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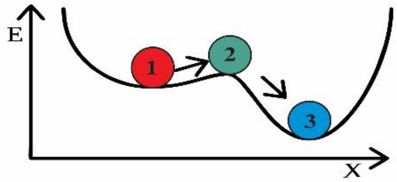

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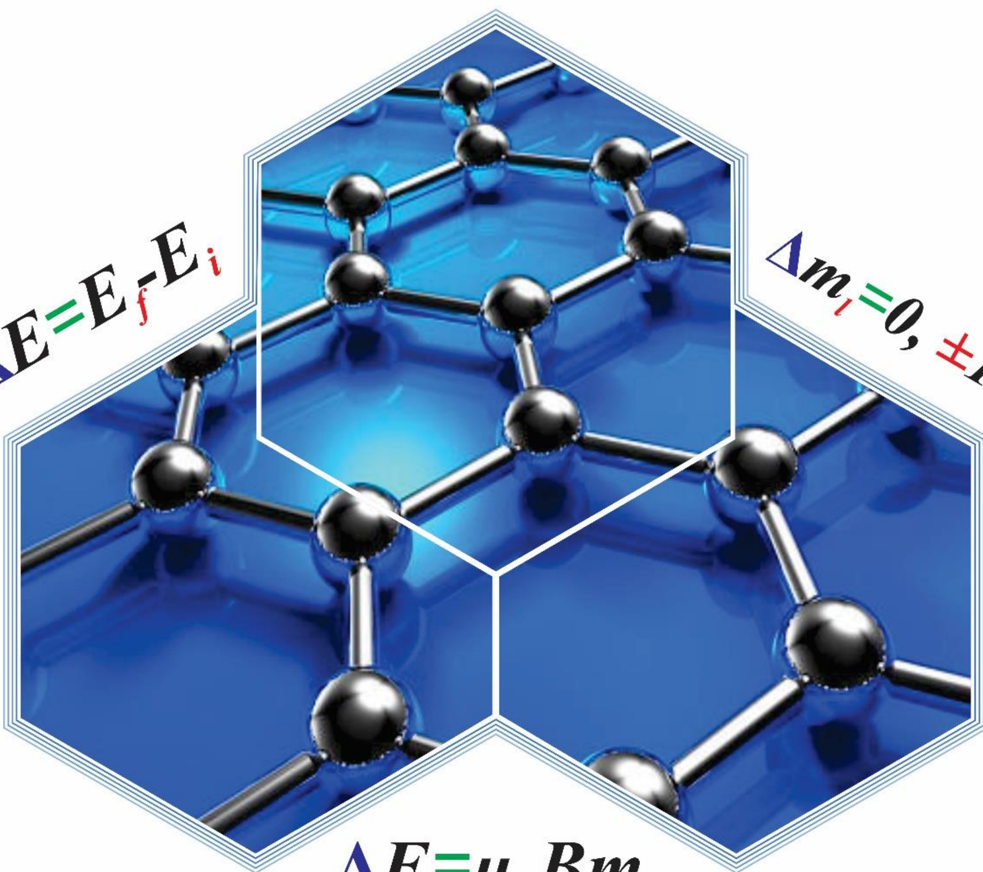
ATOMIC AND MOLECULAR PHYSICS

For BS/M.Sc Physics Programme

$\Delta E = E_f - E_i$

$\Delta m_l = 0, \pm 1$



$\Delta E = \mu_B B m_j$

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Dr. Ejaz Khaira

TEACH YOURSELF

ATOMIC & MOLECULAR PHYSICS

1st Edition

For **BS/M.Sc Physics** students of all Pakistani Universities

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Contents

Chapter 1

Structure of Atoms

SOLVED PROBLEMS

Problem: 1.1- Calculate the linear velocity of an electron in the first, second and third orbit of hydrogen atom.

Solution

$$\begin{aligned} e &= 1.6 \times 10^{-19}\ C \\ \varepsilon_\circ &= 8.85 \times 10^{-12}\ C^2/Nm^2 \\ h &= 6.63 \times 10^{-34}\ Js \\ Z &= 1 \end{aligned}$$

$$v_1 = ? \quad ; \quad v_2 = ? \quad ; \quad v_3 = ?$$

The velocity of an electron in n$^{\text{th}}$ orbit is given by

$$v_n = \frac{e^2 Z}{2\varepsilon_\circ n h}$$

For first orbit, we have $n = 1, Z = 1$.

$$\begin{aligned}
v_1 &= \frac{(1.6\times10^{-19})^2\times1}{2\times8.85\times10^{-12}\times1\times6.63\times10^{-34}}\\
v_1 &= 2.17\times10^{-2}\times10^{-38}\times10^{46}\\
v_1 &= 2.17\times10^{6}\ m/s
\end{aligned}$$

For second orbit, $n = 2$.

$$\begin{aligned}
v_2 &= \frac{(1.6\times10^{-19})^2\times1}{2\times8.85\times10^{-12}\times2\times6.63\times10^{-34}}\\
v_2 &= 1.08\times10^{-2}\times10^{-38}\times10^{46}\\
v_2 &= 1.08\times10^{6}\ m/s
\end{aligned}$$

For third orbit, $n = 3$.

$$\begin{aligned}
v_3 &= \frac{(1.6\times10^{-19})^2\times1}{2\times8.85\times10^{-12}\times3\times6.63\times10^{-34}}\\
v_3 &= 0.72\times10^{-2}\times10^{-38}\times10^{46}\\
v_3 &= 0.72\times10^{6}\ m/s
\end{aligned}$$

Problem: 1.2- Calculate the frequency of revolution of an electron in first and second orbit of hydrogen atom.

Solution

$$\begin{aligned}
m &= 9.1\times10^{-31}\ kg\\
e &= 1.6\times10^{-19}\ C\\
\varepsilon_\circ &= 8.85\times10^{-12}\ C^2/Nm^2\\
h &= 6.63\times10^{-34}\ Js\\
Z &= 1\\
f_1 &= ?\\
f_2 &= ?
\end{aligned}$$

The frequency of revolution of an electron in the n$^{\text{th}}$ orbit is given by

$$f_n = \frac{me^4Z^2}{4\pi\varepsilon_\circ^2h^3}\frac{1}{n^3}$$

For first orbit, $n = 1$.

$$f_1 = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4 (1)^2}{4 \times 3.14 \times (8.85 \times 10^{-12})^2 \times (6.63 \times 10^{-34})^3} \frac{1}{(1)^3}$$

$$f_1 = \frac{59.6377 \times 10^{-107}}{2.8817 \times 10^5 \times 10^{-126}}$$

$$f_1 = 2.06 \times 10^{-4} \times 10^{14}$$

$$f_1 = 2.06 \times 10^{10} \text{ rev/s}$$

for second orbit, $n = 2$.

$$f_2 = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4 (1)^2}{4 \times 3.14 \times (8.85 \times 10^{-12})^2 \times (6.63 \times 10^{-34})^3} \frac{1}{(2)^3}$$

$$f_2 = \frac{59.6377 \times 10^{-107}}{2.8817 \times 10^5 \times 10^{-126}} \frac{1}{8}$$

$$f_2 = \frac{2.06}{8} \times 10^{-4} \times 10^{14}$$

$$f_2 = 0.257 \times 10^{10} \text{ rev/s}$$

Problem: 1.3- An electron collides with a hydrogen atom in its ground state and excites into state of $n = 2$. How much energy was given to hydrogen atom in this inelastic collision?

Solution

Here,

$$n_i = 1$$

$$n_f = 2$$

$$E_1 = -13.6 \ eV$$

$$\Delta E = ?$$

Energy change of an hydrogen atom that goes from its initial state of quantum number n_i to final state of quantum number n_f is

$$\Delta E = E_f - E_i = \frac{E_1}{n_f^2} - \frac{E_1}{n_i^2}$$
$$\Delta E = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right)$$
$$\Delta E = -13.6\left(\frac{1}{4} - \frac{1}{1}\right)$$
$$\Delta E = 10.2\ eV$$

Problem: 1.4- Experimentally it had been proved that energy of order of 13.6 eV is required to separate a hydrogen atom into proton and an electron. Find orbital radius and velocity of electron in a hydrogen atom.

Solution

$$m = 9.1 \times 10^{-31}\ kg$$
$$e = 1.6 \times 10^{-19}\ C$$
$$\varepsilon_\circ = 8.85 \times 10^{-12}\ C^2/Nm^2$$
$$E = -13.6\ eV = -13.6 \times 1.6 \times 10^{-19}\ J$$
$$E = -2.2 \times 10^{-18}\ J$$
$$r = ?$$
$$v = ?$$

Now, we know that

$$r = -\frac{e^2}{8\pi\varepsilon_\circ E}$$
$$r = -\frac{(1.6 \times 10^{-19})^2}{8 \times 3.14 \times 8.85 \times 10^{-12} \times -2.2 \times 10^{-18}}$$
$$r = 5.3 \times 10^{-11}\ m$$

Also, we know that

$$v = \frac{e}{\sqrt{4\pi\varepsilon_\circ m r}}$$

$$
\begin{aligned}
v &= \frac{1.6 \times 10^{-19}}{\sqrt{4 \times 3.14 \times 8.85 \times 10^{-12} \times 9.1 \times 10^{-31} \times 5.3 \times 10^{-11}}} \\
v &= 2.2 \times 10^{6}\ m/s
\end{aligned}
$$

Problem: 1.5- Calculate the ground state energy of electron in the hydrogen atom where $h = 6.63 \times 10^{-34}\ Js, R = 1.097 \times 10^{7}\ m^{-1}$.

Solution

$$
\begin{aligned}
h &= 6.63 \times 10^{-34}\ Js \\
R &= 1.097 \times 10^{7}\ m^{-1} \\
c &= 3 \times 10^{8}\ m/s \\
n &= 1 \\
E_1 &= ?
\end{aligned}
$$

Now, using the formula of energy

$$
\begin{aligned}
E_n &= -\frac{hcR}{n^2} \\
E_1 &= -\frac{6.63 \times 10^{-34} \times 3 \times 10^{8} \times 1.097 \times 10^{7}}{1^2} \\
E_1 &= -2.18 \times 10^{-18}\ J \\
E_1 &= -\frac{2.18 \times 10^{-18}}{1.6 \times 10^{-19}}\ eV \\
E_1 &= -13.6\ eV
\end{aligned}
$$

Chapter 2

One Electron System

SOLVED PROBLEMS

Problem: 2.1- If K, L and M energy levels of platinum are 80000 eV, 14000 eV and 5000 eV respectively. Calculate the wavelength of K_α and K_β lines from platinum.

Solution

$$\begin{aligned}
\text{Energy of } K \text{ level} &= E_K \;=\; 80000\ eV \\
\text{Energy of } K \text{ level} &= E_K \;=\; 80000 \times 1.6 \times 10^{-19}\ J \\
\text{Energy of } L \text{ level} &= E_L \;=\; 14000 \times 1.6 \times 10^{-19}\ J \\
\text{Energy of } M \text{ level} &= E_M \;=\; 5000 \times 1.6 \times 10^{-19}\ J \\
h &= 6.63 \times 10^{-34}\ Js \\
c &= 3 \times 10^{8}\ m/s \\
\text{Wavelength of } K_\alpha \text{ line} &= \lambda_\alpha \;=\; ? \\
\text{Wavelength of } K_\beta \text{ line} &= \lambda_\beta \;=\; ?
\end{aligned}$$

Since, we know that

$$\lambda_\alpha = \frac{hc}{E_K - E_L}$$

$$\lambda_\alpha = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{80000 \times 1.6 \times 10^{-19} - 14000 \times 1.6 \times 10^{-19}}$$

$$\lambda_\alpha = \frac{19.8 \times 10^{-26}}{66 \times 1.6 \times 10^{-16}}$$

$$\lambda_\alpha = 0.1875 \times 10^{-10}\ m$$

$$\lambda_\alpha = 0.1875\ A^\circ$$

Similarly,

$$\lambda_\beta = \frac{hc}{E_M - E_K}$$

$$\lambda_\beta = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5000 \times 1.6 \times 10^{-19} - 80000 \times 1.6 \times 10^{-19}}$$

$$\lambda_\beta = 0.165 \times 10^{-10}\ m$$

$$\lambda_\beta = 0.165\ A^\circ$$

Problem: 2.2- Electrons are accelerated in a television tube through potential difference of 10 kV. Find the highest frequency and minimum wavelength of the electromagnetic waves emitted, when these strike the screen of the tube. In which region of the spectrum will these waves lie?

Solution

$$h = 6.63 \times 10^{-34}\ Js$$

$$c = 3 \times 10^{8}\ m/s$$

$$e = 1.6 \times 10^{-19}\ C$$

$$V = 10\ kV = 10 \times 10^{3}\ V$$

$$\lambda_{\min} = ?$$

$$\nu_{\max} = ?$$

Minimum wavelength is given as

$$\lambda_{\min} = \frac{hc}{eV}$$

$$\lambda_{\min} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 10 \times 10^{3}}$$

$$\lambda_{\min} = \frac{1.242 \times 10^{-6}}{10^{4}}$$

$$\lambda_{\min} = 1.242 \times 10^{-10}$$

$$\lambda_{\min} = 1.242\ A^{\circ}$$

Also, we know that

$$\nu_{\max}\lambda_{\min} = c$$

$$\nu_{\max} = \frac{c}{\lambda_{\min}}$$

$$\nu_{\max} = \frac{3 \times 10^{8}}{1.242 \times 10^{-10}}$$

$$\nu_{\max} = 2.42 \times 10^{18}\ Hz$$

Problem: 2.3- Calculate the wavelength $\lambda_{\min}$ for the continuous spectrum of X-rays emitted when 35 keV electrons fall on a molybdenum target.

Solution

$$h = 6.63 \times 10^{-34}\ Js$$

$$c = 3 \times 10^{8}\ m/s$$

$$K.E = 35\ keV\ =\ 35 \times 10^{3}\ eV$$

$$K.E = 35 \times 10^{3} \times 1.6 \times 10^{-19}\ J\ =\ 5.6 \times 10^{-15}\ J$$

$$\lambda_{\min} = ?$$

Since, we know that

$$K.E = hf\ =\ \frac{hc}{\lambda_{\min}}$$

$$\lambda_{\min} = \frac{hc}{K.E}$$
$$\lambda_{\min} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5.6 \times 10^{-15}}$$
$$\lambda_{\min} = 3.5 \times 10^{-11}\ m$$
$$\lambda_{\min} = 35 \times 10^{-10}\ m$$
$$\lambda_{\min} = 35\ A^{\circ}$$

Problem: 2.4- The wavelength of K_α line from the iron is 19.3 pm. Find the energy difference between the two states of iron atom that give rise to this transition. Find the corresponding energy difference for hydrogen atom. Why is the difference so much greater for iron than for hydrogen?

Solution

$$\lambda = 19.3\ pm = 19.3 \times 10^{-12}\ m$$
$$h = 6.63 \times 10^{-34}\ Js$$
$$c = 3 \times 10^{8}\ m/s$$
$$\Delta E = ?$$

Since,

$$\Delta E = hf = \frac{hc}{\lambda}$$
$$\Delta E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{19.3 \times 10^{-12}}$$
$$\Delta E = 1.03 \times 10^{-14}\ J$$

K_α lines comes when electron jumps from L-shell to K-shell $n = 2, p = 1$.

$$E_n = \frac{-13.6\ eV}{n^2}$$
$$E_1 = -13.6\ eV$$
$$E_2 = \frac{-13.6\ eV}{2^2}$$

$$E_2 = \frac{-13.6\ eV}{4}$$
$$E_2 = -3.4\ eV$$

Now,

$$\Delta E = E_2 - E_1$$
$$\Delta E = -3.4 - (-13.6)$$
$$\Delta E = -3.4 + 13.6$$
$$\Delta E = 10.2\ eV$$

The difference is much greater in iron because its nucleus contains 26 protons which exerts very large force on K and L shell electrons than single proton of hydrogen does.

Problem: 2.5- Calculate the minimum voltage that must be applied to an X-ray tube to produce X-ray photons of wavelength 0.1 A°.

Solution

$$h = 6.63 \times 10^{-34}\ Js$$
$$c = 3 \times 10^{8}\ m/s$$
$$e = 1.6 \times 10^{-19}\ C$$
$$\lambda_{\min} = 0.1\ A^\circ = 0.1 \times 10^{-10}\ m$$
$$V = ?$$

When electrons accelerated through a potential V strike a target the maximum frequency $\nu_{\max}$ of the emitted X-ray photon is given by

$$eV = h\nu_{\max.}$$
$$eV = \frac{hc}{\lambda_{\min}}$$
$$V = \frac{hc}{e\lambda_{\min}}$$

$$
\begin{aligned}
V &= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 0.1 \times 10^{-10}} \\
V &= 1.25 \times 10^{5}\ J/C \\
V &= 1.25 \times 10^{5}\ V
\end{aligned}
$$

Chapter 3

Many Body System

SOLVED PROBLEMS

Problem: 3.1- Statement determine the value of L, the quantum number describing the LL coupling of the angular momentum od two atomic orbital for two d electrons. What are the corresponding later symbols?

Solution

$$L = (\ell_1 + \ell_2), \ell_1 + \ell_2 - 1, \cdots (\ell_1 - \ell_2)$$

For two d electrons, $\ell_1 = 2, \ell_2 = 2$
Corresponding latter symbol?
For two d-electrons $\ell_1 = 2, \ell_2 = 2$

$$\begin{aligned} L &= 2 + 2 = 4 \\ L &= 2 + 2 - 1 = 3 \\ L &= 2 + 2 - 2 = 2 \\ L &= 2 + 2 - 3 = 1 \\ L &= 2 + 2 - 4 = |2 - 2| = 0 \end{aligned}$$

Which correspond to the letters "g, f, d, p, s"

Problem: 3.2- Calculate the spin-orbit interacting splitting of a level corresponding to $n = 2$, and $\ell = 1$ of Hydrogen atom

Solution

For Hydrogen atom

$$\begin{aligned} n =& 2 \\ \ell =& 1 \\ Z =& 1 \\ \varDelta T =& ? \end{aligned}$$

$$\varDelta = 5.84 \frac{Z^4}{n^3 \ell(n+1)} cm^{-1} \tag{3.1}$$

As we know that, for Hydrogen atom $n = 2, \ell = 1$ and $Z = 1$. So by putting values in Eq.(3.1)

$$\begin{aligned} \varDelta T &= 5.84 \frac{(1)^4}{2^3 \times (1 \times 2)} cm^{-1} \\ \varDelta T &= 5.84 \frac{1}{8 \times 1 \times 2} cm^{-1} \\ \varDelta T &= 5.84 \frac{1}{16} \\ \varDelta T &= 0.365 \ cm^{-1} \end{aligned}$$

Problem: 3.3- Radiation coming form transition $n = 2$ to $n = 1$ of Hydrogen atom falls on Helium ions in $n = 1$ and $n = 2$ states. What are the possible transition of Helium?

Solution

Transition is possible on Helium is $n = 1, n = 2$ So $E_1 =?$

Let us check the transition is possible on $He, n = 1, n = 2$

$$E_1 = 4 \times 13.6 \left(1 - \frac{1}{4}\right)$$
$$E_1 = 54.4 \left(\frac{4-1}{4}\right)$$
$$E_1 = 54.4 \times 0.75$$
$$E_1 = 40.8eV \qquad (E_1 > E \text{ is not possible})$$
$$E_2 = 4 \times 13.6 \left(1 - \frac{1}{9}\right)$$
$$E_2 = 48.3eV \qquad (E_2 > E_1 \text{ is not possible})$$

Hence transition is not possible when $n = 1$ and $n = 2$.

Problem: 3.4- Using p-p electron configuration and d-d electron configuration. Find all the possible values of the total angular momentum.

Solution

(1). For p-p electron configuration,

For first electron

$$\ell_1 = 1$$
$$S_1 = \frac{1}{2}$$

For first electron

$$\ell_2 = 1$$
$$S_2 = \frac{1}{2}$$

Resultant orbital momentum is

$$L = (\ell_1 + \ell_2) \text{ to } (\ell_1 - \ell_2)$$
$$L = 0, 1, 2$$

and resultant spin momentum is

$$S = (S_1 + S_2) \text{ to } (S_1 - S_2)$$
$$S = 0, 1$$

Total angular momentum is

$$J = L + S \text{ to } L - S$$

For $S = 0$ multiplicity $(2S + 1) = 1$

$$J = L = 0, 1, 2$$

The singular terms are

$$^1s_0,\ ^1p_1,\ ^1d_2$$

For $S = 1$ multiplicity $(2S + 1) = 3$
For $L = 0, J = 1$ for $L = 1, J = 0, 1, 2$ and for $L = 2, J = 1, 2, 3$
The singlet and triplet terms are $^3s_1,\ ^3p_{0,1,2},\ ^3D_{0,1,2}$
(2). For d-d electron configuration,
For first electron

$$\ell_1 = 2$$
$$S_1 = \frac{1}{2}$$

For second electron

$$\ell_2 = 2$$
$$S_2 = \frac{1}{2}$$

Resultant orbital momentum is

$$L = (\ell_1 + \ell_2) \text{ to } (\ell_1 - \ell_2)$$
$$L = 0, 1, 2, 3, 4$$

and resultant spin momentum is

$$\begin{aligned}S =&(S_1 + S_2) \text{ to } (S_1 - S_2)\\ S =&0, 1\end{aligned}$$

Total angular momentum is

$$J = L + S \text{ to } L - S$$

For $S = 0$ multiplicity $m = (2S + 1) = 1$

$$J = L = 0, 1, 2, 3, 4$$

The singlet terms are

$$^1s_0,\ ^1p_1,\ ^1D_2,\ ^1F_3,\ ^1G_4$$

For $S = 1$ multiplicity $m = (2S + 1) = 3$
when

$$\begin{aligned}L = 0 \quad &, \quad J = 1\\ L = 1 \quad &, \quad J = 0, 1, 2\\ L = 2 \quad &, \quad J = 1, 2, 3\\ L = 3 \quad &, \quad J = 2, 3, 4\\ L = 4 \quad &, \quad J = 3, 4, 5\end{aligned}$$

The singlet and triplet terms are

$$^3s_1,\ ^3p_{0,1,2},\ ^3d_{1,2,3},\ ^3f_{2,3,4},\ ^3g_{3,4,5}$$

Problem: 3.5- A Hydrogen atoms emits ultraviolet radiation of wavelength $102.5nm$. What are the Quantum numbers of states involved in the transition.

Solution

As light is emitted in ultraviolet range the lines lies in Hymen series

$$\begin{aligned}\frac{1}{\lambda} =& R\left(\frac{1}{n_1^2 - \frac{1}{n_2^2}}\right)\\ \frac{1}{102.5\times10^{-9}} =& 1.1\times10^7\left(\frac{1}{2^2}-\frac{1}{n_2^2}\right)\\ \frac{10^9}{102.3} =& 1.1\times10^7\left(1-\frac{1}{n_1^2}\right)\\ 1-\frac{1}{n_2^2} =& \frac{10^2}{102.5\times1.1\times10^7}\\ \frac{1}{n_2^2} =& \frac{1-100}{102.5\times1.1}\\ n_2 =& 2.97 = 3\end{aligned}$$

$$\boxed{n_2 \approx 3}$$

Chapter 4

Interaction with Field

SOLVED PROBLEMS

Problem: 4.1- Calculate the Lande-factor g_j for the level $^3d_3$.

Solution

For $^3d_3$,

$$\begin{aligned} 2s+1 &= 3 \\ 2s &= 3-1 = 2 \\ s &= 1 \\ l &= 2 \\ j &= 3 \\ g_j &= ? \end{aligned}$$

Since the Lande-factor is given as

$$\begin{aligned} g_j &= 1 + \left[\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}\right] \\ g_j &= 1 + \left[\frac{3(3+1)+1(1+1)-2(2+1)}{2(3)(3+1)}\right] \end{aligned}$$

$$g_j = 1 + \left[\frac{12+2-6}{24}\right]$$
$$g_j = 1 + \frac{1}{3}$$
$$g_j = \frac{4}{3}$$

Problem: 4.2- Find the minimum magnetic field needed for Zeeman effect to be observed in a spectral line of 400 nm wavelength, when a spectrometer, whose resolution is 0.010 nm is used.

Solution

$$\begin{aligned} \lambda &= 400\ nm\ =\ 400 \times 10^{-9}\ m \\ \Delta\lambda &= 0.010\ nm\ =\ 0.01 \times 10^{-9}\ m \\ c &= 3 \times 10^{8}\ m/s \\ e &= 1.6 \times 10^{-19}\ C \\ m &= 9.11 \times 10^{-31}\ kg \\ B &= ? \end{aligned}$$

Using the formula

$$\begin{aligned} c &= \nu\lambda \\ \nu &= \frac{c}{\lambda} \\ \Delta\nu &= -\frac{c}{\lambda^2}\Delta\lambda \end{aligned}$$

Ignoring the negative sign,

$$\begin{aligned} \Delta\nu &= \frac{c}{\lambda^2}\Delta\lambda \\ \Delta\nu &= \frac{3 \times 10^{8}}{(400 \times 10^{-9})^2} \times 0.01 \times 10^{-9} \\ \Delta\nu &= \frac{30}{16} \times 10^{10} \\ \Delta\nu &= 1.87 \times 10^{10}\ \text{cycles/sec} \end{aligned}$$

As the separation of Zeeman component is

$$\begin{aligned}
\Delta\nu &= \frac{eH}{4\pi m} \\
B &= \frac{4\pi m \Delta\nu}{e} \\
B &= \frac{4 \times 3.14 \times 9.11 \times 10^{-31} \times 1.87 \times 10^{10}}{1.6 \times 10^{-19}} \\
B &= 133.7 \times 10^{-2} \\
B &= 1.337\ Wb/m^2
\end{aligned}$$

Problem: 4.3- A sample of a certain element is placed in a 0.300 T magnetic field and suitably excited. How far apart are the Zeeman components of the 450 nm spectral line of this element?

Solution

Given data:

$$\begin{aligned}
\lambda &= 450\ nm = 450 \times 10^{-9}\ m \\
B &= 0.300\ T \\
c &= 3 \times 10^{8}\ m/s \\
e &= 1.6 \times 10^{-19}\ C \\
m &= 9.11 \times 10^{-31}\ kg \\
\Delta\lambda &= ?
\end{aligned}$$

The separation of the Zeeman components is

$$\Delta\nu = \frac{eH}{4\pi m}$$

Using the formula

$$\begin{aligned}
c &= \nu\lambda \\
\nu &= \frac{c}{\lambda} \\
\Delta\nu &= -\frac{c}{\lambda^2}\Delta\lambda
\end{aligned}$$

Ignoring the negative sign,

$$\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda\Delta\lambda = \qquad \frac{\lambda^2\Delta\nu}{c}$$

$$\Delta\lambda = \frac{eH\lambda^2}{4\pi mc}$$

$$\Delta\lambda = \frac{1.6\times10^{-19}\times0.300\times(450\times10^{-9})^2}{4\times3.14\times9.11\times10^{-31}\times3\times10^8}$$

$$\Delta\lambda = 2.83\times10^{-12}\ m$$

$$\Delta\lambda = 2.83\ pm$$

Problem: 4.4- A spectral line of wavelength 4500 A° when placed in a magnetic field of 10 T is observed to be a normal Zeeman triplet. Calculate the wavelength separation between composition of triplet.

Solution

$$\lambda = 4500\ A^\circ = 4500\times10^{-10}\ m$$

$$H = 10\ T$$

$$c = 3\times10^8\ m/s$$

$$e = 1.6\times10^{-19}\ C$$

$$m = 9.11\times10^{-31}\ kg$$

$$\Delta\lambda = ?\ ;\quad \lambda_1 = ?\ ;\quad \lambda_2 = ?\ ;\quad \lambda_3 = ?$$

Now, using the formula

$$\Delta\lambda = \pm\frac{eH\lambda^2}{4\pi mc}$$

$$\Delta\lambda = \pm\frac{1.6\times10^{-19}\times10\times(4500\times10^{-10})^2}{4\times3.14\times9.11\times10^{-31}\times3\times10^8}$$

$$\Delta\lambda = \pm 0.943\times10^{-10}\ m$$

$$\Delta\lambda = \pm 0.943\ A^\circ$$

Hence Zeeman triplet has wavelength

$$\lambda_1 = 4500 - \Delta\lambda$$

$$\lambda_1 = 4500 - 0.943$$
$$\lambda_1 = 4499.057\ A^\circ$$
$$\lambda_2 = 4500 - 0 = 4500\ A^\circ$$
$$\lambda_3 = 4500 + \Delta\lambda$$
$$\lambda_3 = 4500 + 0.943$$
$$\lambda_3 = 4500.943\ A^\circ$$

Problem: 4.5- The Zeeman components of a 500 nm spectral line are 0.0106 nm apart when the magnetic field is 0.40 T. Find the ratio $\frac{e}{m}$ for the electron from this data.

Solution

$$\lambda = 500\ nm = 500 \times 10^{-9}\ m$$
$$\Delta\lambda = 0.0106\ nm = 0.0106 \times 10^{-9}\ m$$
$$c = 3 \times 10^8\ m/s$$
$$e = 1.6 \times 10^{-19}\ C$$
$$m = 9.11 \times 10^{-31}\ kg$$
$$H = 0.40\ T$$
$$\frac{e}{m} = ?$$

Using the formula

$$c = \nu\lambda$$
$$\nu = \frac{c}{\lambda}$$
$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$$

Ignoring the negative sign,

$$\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda$$
$$\Delta\nu = \frac{3 \times 10^8}{(500 \times 10^{-9})^2} \times 0.0106 \times 10^{-9}$$
$$\Delta\nu = 1 \times 10^{10}\ \text{cycles/sec}$$

As separation of Zeeman components is

$$\begin{aligned}
\Delta\nu &= \frac{eH}{4\pi m} \\
\frac{e}{m} &= \frac{4\pi\Delta\nu}{B} \\
\frac{e}{m} &= \frac{4\times 3.14\times 1\times 10^{10}}{0.40} \\
\frac{e}{m} &= 31.4\times 110^{10}\ Hz/T
\end{aligned}$$

Chapter 5

Molecules

SOLVED PROBLEMS

Problem: 5.1- In CO the $J = 0 \rightarrow J = 1$ absorption line occurs at a frequency of $1.15 \times 10^{11}\ Hz$. What is the bond length of the CO molecule?

Solution

First we find the moment of inertia of this molecule. As we know that

$$\begin{aligned}
\nu &= \frac{\hbar}{2\pi I_{\text{CO}}}(J+1) \\
I_{\text{CO}} &= \frac{\hbar}{2\pi\nu}(J+1) \\
I_{\text{CO}} &= \frac{1.054 \times 10^{-34}}{2\pi \times 1.15 \times 10^{11}}(0+1) \\
I_{\text{CO}} &= \frac{1.054 \times 10^{-34}}{2\pi \times 1.15 \times 10^{11}} \\
I_{\text{CO}} &= 1.46 \times 10^{-46}\ kgm^2
\end{aligned}$$

Since the reduced mass of the CO molecule is $\mu = 1.14 \times 10^{-26}\ kg$. As we know that

$$I = \mu R^2$$

$$
\begin{aligned}
R &= \sqrt{\frac{I}{\mu}} \\
R &= \sqrt{\frac{1.46 \times 10^{-46}}{1.14 \times 10^{-26}}} \\
R &= 1.13 \times 10^{-10}\ m \\
R &= 0.113\ nm
\end{aligned}
$$

Problem: 5.2- The carbon monoxide (CO) molecule has a bond length R of 0.113 nm and the masses of the ^{12}C and ^{16}O atoms are respectively $1.99 \times 10^{-26}\ kg$ and $2.66 \times 10^{-26}\ kg$. Find the energy and the angular velocity of the CO molecule when it is in its lowest rotational state.

Solution

The reduced mass μ of the CO molecule is

$$
\begin{aligned}
\mu &= \frac{m_1 m_2}{m_1 + m_2} \\
\mu &= \frac{1.99 \times 10^{-26} \times 2.66 \times 10^{-26}}{1.99 \times 10^{-26} + 2.66 \times 10^{-26}} \\
\mu &= 1.14 \times 10^{-26}\ kg
\end{aligned}
$$

And its moment of inertia is

$$
\begin{aligned}
I &= \mu R^2 \\
I &= 1.14 \times 10^{-26} \times (1.13 \times 10^{-10})^2 \\
I &= 1.46 \times 10^{-46}\ kgm^2
\end{aligned}
$$

The lowest rotational energy level corresponds to $J = 1$ and for this level in CO,

$$
\begin{aligned}
E_{J=1} &= \frac{J(J+1)\hbar^2}{2I} \\
E_{J=1} &= \frac{1(1+1)\hbar^2}{2I} && \because J = 1 \\
E_{J=1} &= \frac{2\hbar^2}{2I}
\end{aligned}
$$

$$E_{J=1} = \frac{\hbar^2}{I}$$
$$E_{J=1} = \frac{(1.054 \times 10^{-34})^2}{1.46 \times 10^{-46}}$$
$$E_{J=1} = 7.61 \times 10^{-23}\ J$$
$$E_{J=1} = 4.76 \times 10^{-4}\ eV$$

It is not a lot of energy, and at room temperature, when $kT \approx 2.6 \times 10^{-2}\ eV$, nearly all the molecules in a sample of CO are in excited rotational states. The angular velocity of the CO molecule when $J = 1$ is

$$E = \frac{1}{2} I\omega^2$$
$$\omega = \sqrt{\frac{2E}{I}}$$
$$\omega = \sqrt{\frac{2 \times 7.61 \times 10^{-23}}{1.46 \times 10^{-46}}}$$
$$\omega = 3.23 \times 10^{11}\ \text{rad/s}$$

Problem: 5.3- The spacing between vibrational level of CO molecule is 0.08 eV. Calculate the value of force constant. Take mass of carbon atom 12 and that of oxygen 16 times mass of proton $1.67 \times 10^{-27}\ kg$.

Solution

Reduced mass of CO molecule is given as

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$
$$\mu = \frac{12 \times 16}{12 + 16} \times 10^{-27}$$
$$\mu = 1.14 \times 10^{-26}\ kg$$

Since, we know that

$$\Delta E = E_{\nu+1} - E_{\nu} = h\nu_{\circ}$$

$$
\begin{aligned}
\Delta E &= \; = h\nu_{\circ} = \frac{h}{2\pi}\sqrt{\frac{K}{\mu}} = 0.08\ eV \\
\frac{h}{2\pi}\sqrt{\frac{K}{\mu}} &= 0.08 \times 1.6 \times 10^{-19}\ J \\
\sqrt{\frac{K}{\mu}} &= \frac{0.08 \times 1.6 \times 10^{-19} \times 2\pi}{h} \\
\frac{K}{\mu} &= \left(\frac{0.08 \times 1.6 \times 10^{-19} \times 2\pi}{h}\right)^2 \\
K &= \left(\frac{0.08 \times 1.6 \times 10^{-19} \times 2\pi}{h}\right)^2 \times \mu \\
K &= \left(\frac{0.08 \times 1.6 \times 10^{-19} \times 2\pi}{6.63 \times 10^{-34}}\right)^2 \times 1.14 \times 10^{-26} \\
K &= 167.8\ Nm^{-1}
\end{aligned}
$$

Problem: 5.4- The force constant in CO molecule is 1870 N/m. Calculate the frequency of vibrational of the molecule and spacing between its vibrational energy levels in eV. Given that reduced mass of CO is $1.14 \times 10^{-26}\ kg$, $h = 6.63 \times 10^{-27}\ erg - sec$ and $1\ eV = 1.60 \times 10^{-12}\ erg$.

Solution

Frequency of vibration of the CO molecule is

$$
\begin{aligned}
\nu_{\text{osc}} &= \frac{1}{2\pi}\sqrt{\frac{K}{\mu}} \\
\nu_{\text{osc}} &= \frac{1}{2 \times 3.14}\sqrt{\frac{1870}{1.14 \times 10^{-26}}} \\
\nu_{\text{osc}} &= 6.45 \times 10^{13}\ /sec
\end{aligned}
$$

Now, the vibrational energy of diatomic molecule is

$$
E_V = h\nu_{\text{osc}}\left(V + \frac{1}{2}\right) \qquad \because V = 0, 1, 2, \cdots
$$

The separation between two successive vibrational energy level is

$$\begin{aligned}
\Delta E &= E_{V+1} - E_V \\
\Delta E &= h\nu_{\text{osc}}\left(V + \frac{3}{2}\right) - h\nu_{\text{osc}}\left(V + \frac{1}{2}\right) \\
\Delta E &= h\nu_{\text{osc}} \\
\Delta E &= 6.63 \times 10^{-27} \times 6.45 \times 10^{13} \\
\Delta E &= 42.76 \times 10^{-14}\ erg \\
\Delta E &= \frac{42.76 \times 10^{-14}}{1.60 \times 10^{-12}}\ eV \\
\Delta E &= 26 \times 110^{-2}\ eV
\end{aligned}$$

Problem: 5.5- The $J = 1 \leftarrow 0$ transition in HCl occurs at 20.68 cm^{-1}. Regarding the molecule to be rigid rotator, calculate the wavelength of the transition $J = 15 \leftarrow 14$.

Solution

The wave number of the radiation absorbed in a rotational transition from J to $J+1$ is given by

$$\bar{\nu} = 2B(J+1)$$

where J refers to the lower state. For a transition from $J = 0$ to $J = 1$, we have

$$\begin{aligned}
\bar{\nu} &= 2B \\
2B &= 20.68\ cm^{-1} \\
B &= 10.34\ cm^{-1}
\end{aligned}$$

Again, the wave number of the radiation absorbed in $J = 15 \leftarrow 14$ is given by

$$\bar{\nu} = 2B(J+1)$$

where J refers to the lower state.

$$\begin{aligned}
\bar{\nu} &= 2B(J+1) \\
\bar{\nu} &= 2B(14+1) \\
\bar{\nu} &= 2 \times 10.34 \times 15 \\
\bar{\nu} &= 310.2\ cm^{-1}
\end{aligned}$$

The corresponding wavelength is

$$\begin{aligned}
\lambda &= \frac{1}{\bar{\nu}} \\
\lambda &= \frac{1}{310.2} \\
\lambda &= 32 \times 10^{-4}\ cm
\end{aligned}$$

Chapter 6

LASER

SOLVED PROBLEMS

Problem: 6.1- A He-Ne has light at a wavelength of 6328×10^{-10} m and has an out put power of 2.3×10^{-3} watt. How many photons are emitted each minute by this laser.

Solution

Energy of one photon is E_{Photon}. As, we know that

$$\begin{aligned}
E_{\text{Photon}} &= \frac{hc}{\lambda} \\
E_{\text{Photon}} &= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{6328 \times 10^{-10}} \\
E_{\text{Photon}} &= 3 \times 10^{-19}\ J
\end{aligned}$$

Since the number of photons are

$$\begin{aligned}
\text{No. of photon} = N &= \frac{P \times t}{E_{\text{Photon}}} \\
\text{No. of photon} = N &= \frac{2.3 \times 10^{-3} \times 60}{3 \times 10^{-19}} \\
\text{No. of photon} = N &= 46 \times 10^{16}
\end{aligned}$$

Problem: 6.2- A ruby laser emits light at wavelength 694.4 nm. If a laser pulse is emitted for 12 ps and the energy released per pulse is 150 mJ. What is the length of the pulse. How many photons are in each pulse.

Solution

Given data:

$$\begin{aligned}
\lambda &= 694.4\ nm\ =\ 694.4 \times 10^{-9}\ m \\
t &= 12\ ps\ =\ 12 \times 10^{-12}\ s \\
E &= 150\ mJ\ =\ 150 \times 10^{-3}\ J \\
c &= 3 \times 10^{8}\ m/s \\
S &= ? \\
n &= ?
\end{aligned}$$

Using the formula

$$\begin{aligned}
S &= ct\ =\ 3 \times 10^{8} \times 12 \times 10^{-12} \\
S &= 0.0036\ m
\end{aligned}$$

Number of photons can be determined by

$$\begin{aligned}
E &= \frac{nhc}{\lambda} \\
n &= \frac{E\lambda}{hc} \\
n &= \frac{150 \times 10^{-3} \times 694.4 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} \\
n &= 5.2 \times 10^{17}
\end{aligned}$$

Problem: 6.3- A three level laser emits a laser light at 550 nm near the center of visible band. If optical pumping is shut off, what will be the ratio of the population of the upper level (energy E_2) to that of lower level (energy E_1) at temperature 300 K. At what temperature this condition would the ratio of population e half.

Solution

$$\lambda = 550\ nm = 550 \times 10^{-9}\ m$$
$$T = 300\ K$$
$$h = 6.63 \times 10^{-34}\ Js$$
$$k = 1.38 \times 10^{-23}$$
$$c = 3 \times 10^{8}\ ms^{-1}$$
$$\frac{n(E_2)}{n(E_1)} = ?$$

Since, we know that

$$\frac{n(E_2)}{n(E_1)} = e^{-\frac{E_2 - E_1}{kT}}$$
$$\frac{n(E_2)}{n(E_1)} = e^{-\frac{1}{kT}\left(\frac{hc}{\lambda}\right)}$$
$$\frac{n(E_2)}{n(E_1)} = e^{-\frac{1}{1.38\times10^{-23}\times300}\left(\frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}}\right)}$$
$$\frac{n(E_2)}{n(E_1)} = e^{-87.3}$$
$$\frac{n(E_2)}{n(E_1)} = 1.3 \times 10^{-38}$$

When ratio is half

$$\frac{1}{2} = e^{-\frac{1}{kT}\left(\frac{hc}{\lambda}\right)}$$
$$2 = e^{\frac{1}{kT}\left(\frac{hc}{\lambda}\right)}$$

$$\ln 2 = \ln e^{\frac{1}{kT}\left(\frac{hc}{\lambda}\right)}$$
$$\ln 2 = \frac{hc}{\lambda}\frac{1}{kT}$$
$$T = \frac{hc}{k\lambda \ln 2}$$
$$T = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 550 \times 10^{-9} \times 0.6930}$$
$$T = 37814\ K$$

Problem: 6.4- The wavelength at which the spectral radiancy of the sun is maximum is 500 nm. Calculate the temperature of the sun.

Solution

Given data:

$$\lambda_{\text{max.}} = 500\ nm = 500 \times 10^{-9}\ m$$
$$b = 2898\ \mu m - K = 2898 \times 10^{-6}\ m - K$$
$$T = ?$$

Now, using the formula of temperature

$$\lambda_{\text{max.}} = \frac{b}{T}$$
$$T = \frac{b}{\lambda_{\text{max.}}}$$
$$T = \frac{2898 \times 10^{-6}}{500 \times 10^{-9}}$$
$$T = 5796\ K$$

Problem: 6.5- Calculate the wavelength of maximum spectral radiancy for the sun with surface temperature of 5800 K.

Solution

Given data:

$$T = 5800\ K$$
$$b = 2898\ \mu m - K = 2898 \times 10^{-6}\ m - K$$
$$\lambda_{\text{max.}} = ?$$

Now, using the formula of wavelength

$$\lambda_{\text{max.}} = \frac{b}{T}$$
$$\lambda_{\text{max.}} = \frac{2898 \times 10^{-6}}{5800}$$
$$\lambda_{\text{max.}} = 4.9 \times 10^{-7}\ m$$


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