$$\begin{aligned}
\frac{dP}{dT} &= \frac{L}{T\,(v_2 - v_1)} \\
dP &= \frac{L}{T\,(v_2 - v_1)} dT \\
dP &= \frac{LdT}{T\,(v_2 - v_1)} \\
dP &= \frac{79.6 \times 4.2 \times 10^{7} \times 1}{272\,(1.091 - 1.000)} \\
dP &= 132.8\; atm
\end{aligned}$$

Problem: 3.4- Find the change in vapor pressure of water as boiling point changes from $100°C$ to $110°C$. Latent heat of steam is 540 $kcal\; kg^{-1}$ and specific volume of steam is 1.671 $m^3\; kg^{-1}$.

Solution

$$\begin{aligned}
\frac{dP}{dT} &= \frac{L}{T\,(v_2 - v_1)} \\
dP &= \frac{L}{T\,(v_2 - v_1)} dT \\
dP &= \frac{540 \times 4.2 \times 10^{3}}{373 \times (1.671 - 0.001)} dT
\end{aligned}$$

$$
\begin{aligned}
dP &= 3.64 \times 10^3 dT\ Pa \\
\int_{P_1}^{P_2} dP &= 3.64 \times 10^3 \int_{T_1}^{T_2} dT\ Pa \\
P_2 - P_1 &= 3.64 \times 10^3 \left(T_2 - T_1\right)\ Pa \\
P_2 - P_1 &= 3.64 \times 10^3 \left(110 - 100\right)\ Pa \\
P_2 - P_1 &= 3.64 \times 10^3 \times 10\ Pa \\
P_2 - P_1 &= 3.64 \times 10^4\ Pa
\end{aligned}
$$

Problem: 3.5- A Carnot engine having $100g$ water steam as working substance has at beginning of stroke volume $100cm^3$ and pressure $788mm$, boiling point $101°C$. After a complete isothermal change from water into steam, the volume is $16.7404 \times 10^4 cm^3$ and pressure is then lowered adiabatically to $733.7mm$, boiling point $99°C$. If engine is working between $99°C$ and $101°C$, find latent heat of steam.

Solution

$$
\begin{aligned}
T_2 &= 101\ K \\
T_1 &= 99\ K \\
v_2 &= 167404 \times 10^{-2}\ cc \\
v_1 &= 104 \times 10^{-2}\ cc \\
P_2 &= 788\ mm \\
P_1 &= 733.7\ mm
\end{aligned}
$$

Since, we know that:

$$
\begin{aligned}
\frac{dP}{dT} &= \frac{L}{T\left(v_2 - v_1\right)} \\
L &= T\left(v_2 - v_1\right)\frac{dP}{dT} \\
L &= \left(\frac{1011 + 99}{2} + 273\right) \times \left(167404 - 104\right) \times 10^{-2} \times \frac{\left(788 - 733.7\right) \times 13.6 \times 981}{101 - 99}
\end{aligned}
$$

$$L = \frac{5.43 \times 13.6 \times 981 \times 373 \times 1673}{2} \; erg \; g^{-1}$$
$$L = \frac{5.43 \times 13.6 \times 981 \times 373 \times 1673}{2 \times 4.2 \times 10^7} \; cal \; g^{-1}$$
$$L = 538.3 \; cal \; g^{-1}$$

Chapter 4

Entropy and 2^{nd} Law of Thermodynamics

SOLVED PROBLEMS

Problem: 4.1- The turbine in a steam power plant takes steam from a boiler at $520°C$ and exhausts it into a condenser at $100°C$. What is its maximum possible efficiency?

Solution

$$T_H = 520°C = 520 + 273\ K = 793\ K$$
$$T_L = 100°C = 100 + 273\ K = 373\ K$$
$$\eta = ?$$

The maximum possible efficiency of an engine operating between two temperatures T_H and T_L, can be determine from the relation

$$\eta = 1 - \frac{T_L}{T_H}$$
$$\eta = 1 - \frac{373}{793}$$

$$\eta = \frac{793 - 373}{793}$$
$$\eta = \frac{420}{793}$$
$$\eta = 0.529$$
$$\eta = 0.529 \times 100$$
$$\eta = 53\%$$

Problem: 4.2- An ideal gas undergoes a reversible isothermal expansion at $132^{\circ}C$. The entropy of the gas increases by 46.2 JK^{-1}. How much heat was absorbed?

Solution

The entropy of the system is given by

$$dS = \frac{dQ}{T}$$

Integrating, we get

$$\int_{S_i}^{S_f} dS = \int \frac{dQ}{T}$$
$$S_f - S_i = \int \frac{dQ}{T}$$

We are given that the expansion in isothermal and reversible, that is the temperature remains constant, therefore we bring out the temperature T of the integral. Thus we rewrite above equation as:

$$S_f - S_i = \frac{1}{T}\int dQ$$
$$S_f - S_i = \frac{1}{T}\Delta Q$$
$$\Delta S = \frac{\Delta Q}{T}$$
$$\Delta Q = T\Delta S$$

Given that

$$T = 132°C = 132 + 273\ K$$
$$T = 405\ K$$
$$\Delta S = 46.2\ JK^{-1}$$

So,

$$\Delta Q = 405 \times 46.2$$
$$\Delta Q = 18.7 \times 10^3\ J$$
$$\Delta Q = 18.7\ kJ$$

So, the absorbed heat is 18.7 kJ.

Problem: 4.3- A Carnot engine has an efficiency of 22 %. It operates between heat reservoirs differing in temperature by 75 K. Find the temperatures of the reservoirs.

Solution

$$\eta = 22\ \% = 0.22$$
$$\Delta T = T_H - T_L = 75\ K$$
$$T_H = ?$$
$$T_L = ?$$

The efficiency of a Carnot engine is given data:

$$\eta = \frac{T_H - T_L}{T_H}$$
$$T_H = \frac{T_H - T_L}{\eta}$$
$$T_H = \frac{75}{0.22}$$
$$T_H = 341\ K$$

And,

$$\begin{aligned} T_H - T_L &= 75 \\ 341 - T_L &= 75 \\ T_L &= 341 - 75 \\ T_L &= 266\ K \end{aligned}$$

Problem: 4.4- An air conditioner operating between $93°F$ and $70°F$ is rated at $\frac{4000}{2545}$ hp cooling capacity. Its coefficient of performance is 0.27 of that of Carnot refrigerator operating between same two temperatures. What is required hp of air conditioner motor?

Solution

$$\text{Rate of heat cooling} = \frac{dQ_2}{dt} = \frac{4000}{2545}\ hp$$
$$\text{Temperature of LTR} = T_2 = 70°F$$
$$\text{Temperature of HTR} = T_1 = 93°F$$

As, we know that

$$\begin{aligned} W &= \frac{Q_2}{K} \\ \frac{dW}{dt} &= \frac{1}{K}\frac{dQ_2}{dt} \\ P &= \frac{1}{K}\frac{dQ_2}{dt} \\ P &= \frac{1}{0.27\left(\frac{T_2}{T_1 - T_2}\right)}\frac{dQ_2}{dt} \\ P &= \frac{(T_1 - T_2)}{0.27T_2}\frac{dQ_2}{dt} \\ P &= \frac{(93 - 70)}{0.27 \times 70}\frac{4000}{2545} \\ P &= 0.25\ hp \end{aligned}$$

Problem: 4.5- To make some ice, a freezer extracts 185 J of heat at $-12°C$. The freezer has a coefficient of performance of 5.70. The room temperature is $26°C$. How much heat was delivered to the room and how much work was required to run the freezer.

Solution

$$K = 5.70$$
$$|Q_L| = 185\ J$$
$$|Q_H| = ?$$
$$|W| = ?$$

The coefficient of performance of a refrigerator is

$$K = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

We want to determine $|Q_H|$, so we rewrite above equation as:

$$|Q_H| - |Q_L| = \frac{|Q_L|}{K}$$
$$|Q_H| = \frac{|Q_L|}{K} + |Q_L|$$
$$|Q_H| = \frac{185}{5.70} - 185$$
$$|Q_H| = 217.5\ J$$

So, heat delivered to the room is 217.5 J. Now, the required work is:

$$|W| = |Q_H| - |Q_L|$$
$$|W| = 217.5 - 185$$
$$|W| = 32.5\ J$$

So, the work is 32.5 J.

Chapter 5

Thermoelectricity

SOLVED PROBLEMS

Problem: 5.1- The magnitude of the average electric field normally present in the earth's atmosphere just above the surface of earth is about 150 NC^{-1} directed downward. What is total net surface charge of the earth. Assume earth to be a conductor with a uniform surface charge density.

Solution

$$\begin{aligned}
\text{Surface area of earth } &= 4\pi R^2 \\
E &= 150\ NC^{-1} \\
\epsilon_\circ &= 8.85 \times 10^{-12}\ N^{-1}m^{-2}C^2 \\
R &= 6.4 \times 10^6\ m \\
q &= ?
\end{aligned}$$

Since, we know that

$$\begin{aligned}
E &= \frac{\sigma}{\epsilon_\circ} \\
\sigma &= \epsilon_\circ E
\end{aligned}$$

$$\sigma = 8.85 \times 10^{-12} \times 150$$
$$\sigma = 1.33 \times 10^{-9} \ NC^{-1}$$

So,

$$q = \sigma \times 4\pi R^2$$
$$q = 1.33 \times 10^{-9} \times 4 \times 3.14 \times (6.4 \times 10^6)^2$$
$$q = 680000 \ C$$

Since, electric field point towards surface of earth. So, surface charge of earth is negative, therefore

$$q = -680000 \ C$$

Problem: 5.2- The plates of a parallel plate capacitor are separated by a distance of $10^{-3}\ m$. What must be plate area, if capacitance is to be $1\ F$.

Solution

$$d = 10^{-3} \ m$$
$$\epsilon_\circ = 8.85 \times 10^{-12} \ N^{-1}m^{-2}C^2$$
$$C = 1 \ F$$
$$A = ?$$

Since, we know that

$$C = \epsilon_\circ \frac{A}{d}$$
$$A = \frac{Cd}{\epsilon_\circ}$$
$$A = \frac{1 \times 10^{-3}}{8.85 \times 10^{-12}}$$
$$A = 1.1 \times 10^8 \ m^2$$

Problem: 5.3- An ideal gas undergoes a reversible isothermal expansion at $132°C$. The entropy of the gas increases by $46.2\ JK^{-1}$. How much heat was absorbed?

Solution

The entropy of the system is given by

$$dS = \frac{dQ}{T}$$

Integrating, we get

$$\int_{S_i}^{S_f} dS = \int \frac{dQ}{T}$$

$$S_f - S_i = \int \frac{dQ}{T}$$

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$$S_f - S_i = \frac{1}{T}\Delta Q$$

$$\Delta S = \frac{\Delta Q}{T}$$

$$\Delta Q = T\Delta S$$

Given that

$$T = 132°C = 132 + 273\ K$$

$$T = 405\ K$$

$$\Delta S = 46.2\ JK^{-1}$$

So $$\Delta Q = 405 \times 46.2$$

$$\Delta Q = 18.7 \times 10^3\ J$$

$$\Delta Q = 18.7\ kJ$$

So, the absorbed heat is 18.7 kJ.

Problem: 5.4- The magnitude of magnetic field 0.88 m from the axis of a long straight wire is $7.3 \times 10^{-6}\ T$. Calculate current in wire.

Solution

$$B = 7.3 \times 10^{-6}\ T$$
$$r = 0.88\ m$$
$$\mu_\circ = 4\pi \times 10^{-7}\ Web\ m^{-1}$$
$$I = ?$$

Since, we know that

$$B = \frac{\mu_\circ}{2\pi}\frac{I}{r}$$
$$I = \frac{B \times 2\pi r}{\mu_\circ}$$
$$I = \frac{7.3 \times 10^{-6} \times 2\pi \times 0.88}{4\pi \times 10^{-7}}$$
$$I = \frac{7.3 \times 0.88 \times 10}{2}$$
$$I = 32\ A$$

Problem: 5.5- A generator with an adjustable frequency of oscillations is wired in series to an inductor L of $2.5 \times 10^{-3}\ H$ and a capacitance of $3 \times 10^{-6}\ F$. At what frequency does the generator produce the largest possible current amplitude in the circuit.

Solution

$$L = 2.5 \times 10^{-3}\ H$$
$$C = 3 \times 10^{-6}\ F$$
$$f = ?$$

Since, we know that

$$
\begin{aligned}
f &= \frac{1}{2\pi\sqrt{LC}} \\
f &= \frac{1}{2 \times 3.14\sqrt{2.5 \times 10^{-3} \times 3 \times 10^{-6}}} \\
f &= 1.84 \times 10^{3}\ Hz \\
f &= 1.84\ kHz
\end{aligned}
$$

Chapter 6

Statistical Mechanics

SOLVED PROBLEMS

Problem: 6.1- Find the temperature at which atoms of helium gas have same v_{rms} as the molecules of hydrogen gas at 299 K.

Solution

Let m_{He} be mass of helium atom and m_H be mass of hydrogen molecule, then

$$\frac{m_{He}}{m_H} = \frac{4}{2} = 2$$

$$\text{Temperature of molecules of hydrogen gas } = T_H = 299\ K$$

$$\text{Temperature of molecules of helium gas } = T_{He} = ?$$

Since we know that the root mean square velocity is

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}}$$

Since, hydrogen and helium has same root means square velocity. So,

$$\sqrt{\frac{3k_BT_{He}}{m_{He}}} = \sqrt{\frac{3k_BT_H}{m_H}}$$

$$\frac{3k_B T_{He}}{m_{He}} = \frac{3k_B T_H}{m_H}$$
$$\frac{T_{He}}{m_{He}} = \frac{T_H}{m_H}$$
$$T_{He} = \frac{T_H}{m_H} m_{He}$$
$$T_{He} = \frac{m_{He}}{m_H} T_H$$
$$T_{He} = 2 \times 299$$
$$T_{He} = 598\ K$$

Problem: 6.2- The temperature in outer space is $2.7K$. Find root mean square speed of hydrogen molecules at this temperature.

Solution

$$\text{Temperature} = T = 2.7\ K$$
$$\text{Molar mass} = M = 2.02\ g\ mol^{-1} = 2.02 \times 10^{-3}\ kg\ mol^{-1}$$
$$\text{General gas constant} = R = 8.314\ J\ mol^{-1} K^{-1}$$
$$\text{Root mean square speed} = v_{\text{rms}} = ?$$

Using the relation for the root mean square speed

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$
$$v_{\text{rms}} = \sqrt{\frac{3 \times 8.314 \times 2.7}{2.02 \times 10^{-3}}}$$
$$v_{\text{rms}} = \sqrt{\frac{67.311}{2.02 \times 10^{-3}}}$$
$$v_{\text{rms}} = \sqrt{33.32 \times 10^{3}}$$
$$v_{\text{rms}} = 180\ ms^{-1}$$

Problem: 6.3- Find average translational kinetic energies of one molecule and one mole of hydrogen gas at room temperature.

Solution

$$\text{Temperature} = T = 27^{\circ}C = 27 + 273\ K$$
$$\text{Temperature} = T = 300\ K$$
$$\text{Boltzmann constant} = k_B = 1.38 \times 10^{-23}\ JK^{-1}$$
$$\text{General gas constant} = R = 8.314\ J\ mol^{-1}\ K^{-1}$$
$$\text{Translational kinetic energy} = K.E_{\text{trans.}} = ?$$

For one molecule of hydrogen gas,

$$K.E_{\text{trans.}} = \frac{3}{2}k_B T$$
$$K.E_{\text{trans.}} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$
$$K.E_{\text{trans.}} = \frac{1242}{2} \times 10^{-23}$$
$$K.E_{\text{trans.}} = 621 \times 10^{-23}$$
$$K.E_{\text{trans.}} = 6.21 \times 10^{-21}\ J$$

For one mole of hydrogen gas,

$$K.E_{\text{trans.}} = \frac{3}{2}N_A k_B T$$
$$K.E_{\text{trans.}} = \frac{3}{2}RT$$
$$K.E_{\text{trans.}} = \frac{3}{2} \times 8.314 \times 300$$
$$K.E_{\text{trans.}} = \frac{7482.6}{2}$$
$$K.E_{\text{trans.}} = 3741.3\ J\ mol^{-1}$$

Problem: 6.4- Calculate the volume occupied by 1.00 mol of an ideal gas at STP. And also show that the number of molecules per cubic centimeter under standard conditions is 2.68×10^{19}.

Solution

$$\begin{aligned}
\text{Pressure} &= P = 1.01 \times 10^5\ Nm^{-2} = 1.01 \times 10^5\ Pa \\
\text{Temperature} &= T = 0°C = 0 + 273\ K = 273\ K \\
\text{General gas constant} &= R = 8.314\ Jmol^{-1}K^{-1} \\
\text{Volume} &= V = ?
\end{aligned}$$

From the ideal gas equation

$$\begin{aligned}
PV &= nRT \\
PV &= RT \qquad \because \text{For one mole } (n = 1) \\
V &= \frac{RT}{P} \\
V &= \frac{8.314 \times 273}{1.01 \times 10^5} \\
V &= 2246 \times 10^{-5}\ m^3 \\
V &= 0.02246 = 0.0225\ m^3
\end{aligned}$$

Problem: 6.5- Oxygen at pressures much greater than 1 atm is toxic to lung cells. Assume that a deep sea diver breathes a mixture of oxygen and helium. By weight, what ratio of helium to oxygen must be used if the diver is at an ocean depth of 50 m.

Solution

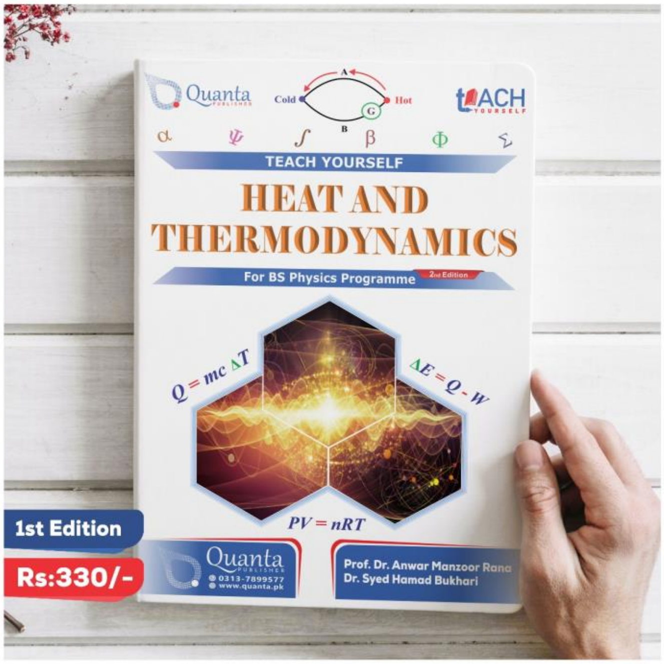
The pressure of the gas in the lungs of the sea diver must must be the same as the absolute pressure of the water at depth of 50 m. So,

$$\begin{aligned}
P_\circ &= 1\ atm = 1.03 \times 10^5\ Pa \\
\rho &= 1.03 \times 10^3\ kgm^{-3} \\
g &= 9.8\ ms^{-2}
\end{aligned}$$

$$h = 50\ m$$
$$P = ?$$

Since, we know that

$$P = P_\circ + \rho g h$$
$$P = 1.03 \times 10^5 + 1.03 \times 10^3 \times 9.8 \times 50$$
$$P = 1.03 \times 10^5 + 5.05 \times 10^5$$
$$P = (1.03 + 5.05) \times 10^5$$
$$P = 6.08 \times 10^5\ Pa$$
$$P = \frac{6.08 \times 10^5}{1.03 \times 10^5}\ atm$$
$$P = 6\ atm$$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