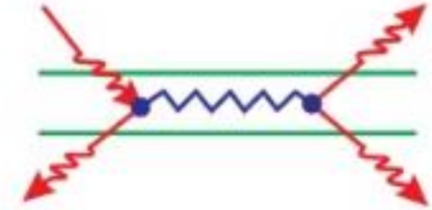

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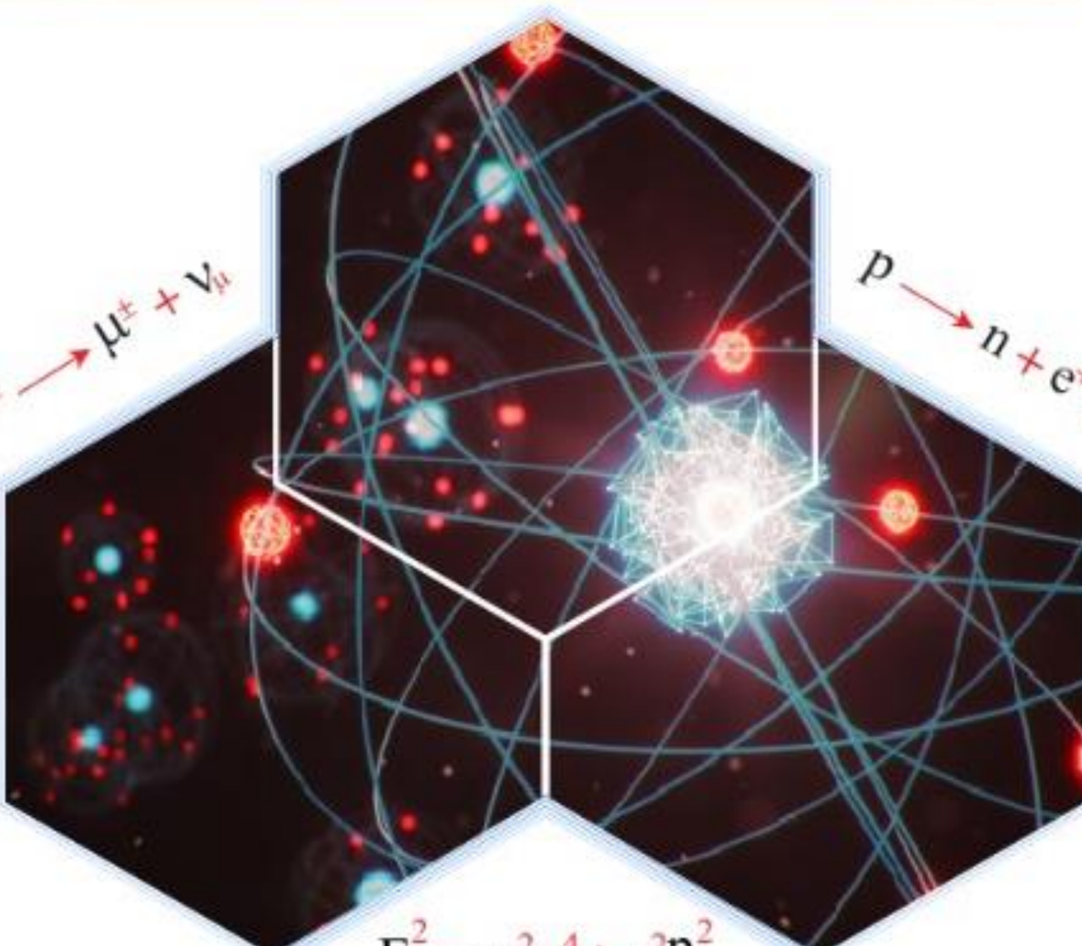

$\pi^{\pm} \longrightarrow \mu^{\pm} + \nu_{\mu}$

$p \longrightarrow n + e^{+} + \nu$

$$E^2 = m_\pi^2 c^4 + c^2 p^2$$

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TEACH YOURSELF

Nuclear Physics

2nd Edition

For **BS/M.Sc Physics** students of all Pakistani Universities

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Contents

Chapter 1

Basic Properties of Nucleus

SOLVED PROBLEMS

Problem: 1.1- Calculate the binding energy and average binding energy per nucleon of ${}_6C^{12}$.

Solution

Since ${}_6C^{12}$ has 6 protons and 6 neutrons, so

$$\begin{aligned} Mass\ of\ 6\ protons &= 6 \times 1.008665 \ = \ 6.05199\ amu \\ Mass\ of\ 6\ neutrons &= 6 \times 1.007825 \ = \ 6.04695\ amu \\ Total\ mass\ of\ constituens &= 12.09894\ amu \\ Mass\ of\ {}_6C^{12} &= 12.0000\ amu \\ Mass\ of\ deficiency &= 12.09894 - 12.0000 \ = \ 0.09894\ amu \\ 1\ amu &= 931.5\ MeV \end{aligned}$$

Then

$$= \ 0.09894 \times 931.5 \ = \ 92.16\ MeV$$

and average binding energy per nucleon is,

$$= \ \frac{92.16}{12} \ = \ 7.68\ MeV$$

Problem: 1.2- Estimate the value of A and identify the nucleus if its radius is given to be 3.46 fm.

Solution

Radius of the nucleus is $3.46 \times 10^{-15} m$. As we know

$$
\begin{aligned}
R &= R_{\circ} A^{1/3} \\
A^{1/3} &= \frac{R}{R_{\circ}} \\
&= \frac{3.46 \times 10^{-15}}{1.2 \times 10^{-15}} = 2.88 \\
A &= (2.883)^3 \\
A &= 23.97 \cong \ 24
\end{aligned}
$$

Problem: 1.3- Group the following nuclides as isotopes, isotones and isobars: ${}^{12}_{6}C, {}^{13}_{6}C, {}^{14}_{6}C, {}^{14}_{7}N, {}^{14}_{8}O, {}^{15}_{7}N, {}^{15}_{8}O, {}^{16}_{6}C, {}^{16}_{7}N, {}^{16}_{8}O, {}^{17}_{7}N, {}^{17}_{8}O$

Solution

Isotopes:

$$
\begin{aligned}
&{}^{12}_{6}C,\ {}^{13}_{6}C,\ {}^{14}_{6}C,\ {}^{16}_{6}C \\
&{}^{14}_{7}N,\ {}^{15}_{7}N,\ {}^{16}_{7}N,\ {}^{16}_{7}N \\
&{}^{14}_{8}O,\ {}^{15}_{8}O,\ {}^{16}_{8}O,\ {}^{17}_{8}O
\end{aligned}
$$

Isotones:

$$
\begin{aligned}
&{}^{12}_{6}C,\ {}^{14}_{8}O \\
&{}^{13}_{6}C,\ {}^{14}_{7}N,\ {}^{15}_{8}O \\
&{}^{14}_{6}C,\ {}^{15}_{7}N,\ {}^{16}_{8}O \\
&{}^{16}_{7}N,\ {}^{17}_{8}N
\end{aligned}
$$

Isobars:

$$
{}^{14}_{6}C,\ {}^{14}_{7}N,\ {}^{14}_{8}O
$$

$$^{15}_{7}N,\ ^{15}_{8}O$$
$$^{16}_{6}C,\ ^{16}_{7}N,\ ^{16}_{8}O$$
$$^{17}_{7}N,\ ^{17}_{8}O$$

Problem: 1.4- Assume 1 $amu = 1.66 \times 10^{-27}\ kg$, estimate the density of nuclear matter.

Solution

Protons and neutrons present in the nucleus constitute nuclear matter. Let mass of a nucleus $A = 40\ amu$

$$\begin{aligned}
Mass\ of\ this\ nucleus\ m &= 40 \times 1.66 \times 10^{-27}\ kg \\
&= 6.64 \times 10^{-26}\ kg \\
Radius\ r &= r_0 A^{\frac{1}{3}} \qquad where\ r_0 = 1.2\ fm\ and\ A = 40 \\
r &= 1.2 \times 40^{\frac{1}{3}} \times 10^{-15}\ m \\
r &= 4.1 \times 10^{-15}\ m \\
Volume\ of\ nucleons\ V &= \frac{4}{3} \times 3.1415926 \times (4.1 \times 10^{-15})^3\ m^3 \\
&= 2.886 \times 10^{-43}\ m^3
\end{aligned}$$

$$\begin{aligned}
Density &= \frac{Mass}{Volume} = \frac{m}{V} \\
&= \frac{6.65 \times 10^{-26}}{2.886 \times 10^{-43}} \\
Density\ of\ nuclear\ matter &= 2.3 \times 10^{17}\ \frac{kg}{m^3}
\end{aligned}$$

Problem: 1.5- Find the nuclear density of ^{235}U, if $r_\circ = 1.2\ fm$.

Solution

$$\begin{aligned}
Mass\ of\ 1\ nucleon &= 1.66 \times 10^{-27}\ kg \\
Mass\ of\ 235\ nucleons\ m &= 235 \times 1.66 \times 10^{-27}\ kg \\
&= 3.901 \times 10^{-25} kg \\
Radius\ of\ ^{235}U\ nucleons\ r &= r_0 A^{\frac{1}{3}} \\
&= 1.2 \times 10^{-15} \times 235^{\frac{1}{3}} \\
&= 7.405 \times 10^{-15}\ m \\
Volume\ of\ ^{235}U\ nucleus\ V &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3} \times 3.1415926 \times (7.405 \times 10^{-15})^3\ m^3 \\
&= 1.701 \times 10^{-42}\ m^3 \\
Desity\ of\ ^{235}U &= \frac{Mass}{Volume} \\
&= \frac{3.901 \times 10^{-25}}{1.701 \times 10^{-42}} \frac{kg}{m^3} \\
&= 2.29 \times 10^{17} \frac{kg}{m^3}
\end{aligned}$$

Problem: 1.6- Calculate the energy of electron at rest.

Solution

$$\begin{aligned}
\text{Mass of electron } m &= 9.1 \times 10^{-31}\, kg \\
\text{Energy } mc^2 &= 9.1 \times 10^{-31} \times (3 \times 10^8)^2\, J = 8.19 \times 10^{-14}\, J \\
1.6 \times^{-13}\ J &= 1 MeV \\
81.9 \times 10^{-15}\, J &= \frac{8.19 \times 10^{-14}}{1.6 \times 10^{-13}}\, MeV \\
&= 0.511\, MeV
\end{aligned}$$

Problem: 1.7- Estimate A value and identify the nucleus if its radius is given to be 3.46 fm.

Solution

Radius of the nucleus $R = 3.46\,fm$

We know,

$$r = r_\circ A^{1/3}; \qquad \text{and} \qquad r_\circ = 1.2\,fm$$

Therefore,

$$\frac{r}{r_\circ} = A^{1/3}; \qquad \Rightarrow \quad A^{1/3} = \frac{r}{r_\circ}$$
$$= \frac{3.46}{1.2} = 2.883;$$
$$A = (2.883)^3 = 23.97 \approx 24$$

Therefore, $A = 24$, the nucleus is ^{24}Mg.

Problem: 1.8- Calculate the binding energy of α-particle and express it in MeV and joules. Given $m_p = 1.00758\,amu$, $m_n = 1.00897\,amu$ and $m_{|textHe} = 4.0028\,amu$

Solution

Binding energy of α-particle is as,

$$BE = (Z_{m_p} + N_{m_n} - m_{\text{He}})\,amu$$
$$= (2 \times 1.00758 + 2 \times 1.00897 - 4.0028)amu = 0.0293\,amu$$

Now,

$$1\,amu = 931.49\,MeV$$
$$0.0293amu = 0.0293 \times 931\,MeV = 27.29\,MeV$$

Therefore, The binding energy of α-particle $= 27.29\,MeV$

$$1\,amu = 1.49239 \times 10^{-10} JS$$
$$0.0293\,amu = 0.0293 \times 1.49239 \times 10^{-10} = 4.37 \times 10^{-12} J$$

Problem: 1.9- Find the energy required in joules to break ^{12}C into 3 α-particles. The atomic mass of $^{12}C = 12\,amu$ and $m_{\text{He}} = 4.0026\,\text{amu}$.

Solution

We have the equation

$$^{12}_{6}\text{C} \longrightarrow 3\,^{4}_{2}\text{He}$$

Given

$$\begin{aligned}
\text{Mass of } ^{12}\text{C} &= 12\,amu \\
\text{Mass of } \alpha\text{-particle} &= 4.0026\,amu \\
\text{Mass of } 3\alpha\text{-particle} &= 3 \times 4.0026\,amu = 12.0078\,amu \\
\text{Difference in two masses} &= 12 - 12.0078 = -0.0078\,amu \\
1\,amu &= 1.49239 \times 10^{-10}\,J \\
0.0078\,amu &= -0.0078 \times 1.49239 \times 10^{-10} = -1.16 \times 10^{-12}\,J
\end{aligned}$$

Therefore, 1.16×10^{-12}J of energy is required to break up ^{12}C into 3 α-particles.

Problem: 1.10- Masses of He nucleus, proton and neutron in amu are 4.0026, 1.007895, and 1.008665. Find the energy required to knock out nucleons from the He nucleus.

Solution

We have the equation

$$^{4}_{2}\text{He} \longrightarrow 2p + 2n$$

Given:

$$\begin{aligned}
m_{\text{He}} &= 4.0026\,amu \\
m_p &= 1.007895\,amu \\
m_n &= 1.008665\,amu \\
\text{Mass of 2 protons and 2 neutrons} &= 2(1.007895) + 2(1.008665) = 4.03312\,amu \\
\Rightarrow \quad \text{Mass defect} &= 4.0026 - 4.03312 = -0.03052\,amu \\
\text{Equivalent energy} &= -0.03052 \times 931.49 = -28.43\,MeV
\end{aligned}$$

Energy required to knock out 2 protons and neutrons from He nucleus= $28.43\,MeV$.

Problem: 1.11- Find the binding energy of $^{56}_{26}$Fe in MeV. Given: $m_p = 1.007825\,amu$, $m_n = 1.008665\,amu$ and $m_{\text{Fe}} = 55.934939\,amu$

Solution

Binding energy (BE) is given as

$$BE = (26m_p + 30m_n - m_{\text{Fe}})\,amu$$
$$= (26 \times 1.007825 + 30 \times 1.008665 - 55.934939)\,amu = 0.5285\,amu$$

or Binding energy $BE = 0.5285 \times 931.49 = 492.29\,MeV$

Problem: 1.12- A singly charged positive ion is accelerated through a potential difference of 1000 V in a mass spectrograph. It then passes through a uniform magnetic field $B = 1500$ gauss, and then deflected intio a circular path of radius $0.122\,m$.
(i)- What is the speed of the ion?
(ii)- What is the mass of the ion?
(iii)- What is the mass number of the ion?

Solution

(i)- Using the relations $\frac{mv^2}{R} = eVB$ and $\frac{1}{2}mv^2 = eV$, where e is ionic charge and V is the potential applied, we get

$$v = \frac{2V}{RB}$$

Now,

$$V = 1000\,V\ ; \qquad R = 0.122\,m\ ; \qquad B = 1500 \times 10^{-4}\,tesla$$

Therefore,

$$v = \frac{2 \times 1000}{0.122 \times 1500 \times 10^{-4}}\,m = 1.093 \times 10^5\,m/s$$

(ii)- Mass of the ion (in amu),

$$M = \frac{2eV}{v^2} = \frac{2 \times 1.602 \times 10^{-19} \times 1000}{(1.093 \times 10^5)^2} = 2.682 \times 10^{-26} kg$$

(iii)-

$$\text{Mass number} = \frac{2.682 \times 10^{-26}}{1.673 \times 10^{-27}} \approx 16.03 \approx 16$$

Problem: 1.13- The mass of a deuteron is 2.014103 amu. If the masses of a proton and neutron are respectively 1.007825 amu and 1.008663 amu, find the mass defect and packing fraction.

Solution

$$\begin{aligned}\text{Mass defect } \mathit{\Delta} M &= m_p + m_n - m_d \\ &= 1.008825 + 1.008663 - 2.014104 = 0.002385\, amu = 2.22\, MeV\end{aligned}$$

$$\text{Packing fraction } f = \frac{\text{Mass} - A}{A} = \frac{2.014103 - 2}{2} = 0.00705$$

Problem: 1.14- Calculate the average binding energy per nucleon of ^{4}He nucleus. Given $m_{\text{He}} = 4.002643\, amu$ and $m_p = 1.007825\, amu$ and $m_n = 1.008665\, amu$.

Solution

Binding energy is given by

$$\begin{aligned}BE &= (2m_p + 2m_n - m_{\text{He}}) \times 931.47\, MeV \\ &= (2(1.007825) + 2(1.008665) - 4.002634) \times 931.47\, MeV \\ &= 0.03034 \times 931.47 = 28.3\, MeV\end{aligned}$$

$$\text{Binding energy per nucleon} = \frac{28.3}{4} = 7.07\, MeV$$

Chapter 2

Nature of Nuclear Forces

SOLVED PROBLEMS

Problem: 2.1- The speed of electron in a uniform electric field changes from $v_1 = 0.98\ c$; $v_2 = 0.99\ c$

1. Compute the change in mass.
2. Compute the work done on electron to change its velocity.
3. Calculate the accelerating potential in volts.

Solution

(1). From the theory of relativity, $m = \frac{m_0}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}$. So,

$$m_1 = \frac{m_0}{[1-(0.98)^2]^{\frac{1}{2}}}$$
$$m_1 = 5.025\ m_0$$

$$m_2 = \frac{m_0}{[1-(0.99)^2]^{\frac{1}{2}}}$$
$$m_2 = 7.088\ m_0$$
$$\Delta m = m_2 - m_1 = 2.063 m_0$$

(2). The work done will be equal to change in kinetic energy $K.E = k_2 k_1 = \Delta mc^2$

$$\begin{aligned} &= 1.88 \times 10^{28} \times (3 \times 10^8)^2 \\ &= 1.692 \times 10^{-13}\ J \\ &= \frac{1.692 \times 10^{-13}}{1.6 \times 10^{-13}} = 1.056 MeV \end{aligned}$$

(3). As $\Delta E = qV$ where V is the accelerating potential

$$\begin{aligned} V &= \frac{1.056 \times 10^6}{1.6 \times 10^{-19}} \\ &= 1.056 \times 10^6\ Volts = 1.056\ MeV \end{aligned}$$

Problem: 2.2- The meson theory of nuclear force assume that virtual exchange of pions. If a nucleon emit a virtual pion of rest mass $270me$. Show that the range of nuclear force is 1.43 fm.

Solution

We have $\Delta E = \Delta mc^2$ from uncertainty relation

$$\begin{aligned} \Delta E.\Delta t &= 1 \\ \Delta t &= \frac{1}{\Delta E} \\ \Delta t &= \frac{1}{\Delta mc^2} \end{aligned}$$

Assume that the emitted pion travels at the speed of light, distance traveled by it during this time is given by

$$\begin{aligned} R_0 &= c\Delta t \\ &= \frac{c}{\Delta mc^2} = \frac{1}{\Delta mc} \\ &= \frac{1.05 \times 10^{-34}}{270} \times (9.11 \times 10^{-31}) \times (3 \times 108) \\ &= 1.43 \times 10^{-15}\ m = 1.43\ fm \end{aligned}$$

Problem: 2.3- Calculate the mass of exchange particle, it the range of the force is about 0.2 fm

Solution

As we know the equation,

$$R = \frac{\hbar}{\Delta mc} \quad \Rightarrow \quad \Delta m = \frac{\hbar}{Rc}$$
$$= 1.95 \times 10^{-34} \times 0.25 \times 10^{-15} \times 3 \times 10^{8}$$
$$= 1.4 \times 10^{-27}\ kg$$

Problem: 2.4- Determine the radius of ^{208}Pb.

Solution

For ^{208}Pb, $A = 208$

$$r = r_0 A^{\frac{1}{3}}$$
$$where\ r_0 = 1.2 \times 10^{-15}\ m$$
$$Radis\ r\ of\ ^{208}Pb = 1.2 \times 10^{-15} \times 208^{\frac{1}{3}}$$
$$= 7.11 \times 10^{-15}\ m$$
$$Radius\ of\ ^{208}Pb = 7.11\ fm$$

Problem: 2.5- The experimentally measured mass of π-meson is 140 MeV/c^2. Show tat the range of the nuclear force is $\sim 1.4\,fm$.

Solution

The range of nuclear force is

$$r = \frac{\hbar}{m_\pi c} = \frac{\hbar c}{m_\pi c^2}$$

Given is $\quad c = 3 \times 10^8 ms^{-1}; \qquad \hbar = 1.05 \times 10^{-34}\, J - s$

$$m_\pi = 140\, MeV/c^2 = 140\, MeV/c^2 \times 1.6 \times 10^{-13}\, J/MeV/c^2 = 140 \times 1.6 \times 10^{-13}\, J$$

Now $\quad = \dfrac{1.05 \times 3 \times 10^{-26}\, J - m}{140 \times 1.6 \times 10^{-13}\, J}\, m = 1.4 \times 10^{-15} m = 1.4\, fm$

Problem: 2.6- What range of nuclear force is mediated by η° having a mass corresponding to $550\,MeV$?

Solution

Mass(energy) of the particle is,

$$\Delta E = 550\,MeV = 550 \times 1.6 \times 10^{-13}\,J$$

If we assume that the emitted particle moves with the speed of light, then the range of nuclear force is,

$$R = c\Delta t \cong c\frac{\hbar}{\Delta E} \qquad \because\ \Delta E\ \Delta t \cong \hbar$$

$$\Rightarrow\ R \cong \frac{ch}{2\pi\Delta E} = \frac{3 \times 10^{8} ms^{-1} \times 6.63 \times 10^{-34}\,Js}{2\pi \times 550 \times 1.6 \times 10^{-13}\,J} = 0.36 \times 10^{-15}\,m$$

Chapter 3

Nuclear Models

SOLVED PROBLEMS

Problem: 3.1-

The atomic mass of Zinc isotope ${}^{64}_{30}Zn$ is 63.929 u compare it's binding energy with particle.

Solution

The binding energy of ${}^{64}_{30}Zn$ is,

$$E_b = [(30)(1.007825\ u) + (34)(1.008665) - 63.939\ u]\left(931.49\ \frac{MeV}{u}\right)$$
$$= 559.1\ MeV$$

The semi empirical binding energy formula, using the coefficient in the text gives,

$$E_b = (14.1\ MeV)(64) - (13.0\ MeV)(64)^{\frac{2}{3}} - \frac{(0.595\ MeV)(30)(29)}{(64)^{\frac{1}{3}}} - \frac{(19.0\ MeV)(16)}{(64)} + \frac{33.5\ MeV}{(64)^{\frac{3}{4}}} = 561.7\ MeV$$

The plus sign is used for the last term because ${}^{64}_{30}Zn$ is an even-even nucleus. The difference between observed and calculated binding energy is less than 0.5%

Problem: 3.2- Calculate the atomic number of the most stable nucleus for given mass number A.

Solution

The most stable nucleus with a given mass number A is that which has a maximum value of the binding energy. Obviously one has to compute $\frac{\partial E_b}{\partial Z}$ with A constant, and equate it to zero. The expression for binding energy is,

$$E_b = a_1 A a_2 A^{\frac{2}{3}} + a_3 z^2 A^{\frac{-1}{3}}$$

Where a_1, a_2 and a_3 are constant. Using $N - Z = A - 2Z$, one finds,

$$\frac{\partial E_b}{\partial Z} = -2a_3 Z A^{\frac{-1}{3}} + 4a_4 (A2Z)^{-1} = 0$$

Substituting the numerical values of a_3 and a_4, one obtains;

$$Z = \frac{A}{\left(Z + 0.0157\ A^{\frac{2}{3}}\right)}$$

For all light nuclei, i.e. for small A, one can neglect the second term in the denominator, and obtains approximately $Z = \frac{1}{2}A$. This result is confirm experimentally.

Problem: 3.3- Isobars are nuclides that have the same mass number A. Drive a formula for atomic number of the most stable isobars of a given A and use it to find more stable isobars $A = 25$.

Solution

To find the value of Z for which binding energy E_b is maximum, which corresponds to maximum stability we must solve $\frac{dE_b}{dZ} = 0$ for Z. We have

$$\frac{dE_b}{dZ} = -\frac{a_3}{A^{\frac{1}{3}}}(2Z - 1) + \frac{4a_4}{A}(A - 2Z) = 0$$

$$Z = \frac{a_3 A^{\frac{-1}{3}} + 4a_4}{a_3 A^{\frac{-1}{3}} + 8a_4 A^{-1}}$$

$$Z = \frac{0.595 A^{\frac{-1}{3}} + 76}{1.19 A^{\frac{-1}{3}} + 152 A^{-1}}$$

For $A = 24$ this formula gives $Z = 11.7$ from which we conclude that $Z = 12$ should be atomic number of most stable isobar $A = 25$, this nuclide $^{25}_{12}Mg$ which is infact the only stable $A = 25$ isobar. The other isobar $^{25}_{11}Na,^{25}_{13}Al$ both are radioactive.

Problem: 3.4- Using the shell model, predict the characteristics of ground state of $^{15}_{8}O,^{16}_{8}O$ and $^{17}_{8}O$.

Solution

$^{15}_{8}O$**:**

According to shell model, 8 protons pair to give no contribution to the spin. Seven neutrons will be distributed in different shells as

$$\left(1s_{\frac{1}{2}}\right)^2 \mid \left(1p_{\frac{3}{2}}\right)^4 \left(1p_{\frac{1}{2}}\right)^1 \mid$$

Last unpaired neutron is in $p_{\frac{3}{2}}$ shell. Therefore, spin of $^{15}_{8}O$ is $\frac{1}{2}$. Since the last unpaired neutron is in p shell for which $l = 1$, therefore, parity is odd. Therefore, spin parity of $^{15}_{8}O$ is $\frac{1}{2}$.

$^{16}_{8}O$**:**

According to shell model, 8 protons pair to give no contribution to the spin. Similarly, 8 neutrons also pair together giving no contribution to spin. Therefore, spin parity of $^{16}_{8}O$ is 0^+.

$^{17}_{8}O$**:**

According to shell model, 8 protons pair to give no contribution to the spin. Nine neutrons will be distributed in different shells as

$$\left(1s_{\frac{1}{2}}\right)^2 \mid \left(1p_{\frac{3}{2}}\right)^4 \left(1p_{\frac{1}{2}}\right)^2 \mid \left(1d_{\frac{5}{2}}\right)^1 \left(2s_{\frac{1}{2}}\right)^0 \left(1d_{\frac{3}{2}}\right)^0$$

Last unpaired neutron is in $d_{\frac{1}{2}}$ shell. Therefore, spin of $^{17}_{8}O$ is $\frac{1}{2}$. Since the last unpaired neutron is in d shell for which $l = 2$, therefore, parity is even. Therefore, spin parity of $^{17}_{8}O$ is $+\frac{5^+}{2}$

Problem: 3.5- Use single particle shell model to predict the ground state spins and parities of ${}^{63}_{29}Cu$ and ${}^{40}_{18}Ar$.

Solution

${}^{68}_{29}Cu_{34}$:

According to shell model, 34 neutrons pair to give no contribution to the spin. Twenty-nine protons will be distributed in different shells

$$\left(1s_{\frac{1}{2}}\right)^2 | \left(1p_{\frac{3}{2}}\right)^4 \left(1p_{\frac{1}{2}}\right)^2 | \left(1d_{\frac{5}{2}}\right)^6 \left(2s_{\frac{1}{2}}\right)^2 \left(1d_{\frac{3}{2}}\right)^4 | \left(1f_{\frac{7}{2}}\right)^8 | \left(2p_{\frac{3}{2}}\right)^1 \left(1f_{\frac{5}{2}}\right)^0$$

Last unpaired proton is in $p_{\frac{3}{2}}$ shell. Therefore, spin of ${}^{68}_{29}Cu_{34}$ is $\frac{3}{2}$. Since the last unpaired proton is in p shell for which $l = 1$, therefore, parity is odd. Therefore, spin parity of ${}^{68}_{29}Cu_{34}$ is$\frac{3^-}{2}$.

${}^{40}_{18}Ar_{22}$:

According to shell model, 18 protons pair to give no contribution to the spin. Similarly, 22 neutrons also pair together giving no contribution to spin. Therefore, spin parity of ${}^{40}_{18}Ar$ is 0^+.

Problem: 3.6- Use the semiempirical mass formula to calculate the binding energy of ${}^{40}_{20}$Ca. What is the percentage discrepency between this value and the actual value?

Solution

Binding energy $= [M(Z, N) - Zm_p - Nm_n] \times 931.49\, MeV$

For ${}^{40}_{20}$Ca, actual binding energy

$$= [M(20, 20) - 20m_p - 20m_n] \times 931.49\, MeV$$
$$= (39.962591 - 20 \times 1.007825 - 20 \times 1.008665) \times 931.49\, MeV = -342.05\, MeV$$

Binding energy as per semiempirical mass formula

$$B = a_v - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm a_p A^{-3/4}$$

Here, $A = 40$, $Z = 20$, $N = 20$, $a_v = 15.5\, MeV.$ $a_s = 16.8\, MeV,$ $a_c = 0.7\, MeV,$ $a_a = 23.0\, Mev$ and $a_p = 34.0\, MeV$

Substituting various values, we get

$$B = 40 \times 15.5 - 16.8 \times 40^{2/3} - 0.7 \times \frac{20 \times 19}{40^{1/3}} - 23.0 \frac{40 - 2 \times 20}{40} - 30 \times 40^{-3/4}$$
$$= 620 - 196.4939 - 77.77887 - 1.8862 = 343.84\, MeV$$

Therefore, percentage discrepancy between the actual and semiempirical mass formula value

$$\frac{343.84 - 342.05}{342.05} = 0.5\%$$

Problem: 3.7- Find the energy needed to remove a neutron from ^{81}Kr, ^{82}Kr and ^{83}Kr.

Solution

$$\text{Separation energy of neutron for } ^{81}\text{Kr} = [BE \text{ of } ^{81}\text{Kr} - BE \text{ of } ^{80}\text{Kr}]$$
$$\text{Binding energy} = [Zm_p + Nm_n - M(Z, N)] \times 931.49\, MeV$$
$$\text{For } ^{81}\text{Kr, binding energy} = (36 \times 1.007825 + 45 \times 1.008665 - 80.91661) \times 931.49\, MeV$$
$$= 703.28\, MeV$$
$$\text{For } ^{80}\text{Kr, binding energy} = (36 \times 1.007825 + 44 \times 1.008665 - 80.91661) \times 931.49\, Mev$$
$$= 695.50\, MeV$$

Therefore,

$$\text{Separation energy of neutron for } ^{81}\text{Kr} = 703.27 - 695.49 = 7.78\, MeV$$
$$\text{Separation energy of neutron for } ^{82}\text{Kr} = [BE \text{ of } ^{82}\text{Kr} - BE \text{ of } ^{81}\text{Kr}]$$
$$\text{For } ^{81}\text{Kr, binding energy} = (36 \times 1.007825 + 46 \times 1.008665 - 81.913482) \times 931.49\, MeV$$
$$= 714.28\, MeV$$

Therefore,

$$\text{Separation energy of neutron for } ^{82}\text{Kr} = 714.28 - 703.28 = 11.00\, MeV$$

$$\begin{aligned}
\text{Separation energy of neutron for } {}^{83}\text{Kr} &= [BE \text{ of } {}^{83}\text{Kr} - BE \text{ of } {}^{82}\text{Kr}] \\
\text{For } {}^{83}\text{Kr},\ \text{binding energy} &= (36 \times 1.007825 + 47 \times 1.008665 - 82.914134) \times 931.49\, MeV \\
&= 721.74\, MeV
\end{aligned}$$

Therefore,

$$\text{Separation energy of neutron for } {}^{83}\text{Kr} = 721.74 - 714.28 = 7.46\, MeV$$

Problem: 3.8- Which isobars of $A = 75$ does the liquid drop model suggest to be the most stable nucleus?

Solution

According to liquid drop model, binding energy is given by,

$$B = a_v - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm a_p A^{-3/4}$$

Here, $a_v = 15.5\, MeV$, $a_s 16.8\, MeV$, $a_c 0.7\, MeV$, $a_a = 23.0\, MeV$, and $a_p = 34.0\, MeV$

In order to find most stable isotope for $A = 75$, we take the partial derivative of B with respect to Z and equate that to zero.

$$\frac{\partial B}{\partial Z} = -a_c \frac{2Z-1}{A^{1/3}} + 4a_a \frac{A-2Z}{A} = 0$$

Substituting various values

$$= -\frac{0.7}{(75)^{1/3}}(2Z-1) + \frac{4 \times 23}{75}(75 - 2Z) = 0$$

which gives

$$Z = 32.89 \approx 33$$

Therefore, according to liquid drop model, most stable isotope for $A = 75$ is with $Z = 33$.

Problem: 3.9- Using the liquid drop model, find the most stable isobar for $A = 27$, $A = 118$ *and* $A = 238$.

Solution

Binding energy is given by

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm a_p A^{-3/4}$$

For most stable isobar, $\frac{\partial B}{\partial Z} = 0$

or $$\frac{\partial B}{\partial Z} = \frac{a_c}{A^{1/3}}(2Z-1) + \frac{4a_a}{A}(A-2z) = 0$$

which gives

$$Z = \frac{4a_a + \frac{a_c}{A^{1/3}}}{\frac{2a_c}{A^{1/3}} + \frac{8a_a}{A}}$$

Substituting $a_c = 0.7\,MeV$ and $a_a = 23.0\,MeV$ and $A = 27$, we get

$$Z = \frac{4(23) + \frac{0.7}{(27)^{1/3}}}{\frac{2(0.7)}{(27)^{1/3}} + \frac{8(23)}{27}} \approx 12.6 \qquad \text{or} \qquad Z = 13$$

Similarly, for $A = 118$, we get $Z = 49.8$ or $Z = 50$.

And, for $A = 238$, we get $Z = 91.98$ or $Z = 92$

Problem: 3.10- Calculate the contribution of Coulomb energy and surface energy terms for $^{236}_{92}$U nucleus.

Solution

Contribution due to Coulomb energy term for $^{236}_{92}$U nucleus.

$$= -\frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_\circ r} = -\frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_\circ r_\circ A^{1/3}}$$

Substituting various values, we get

$$= -\frac{3}{5} \times 8.98 \times 10^9 \times \frac{(92 \times 91)(1.6 \times 10^{-19})^2}{1.2 \times 10^{-15}(236)^{1/3}} = -1.557 \times 10^{-10}\,J = -972.0\,MeV$$

Contribution due to surface energy term for $^{236}_{92}$U nucleus

$$= a_s A^{2/3} = -16.8 \times (236)^{2/3} = -641.6\, MeV$$

Chapter 4

Theories of Radioactive Decay

SOLVED PROBLEMS

Problem: 4.1- The half life of a nucleus is 2.1 minute. What is its decay constant?

Solution

We know the equation,

$$_{7}N^{12} \rightarrow {}_{6}C^{12} + e^{+} + \nu$$

The Q-value for this process is

$$T_{1/2} = 0.693 \ / \ \lambda$$
$$2.1 \times 60 = 0.693 \ / \ \lambda$$
$$126 = 0.693 \ / \ \lambda$$
$$\lambda = 0.693 \ / \ 126$$
$$= 5.5 \times 10^{-3} sec^{-1}$$

Problem: 4.2- How many curies are there in $10^{10}Bq$?

Solution

we know that $3.7 \times 10^{10} Bq = 1Ci$. Therefore,

$$\begin{aligned} 10^{10} Bq &= \frac{1}{3.7 \times 10^{10}} \times 10^{10}\ ci \\ &= 0.27ci \end{aligned}$$

Problem: 4.3- The half-life of ${}^{238}_{92}U$ against a-decay is 4.5×10^9 years. Find the activity of 1 kg of ${}^{238}_{92}U$.

Solution

Half-life $t_{\frac{1}{2}} = 4.5 \times 10^9\ years = 1.419 \times 10^{17} s$. Therefore,

$$\begin{aligned} Decay\ constant\ \lambda &= \frac{0.6931}{t_{\frac{1}{2}}} \\ &= \frac{0.6131}{1.419 \times 10^{17}} \\ &= 4.88 \times 10^{-18} s^{-1} \\ 238g\ of\ {}^{238}_{92}U\ contains &= 6.023 \times 10^{23} atoms \\ 1000g\ of\ {}^{238}_{92}U\ contains\ N &= \frac{6.023 \times 10^{23} \times 1000}{238} atoms \end{aligned}$$

Now, we have

$$Activity\ A = l \times N$$

Therefore,

$$\begin{aligned} A &= 4.88 \times 10^{-18} \times 2.53 \times 10^{24} \\ &= 1.236 \times 10^{7} dps \\ &= 3.34 \times 10^{-4} Ci \end{aligned}$$

Problem: 4.4- Complete the following reactions by putting appropriate particle on the arrows:

$$
\begin{array}{l}
{}^{238}_{92}U \longrightarrow {}^{234}_{90}Th \longrightarrow {}^{234}_{91}Pa \longrightarrow {}^{234}_{92}U \\
{}^{211}_{82}Pb \longrightarrow {}^{211}_{83}Bi \longrightarrow {}^{207}_{81}Tl \longrightarrow {}^{207}_{82}Pb \\
\downarrow \longrightarrow^{211}_{84} Pa \longrightarrow \uparrow
\end{array}
$$

Solution

$$
\begin{array}{l}
{}^{238}_{92}U \longrightarrow {}^{234}_{90}Th \longrightarrow {}^{234}_{91}Pa \longrightarrow {}^{234}_{92}U \\
{}^{211}_{82}Pb \longrightarrow {}^{211}_{83}Bi \longrightarrow {}^{207}_{81}Tl \longrightarrow {}^{207}_{82}Pb \\
\downarrow \longrightarrow^{211}_{84} Pa \longrightarrow \uparrow
\end{array}
$$

Problem: 4.5- ${}^{238}_{92}$U decays to Pb through the successive emission of 6 electrons and 8 α-Particles. What is the mass number of Pb isotope? What is the total energy evolved in the decay?

Given:

$$
\begin{array}{lll}
M({}^{238}U) = 238.050786 \text{ amu} & \text{and} & M(Pb) = 205.9744550 \text{ amu} \\
M(\alpha) = 4.002603 \text{ amu} & \text{and} & M(e) = 5.486 \times 10^{-4} \text{ amu}
\end{array}
$$

Solution

$$\text{Number of nucleons in 8 } \alpha - \text{particles} = 8 \times 4 = 32$$

$$\text{Therefore, the mass of Pb} = 238 - 32 = 206$$

or

The daughter nucleus is ${}^{206}_{82}$Pb

$$\begin{aligned}\text{Total mass of products} &= M(^{206}\text{Pb}) + 8M(\alpha) + 6M(e)\\ &= 205.974455 + 8 \times 4.002603 + 6 \times 5.486 \times 10^{-4}\\ &= 237.9985706 \text{ amu}\end{aligned}$$

Therefore, decrease in mass = $238.050786 - 237.9985706 = 0.0522154$ amu

or

$$\text{Energy evolved} = 0.0522154 \times 931.47 = 48.6\ MeV$$

Problem: 4.6- How many curies are there in 10^{10} Bq?

Solution

We know that 3.7×10^{10} Bq = 1 Ci. Therefore,

$$10^{10}\ \text{Bq} = \frac{1}{3.7 \times 10^{10}} \times 10^{10}\ \text{Ci} = 0.27\,\text{Ci}$$

Problem: 4.7- Calculate the activity of 10 g of ^{232}Th. Given: $\lambda_{^{232}\text{Th}} = 1.58 \times 10^{-18}\ s^{-1}$.

Solution

$$\begin{aligned}1 \text{ g mole of } ^{232}\text{Th} &= 232\,\text{g of } ^{232}\text{Th}\\ 232 \text{ g of } ^{232}\text{Th contains} &= 6.023 \times 10^{23} \text{ atoms of } ^{232}\text{Th}\\ 10 \text{ g of } ^{232}\text{Th contains} &= \frac{6.023 \times 10^{23}}{232} \times 10 = 2.596 \times 10^{22} \text{ atoms}\end{aligned}$$

Therefore,

$$\begin{aligned}N &= 2.596 \times 10^{2}\\ \text{Activity } N\lambda &= 2.596 \times 10^{22} \times 1.58 \times 10^{-18} = 4.104 \times 10^{4}\,\text{dps}\end{aligned}$$

Problem: 4.8- What will be the mass of 1 curie sample of a radioactive substance of atomic mass 214? Its half life is 26.8 min.

Solution

Given:

$$\text{Activity } A = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ dps}$$
$$\text{Half-life } t_{1/2} = 26.8 \text{ min} = 1608\, s$$
$$\text{Decay constant } \lambda = \frac{0.6931}{t_{1/2}} = \frac{0.6931}{1608} = 4.31 \times 10^{-4}\, S^{-1}$$

Let m be the mass of sample having activity 1 Ci. Therefore,

$$214 \text{ g of raqdioactive substance contains} = 6.023 \times 10^{23} \text{ atoms}$$
$$m \text{ g of radioactive substance contains } N = \frac{6.023 \times 10^{23} \times m}{214} \text{ atoms}$$

Now, we have

$$\text{Activity } A = \lambda \times N$$

Therefore,

$$3.7 \times 10^{10} = 4.31 \times 10^{4} \times \frac{6.023 \times m}{214}$$
$$m = \frac{3.7 \times 10^{10} \times 214}{4.31 \times 10^{-4} \times 6.023 \times 10^{23}} = 3.05 \times 10^{-8} \text{ g}$$

Problem: 4.9- The half life of $^{238}_{92}$U against α-decay is 4.5×10^{9} years. Find the activity of 1 kg of $^{238}_{92}$U.

Solution

Half-life $t_{1/2} = 4.5 \times 10^{9}$ years $= 1.419 \times 10^{17}$ s

$$\text{Decay constant } \lambda = \frac{0.6931}{t_{1/2}} = \frac{0.6931}{1.419 \times 10^{17}} = 4.88 \times 10^{-18}\ s^{-1}$$
$$238 \text{ g of } ^{238}_{92}\text{U contains} = 6.023 \times 10^{23} \text{ atoms}$$
$$1000 \text{ g of } ^{238}_{92}\text{U contains } N = \frac{6.023 \times 10^{23} \times 1000}{238} \text{ atoms}$$

Now, we have

$$\text{Activity } A = \lambda \times N$$

Therefore,

$$A = 4.88 \times 10^{-18} \times 2.53 \times 10^{24} = 1.236 \times 10^{6} \text{ dps} = 3.36 \times 10^{-4} \text{ Ci}$$

Problem: 4.10- One milligram of a radioactive material with half-life of 1600 years is kept for 2000 years. Calculate the mass, which would have decayed by this time.

Solution

Given:

$$\text{Half-life } t_{1/2} = 1600 \text{ years}$$

Therefore,

$$\text{Decay constant } \lambda = \frac{0.6931}{t_{1/2}} = \frac{0.6931}{1600} = 4.332 \times 10^{-4} \text{ years}^{-1}$$

Now,

$$N = N_{\circ}e^{-\lambda t} \text{ or } \frac{N_{\circ}}{N} = e^{\lambda t}$$

Therefore,

$$\frac{N_{\circ}}{N} = e^{6.332\times 10^{-4}\times 2000} = e^{0.8663} = 2.378$$

If $N_{\circ}$ is the initial number of nuclei at time $t = 0$, and the corresponding mass of the radioactive substance is $m_{\circ}$. Similarly, if N is the number of nuclei left at time $t =$ (2000 years), the corresponding mass be m, then we must have

$$\frac{N_{\circ}}{N} = \frac{m_{\circ}}{m} \qquad \text{or} \qquad \frac{N_{\circ}}{N} = \frac{m_{\circ}}{m} = 2.378$$

Given that $m_{\circ} = 1$ mg. Therefore,

$$\frac{m_{\circ}}{m} = 2.378 \qquad \text{or} \qquad m = \frac{1}{2.378} = 0.4205 \text{ mg}$$

Therefore, the amount of radioactive substance decayed is

$$= 1 - 0.4205 = 0.5795 \text{ mg}$$

Problem: 4.11- Calculate the activity of ^{40}K in a human body weighing 100 kg. Assume that 0.35% of the body weight is potassium. In natural potassium, the abundance of ^{40}K is 0.012%. Half-life of $^{40}\text{K} = 1.31 \times 10^9$ years

Solution

Total weight of potassium in 100 kg human body $= 100 \times 0.35 \times 10^{-2} = 0.35$ kg
Weight of ^{40}K with abundance 0.012% $= 0.35 \times 0.012 \times 10^{-2} = 4.2 \times 10^{-5}$ kg
From Avogadro's hypothesis, number of atoms in 4.2×10^{-5} kg of ^{40}K

$$N = \frac{6.023 \times 10^{26}}{40} \times 4.2 \times 10^{-5} = 6.324 \times 10^{20} \text{ atoms}$$

$$\text{Therefore, activity of } {}^{40}K = \lambda \times N = \frac{0.693 \times N}{\lambda}$$

$$= \frac{0.693 \times 6.324 \times 10^{20}}{1.31 \times 10^9 \times 365 \times 24 \times 60 \times 60} = 1.061 \times 10^4 \text{ dps}$$

$$\text{Activity in human body} = 1.061 \times 10^4 \text{ dps} = 0.287\,\mu\text{Ci}$$

Problem: 4.12- Calculate the height of potential barrier faced by an α-particle inside the $^{226}_{88}$Ra nucleus.

Solution

The height of Coulomb barrier is given by the relation

$$B = \frac{2(Z-2)e^2}{4\pi\varepsilon_\circ R} = \frac{2(Z-2)e^2}{4\pi\varepsilon_\circ R_\circ A^{1/3}}$$
$$= \frac{8.98 \times 10^9 \times 2 \times 86 \times (1.6 \times 10^{-19})^2}{1.3 \times 10^{-15} \times (226)^{1/3}} = 4.9936 \times 10^{-12} \text{ J} = 31.2\ MeV$$

Problem: 4.13- Rutherford bombarded 7.7 MeV α-particles from ^{214}Po on ^{14}N to initiate the nuclear reaction $^{14}\text{N}(\alpha,\ p)\ ^{17}\text{O}$. Find the height of Coulomb barrier faced by α-particles.

Solution

Height of Coulomb barrier is given by the relation

$$V_c = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_\circ (R_1 + R_2)}$$

where Z_1 is the charge of the incident projectile, Z_2 is the charge of the target. R_1 and R_2 are the radii of the projectile and target nuclei respectively. We have $R = R_\circ A^{1/3}$, Therefore,

$$V_c = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_\circ R_\circ (A_1^{1/3} + A_2^{1/3})}$$

Substituting various values, we get

$$V_c = \frac{8.98 \times 10^{9\times 2\times 7\times (1.6\times 10^{-19})^2}}{1.5\times 10^{-15} \times ((4)^{1/3} + (14)^{1/3})} = 5.367\times 10^{-13}\ \text{J} = 3.35\ MeV$$

Problem: 4.14- Find the kinetic energy required by a proton to penetrate Coulomb barrier of a hydrogen nucleus.

Solution

When a proton is bombarded on hydrogen nucleus, the Coulomb barrier E_b is given by

$$E_b = \frac{Z_1 Z_2 e^2}{r}$$

Problem: 4.15- Calculate the kinetic energy of α-particles in the following decay

$$^{239}\text{Pu} \longrightarrow {}^{235}\text{U} + {}^{4}\text{He}$$

Given:

$M(^{239}\text{Pu}) = 239.052158$ amu, $M(^{235}\text{U}) = 235.043925$ amu, $M(^{4}\text{He}) = 4.002603$ amu

Solution

$$^{239}\text{Pu} \longrightarrow {}^{235}\text{U} + {}^{4}\text{He}$$

The total disintegration energy Q is the difference in masses of the initial nucleus and the final products, or

$$
\begin{aligned}
Q &= M(^{239}\text{Pu}) - M(^{235}\text{U}) - M(^{4}\text{He}) \\
&= 239.052158 - 235.043925 - 4.002603 = 0.00563 \text{ amu} \\
&= 0.00563 \times 931.47 = 5.244\ MeV
\end{aligned}
$$

Now,

$$K_\alpha = Q\frac{A-4}{A} = 5.244 \times \frac{237}{241} = 5.16\ MeV$$

Therefore, the kinetic energy of α-particle =5.16 MeV

Problem: 4.16- ^{14}C decays by β^--emission. The end point energy of β-particles in this decay is 0.156 MeV. Given the mass of ^{14}C$=$ 14.007685 amu, find the mass of daughter nucleus.

Solution

The given decay is

$$^{14}\text{C} \longrightarrow {}^{14}\text{N} + \beta^- + \bar{\nu}_e$$

For β^--emission with energy equal to end point energy, the energy carried by $\bar{\nu}_e$ = 0 MeV and mass of $\bar{\nu}_e \approx 0\ MeV$.

Now,

$$
\begin{aligned}
M(^{14}\text{N}) &= M(^{14}\text{C}) - \text{K.E of } \beta^- \text{ (in amu)} \\
&= 14.007685 - \frac{0.156}{931.47} = 14.007517 \text{ amu}
\end{aligned}
$$

Therefore, the mass of ^{14}N = 14.007517 amu

Chapter 5

Nuclear Reactions

SOLVED PROBLEMS

Problem: 5.1- The isotope ${}_{7}N^{12}$ beta decays(positron) to an excited state of ${}_{6}C^{12}$ which decays to the ground state with the emission of a 4.43 MeV gamma ray. Calculate the maximum kinetic energy of the beta particle. Mass of ${}_{7}N^{12} = 12.01861$u.

Solution

The process for positive beta emission is

$${}_{7}N^{12} \;\; \rightarrow \; {}_{6}C^{12} + \; e^{+} \; + \; \nu$$

The Q-value for this process is

$$\begin{aligned} \text{Q} = \; & \left[m\left({}_{7}N^{12}\right) m\left({}_{6}C^{12}\right) - 2m_e\right] c^2 \\ = \; & (12.018613 - 12.0000 - 2 \times 0.000549) \times 931.5 \; MeV \;\; = \; 16.32\text{MeV} \end{aligned}$$

The energy of the emitted gamma ray = 4.43 MeV
The maximum energy of beta particle = $(16.32 - 4.43)\, MeV \;\; = \; 11.89 \; MeV$

Problem: 5.2- Complete the following reactions:

1. ${}^{59}_{27}Co + {}^{1}_{0}n \longrightarrow ? + \gamma$
2. ${}^{14}_{7}N + {}^{1}_{1}H \longrightarrow {}^{11}_{6}C + ?$
3. ${}^{64}_{30}Zn + {}^{1}_{0}n \longrightarrow ? + {}^{1}_{0}n + {}^{1}_{0}n$
4. ${}^{238}_{92}U + {}^{14}_{7}N \longrightarrow {}^{243}_{97}Bk + 5{}^{1}_{0}n + ?$

Solution

(1) Here total number of protons of the reactants is 27 and total number of nucleons is 60. Therefore, the resultant nucleus will have 27 protons and 60 nucleons. So, the missing nucleus is ${}^{60}_{27}Co$ and the reaction is

$$ {}^{59}_{27}Co + {}^{1}_{0}n \longrightarrow {}^{60}_{27}Co + \gamma $$

Similarly, for other reactions.

(2) The missing particle is ${}^{4}_{2}He$ and the complete reaction is

$$ {}^{14}_{7}N + {}^{1}_{1}H \longrightarrow {}^{11}_{6}C + {}^{4}_{2}He $$

(3) The missing nucleus is ${}^{63}_{30}Zn$ and the complete reaction is

$$ {}^{64}_{30}Zn + {}^{1}_{0}n \longrightarrow {}^{62}_{30}Zn + {}^{1}_{0}n + {}^{1}_{0}n $$

(4) The missing particle is ${}^{4}_{2}He$ and the complete reaction is

$$ {}^{238}_{92}U + {}^{14}_{7}N \longrightarrow {}^{243}_{97}Bk + 5{}^{1}_{0}n + {}^{4}_{2}He $$

Problem: 5.3- Are the following reactions/decays possible? If not, write the correct relation.

(i) ${}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \rightarrow {}^{30}_{14}\text{Si} + {}^{1}_{0}n$ (ii) ${}^{235}_{92}\text{U} + {}^{1}_{0}n \rightarrow {}^{143}_{56}\text{Ba} + {}^{90}_{36}\text{Kr} + 2\,{}^{1}_{0}n$

(iii) ${}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}e + \nu_e$

Solution

(i)-

$$ {}^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \rightarrow {}^{30}_{14}\text{Si} + {}^{1}_{0}n $$

In the reaction, the number of protons in the reactants is $13 + 2 = 15$. In the product, the number of protons is $14 + 0 = 14$. Hence, number if nucleons is not conserved. In the products, ${}^{1}_{0}n$ should be replaced by ${}^{1}_{1}\text{H}$. The correct equation is

$$^{27}_{13}\text{Al} \ + \ ^{4}_{2}\text{He} \ \rightarrow \ ^{30}_{14}\text{Si} \ + \ ^{1}_{1}\text{H}$$

(ii)-

$$^{235}_{92}\text{U} \ + \ ^{1}_{0}n \ \rightarrow \ ^{143}_{56}\text{Ba} \ + \ ^{90}_{36}\text{Kr} \ + 2 \ ^{1}_{0}n$$

In the reaction, the number of nucleons in the reactants is $235 + 1 = 236$. In the products, the number of nucleons is $143 + 90 + 2 = 235$. Hence, number of nucleons is not conserved. In the products, $2 \ ^{1}_{0}n$ should be replaced by $3 \ ^{1}_{0}n$. The correct equation is

$$^{235}_{92}\text{U} \ + \ ^{1}_{0}n \ \rightarrow \ ^{143}_{56}\text{Ba} \ + \ ^{90}_{36}\text{Kr} \ + 3 \ ^{1}_{0}n$$

(iii)-

$$^{32}_{15}\text{P} \ \rightarrow \ ^{32}_{16}\text{S} \ + \ ^{0}_{-1}e \ + \ \nu_e$$

In the reaction, the number of leptons in the reactants is $0 + 0 = 0$. In the products, the number of leptons is $1 + 1 = 2$. Hence, lepton number is not conserved. In the products, ν_e should be replaced by $\bar{\nu}_e$. The correct equation is

$$^{32}_{15}\text{P} \ \rightarrow \ ^{32}_{16}\text{S} \ + \ ^{0}_{-1}e \ + \ \bar{\nu}_e$$

Problem: 5.4- Prove that ^{252}Cf is unstable and decays by α-emission as under

$$^{252}\text{Cf} \ \rightarrow \ ^{248}\text{Cm} \ + \ ^{4}\text{He}$$

Given:
$M(^{252}\text{Cf}) = 252.081621$ amu $\qquad$ $M(^{248}\text{Cm}) = 248.072343$ amu
$M(^{4}\text{He}) = 4.002603$ amu

Solution

Let us calculate the Q-value for this decay

$$\begin{aligned} Q &= [M(^{252}\text{Cf}) - M(^{248}\text{Cm}) - M(^{4}\text{He})] \\ &= [252.081621 - 248.072343 - 4.002603] = 0.006675 \text{ amu} \\ &= 0.006675 \times 931.49 = 6.21 \ MeV \end{aligned}$$

Since Q-value for thisb reaction is $+6.21\ MeV$, therefore, ^{252}Cf will decay by α-emission.

Problem: 5.5- Find out the Q-value for the reaction

$$^{208}\text{Pb}\,(^{56}\text{Fe},\ ^{54}\text{Fe})\ ^{210}\text{Pb}$$

Given:

$M(^{208}\text{Pb}) = 207.976641$ amu $\qquad M(^{56})\text{Fe} = 55.934939$ amu

$M(^{210}\text{Pb}) = 209.984178$ amu $\qquad M(^{54}\text{Fe}) = 53.939612$ amu

Also find the treshold for this reaction

Solution

In this reaction the projectile is ^{56}Fe and the reaction is

$$^{208}\text{Pb} + {}^{56}\text{Fe} \rightarrow {}^{210}\text{Pb} + {}^{54}\text{Fe}$$

The Q-value for this reaction is given by

$$\begin{aligned} Q &= [M(^{208}\text{Pb}) + M(^{56}\text{Fe}) - M(^{210}\text{Pb}) - M(^{54}\text{Fe})] \text{ amu} \\ &= [207.976641 + 55.934939 - 209.984178 - 53.939612] \text{ amu} \\ &= -0.001221 \text{ amu} = -0.01221 \times 931.49\, MeV = -11.37\, MeV \end{aligned}$$

Therefore, the Q-value for this reaction is $-11.37\, MeV$.
Threshold for this reaction is

$$E_{\text{th}} = -Q\frac{M(\text{projectile}) + M(\text{target})}{M(\text{target})} = -Q\frac{M(^{56}\text{Fe}) + M(^{208}\text{Pb})}{M(^{208}\text{Pb})}$$

In this relation atomic masses can be replaced by respective mass numbers. The error introduced by this is negligible. Therefore,

$$E_{\text{th}} = 11.37\frac{56 + 208}{208} = 14.43\ MeV$$

So, the threshold for the reaction is $14.43\ MeV$.

Problem: 5.6- A 0.01 mm thik ${}^{7}_{3}$Li target is bombarded with a beam of flux of 10^{13} particles/cm^2-s. As a result 10^8 neutrons/s are produced. Calculate the cross-section for this reaction. Given density of lithium = 500 kg/m^3.

Solution

$$\text{Thikness of } {}^{7}_{3}\text{Li} = t = 0.01 \text{ mm} = 10^{-5} \text{ m} \; ; \qquad \text{Density} = 500 \text{ kg/m}^3$$

$$\text{Number of } {}^{7}_{3}\text{Li nuclei/volume } n = \frac{\rho N}{M} = \frac{500 \times 6.023 \times 10^{26}}{7} = 4.302 \times 10^{28}/\text{m}^3$$

$$\begin{aligned} \text{Number of } {}^{7}_{3}\text{Li nuclei / area} &= 4.302 \times 10^{28} \times t \\ &= 4.302 \times 10^{28} \times 10^{-5} = 4.302 \times 10^{23} \end{aligned}$$

$$\begin{aligned} \text{Number of nuclei undergoing interactions/s} &= \text{Number of neutrons produced/s} \\ \text{Number of neutrons produced/s} &= 10^8 \\ \text{Number of incident particles striking/unit area of target } N_\circ &= 10^{13}/\text{unit area of target.} \\ \text{cross-section } \sigma &= \frac{\text{Number of neutrons produced/s}}{N_{\circ \times n}} \\ &= \frac{10^8}{10^{13} \times 4.302 \times 10^{23}} = 2.32 \times 10^{-29} \text{ m}^2 \\ &= 0.232 \text{ b } (1 \text{ b} = 10^{-28} \text{ m}^2) \end{aligned}$$

Problem: 5.7- Calculate the Q-value for the reaction

$$^{2}\text{H} + {}^{3}\text{H} \rightarrow {}^{4}\text{He} + {}^{1}n$$

Given:

$M(^1n) = 1.00866501$ amu $\qquad M(^H) = 2.014102$ amu

$M(^3\text{H}) = 3.016049$ amu $\qquad M(^4\text{He}) = 4.002603$ amu

Solution

Q-value is given by

$$
\begin{aligned}
Q &= [M(^2\text{H}) + M(^3\text{H}) - M(^4\text{He}) - M(^1n)] \\
&= [2.014102 + 3.016049 - 4.002603 - 1.00866501] = 0.01888299 \text{ amu} \\
&= 0.01888299 \times 931.49 = 17.6 \; MeV
\end{aligned}
$$

Therefore, the Q-value for this reaction is 17.6 MeV.

Problem: 5.8- When a proton captures a neutron to produce a deuteron nucleus. a γ-ray of energy 2.230 MeV is released based on the following equation:

$$ {}^1_1\text{H} \;+\; {}^1_0n \;\rightarrow\; {}^2_1\text{H} \;+\; \gamma $$

Calculate the mass of neutron. Given:

Mass of ${}^1_1$H = 1.008142 amu   Mass of ${}^2_1$H = 2.014735 amu

Solution

$$ \text{Total mass of the reactants} \;=\; \text{Total mass of the products} $$

Let us take all the masses in MeV. Therefore,

$$ 1.008142 \times 931.47 + m \times 931.47 = 2.014735 \times 931.47 + 2.230 $$

where m is the mass of neutron.

$$ 931.47 \times m = 939.8613 \qquad \Rightarrow \qquad m = \frac{939.8412}{931.47} = 1.008987 \text{ amu} $$

Problem: 5.9- Find out whether the following reaction is exoergic or endoergic.

$$ {}^6\text{Li} \;+\; {}^1n \;\rightarrow\; {}^4\text{He} \;+\; {}^3\text{H} $$

Given:

$M(^6\text{Li}) = 6.0151234$ amu $M(^1n) = 1.0086654$ amu

$M(^4\text{He}) = 4.0026034$ amu $M(^3\text{H}) = 3.0160294$ amu

Solution

Q-value for this reaction is given by

$$
\begin{aligned}
Q &= [M(^6\text{Li}) + M(^1n) - M(^4\text{He}) - M(^3\text{H})] \\
&= [6.0151234 + 1.0086654 - 4.0026034 - 3.0160294] = 0.005156 \text{ amu} \\
&= 0.005156 \times 931.47 = 4.8 \; MeV
\end{aligned}
$$

Therefore, the Q-value for this reaction is 4.8 MeV and the reaction is exoergic

Problem: 5.10- Prove that ^{252}Cf can undergo spontaneous fission as under:

$$^{252}\text{Cf} \rightarrow {}^{98}\text{Zr} + {}^{145}\text{Ce} + 9\ {}^{1}n$$

Given:

$M(^{252}\text{Cf}) = 252.081621$ amu $\quad M(^{98}\text{Zr}) = 97.912735$ amu

$M(^{145}\text{Ce}) = 144.917230$ amu $\quad M(^{1}n) = 1.008664916$ amu

Solution

Q-value for spontaneous fission is given by

$$\begin{aligned} Q &= M(^{252}\text{Cf}) - M(^{98}\text{Zr}) - N(^{145}\text{Ce}) - 9 \times M(^{1}n) \\ &= 0.1737009 \times 931.47 = 161.8\ MeV \end{aligned}$$

Since Q-value is positive,. so spontaneous decay of ^{252}Cf is possible.

Problem: 5.11- Calculate the separation energy to remove one proton from $^{16}_{8}$O. Given:

$m(\text{H}) = 1.007825$ amu , $\quad m(^{15}_{7}\text{N}) = 15.000108$ amu , $\quad m(^{16}_{8}\text{O}) = 16$ amu

Solution

Given reaction is

$$^{16}_{8}\text{O} \rightarrow {}^{15}_{7}\text{N} + {}^{1}_{1}\text{H}$$

Q-value for this reaction is

$$\begin{aligned} Q &= M_{\text{O}} - M_{\text{N}} - M_{\text{H}} = 16 - 15.000108 - 1.007825 = -0.007933 \\ &= -0.007933 \times 931.47 = -7.4\ MeV \end{aligned}$$

The energy required to remove one proton from $^{16}_{8}\text{O} = 7.4\ MeV$

Problem: 5.12- The Q-value of the $^{23}\text{Na}\,(n,\,\alpha)\ ^{20}\text{F}$ reaction is $-5.4\ MeV$. Determine the threshold energy of the neutrons for this reaction. Given:

Mass of neutron = 1.00866 amu $\quad$ Mass of ^{23}Na = 22.99097 amu

Solution

Given reaction is

$$^{23}\text{Na} + n \rightarrow {}^{20}\text{F} + {}^{4}_{2}\text{He}$$

$$Q\text{-value} = -5.4\ MeV\ ; \qquad \text{Therefore, threshold energy} = -Q\frac{M_{\text{Na}} + M_n}{M_{\text{Na}}}$$

$$= -5.5\frac{22.99097 + 1.00866}{22.99097} = 5.64\ MeV$$

Problem: 5.13- Calculate the Q-value in MeV for the following nuclear reaction:

$$^4_2\text{He} \ + \ ^{14}_7\text{N} \ \rightarrow \ ^{17}_8\text{O} \ + \ ^1_1\text{H}$$

Given masses of:
$^4_2$He = 4.00388 amu, $^{14}_7$N = 14.00755 amu, $^{17}_8$O = 17.00452 amu, $^1_1$H = 1.0815 amu

Solution

Q-value is given by the relation

$$Q = M_{\text{N}} + M_{\text{He}} - M_{\text{O}} - M_{\text{H}} = 14.00755 + 4.00388 - 17.00452 - 1.00866$$

$$= -0.00124 \text{ amu} \qquad \Rightarrow \qquad Q = -0.00124 \times 931.47 = -1.15\ MeV$$

Therefore, Q-value is $-1.15\ MeV$

Problem: 5.14- Calculate the minimum energy of γ-rays necessary to disintegrate deuteron into a proton and neutron. Given masses of;
$^2_1$H = 2.014735 amu $^1_1$H = 1.008142 amu ^1_0n = 1.008987 amu

Solution

The given reaction is

$$^2_1\text{H} \ + \ \gamma \ \rightarrow \ ^1_1\text{H} \ + \ ^1n$$

Let E_γ be the energy of γ-ray to start this reaction.
Total mass of the reactants = Total mass of the products
Let us take all masses in MeV. Therefore,

$$2.014735 \times 931.47 + E_\gamma = 1.008142 \times 931.47 + 1.008987 \times 931.47$$

or

$$E_\gamma = 2.2299\ MeV$$

Chapter 6

Elementary Particles

SOLVED PROBLEMS

Problem: 6.1- An antiproton comes to rest and annihilates with a proton. They produce P^{+}, P^{-} and P^{0} equal energy. What is the average K.E of each pion in MeV ?

Solution

$$P + P^{-} \longrightarrow P^{0} + P^{-} + P^{+}$$

$$\text{Total energy of } P \text{ and } P^{-} \text{ system} = 938 + 938 = 1876\ MeV$$

$$\text{Out of this, the energy consumed in creating 3 Pions} = 134.9 + 139.5 + 139.5 = 413.9\ MeV$$

$$\text{Energy left to be shared by 3 Pions} = 1876 - 413.9 = 1462.1\ MeV$$

$$\text{Therefore, average K.E of each Pion} = \frac{1462.1}{3} = 487.4\ MeV$$

Problem: 6.2- Indicate with an explanation, whether the following interactions proceed through the strong, electromagnetic or weak interactions, or whether they do not occur.

1. $\pi \longrightarrow \mu^- + \nu^\mu$
2. $\tau^- \longrightarrow \mu^- + \nu^\tau$
3. $P \longrightarrow n + e^+ + \nu^e$
4. $\pi^- + P \longrightarrow \pi^0 + \Sigma^0$
5. $e^+ + e^- \longrightarrow \mu^+ + \mu^-$

Solution

1. Weak interactions because neutrino is involved.
2. Does not occur because lepton number is violated.
3. Allowed as a weak decay if proton is bound but forbidden when is lighter than the sum of masses of the product particle.
4. Does not occur as a strong or electromagnetic interaction because $\Delta S \neq 0$ and other quantum numbers are conserved.
5. Strong interaction because $\Delta S \neq 0$ and other quantum numbers are conserved.
6. Strong interactions because a lepton-antilepton pair is involved.

Problem: 6.3- Consider the decay of K^0 meson of momentum P_0 into π^+ and π^- of momentum P_+ and P_- in the opposite direction such that $P_+ = 20_-$. Determine P_0.

Solution If P_0 is the momentum of Kaon, P_1 and P_2 the momentum of Pion, then momentum conservation is required.

$$P_0 = P_+ - P_{-2} = 2P_{-2} - P_{-2} = P_{-2}$$

Energy conservation requires $E_2 + E_1 = E_0$,

$$\sqrt{P_-^2 + m_\pi^2} + \sqrt{P_+^2 + m_\pi^2} = \sqrt{P_0^2 + m_K^2} \tag{6.1}$$

$$P_- = P_0$$

$$P_+ = 2P_- = 2P_0 \tag{6.2}$$

Using Eq.(6.2) in Eq.(6.1) and solving the resultant equations

$$P_0 =` \frac{m_K}{2}\left(\frac{m_K^2 - 4m_\pi^2}{2m_K^2 + m_\pi^2}\right)^{\frac{1}{2}}$$

substituting $m_K = \ 498\frac{MeV}{e^2}$ and $m_\pi = \ 140\ \frac{Mev}{e^2}$, we get

$$P_0 = 142.8\ \frac{MeV}{e^2}$$

Problem: 6.4- Given that the π-meson has a width of 158 $\frac{MeV}{e^2}$ in its mass, how would you classify the interaction for its decay ?

Solution

$$\tau = \frac{h}{\Gamma} = \frac{hc}{\Gamma c} = \frac{197.3 \times 10^{-15} - fm}{158(MeV) \times 3 \times 10^8 \left(\frac{m}{s}\right)}$$
$$= \frac{197.3 \times 10^{-15}(MeV - m)}{474(MeV) \times 10^8 \left(\frac{m}{s}\right)} = \ 4 \times 10^{-24}\ sec$$

Therefore, the π-meson decays via strong interaction.

Problem: 6.5- Use the quark model to determine the quark composition of

1. Σ^+, Σ^-,n and P
2. K^+, K^-, π^+, π^- mesons

Solution

1. Σ^+:uus, Σ^-:dds,n:udd and P:uud
2. $K^+ : u\bar{s}$, K^-:ds, $\pi^+ u\bar{d}$, $\pi^- \bar{u}d$ mesons

Chapter 7

Detectors and Accelerators

SOLVED PROBLEMS

Problem: 7.1- What potential must be developed across the capacitor of 318 pF of Si detector by the absorption of α-particles of energy 4.5 MeV which produces one ion pair for each 3.5 eV?

Solution

No. of ion pairs produced by 3.5 eV = 1

No. of ion pairs produced by 1 eV = $\frac{1}{3.5}$

No. of ion pairs produced by 4 MeV = $1/3.5 \times (4.5 \times 106)$

Or $\quad N_e = 1.286 \times 10^6$

Q = charge on Ne ions (electrons)

$$Q = N_e \times e$$

$$Q = (1.286 \times 106) \times (1.6 \times 10 - 19)$$

$$Q = 2.06 \times 10^{-13} C$$

Given $\quad C = 318$ pF

$$C = 318 \times 10^{-12} \text{ F } V = \frac{Q}{C} = N_e \frac{e}{C}$$

$$V = 6.48 \times 10^{-14} \text{ volts}$$

$$V = 0.65 \text{ mV}$$

Problem: 7.2- A 20 MeV charge particle will produce $\approx 600,000$ ion pairs in the gas if it is completely stopped inside the chamber. If the total capacitance of the chamber is C = 100 pF, then find the total voltage ?

Solution

$$N_e = 600,000$$

$$Q = N_e \times e = (600,000)(1.6 \times 10^{-19})$$

$$Q = 9.6 \times 10^{-14} C$$

$$C = 100 \text{ pF} = 100 \times 10^{-12} F$$

$$V = \frac{Q}{C} = \frac{9.6 \times 10^{-14}}{100 \times 10^{-12}} = 0.96 \text{ mV}$$

Problem: 7.3- Calculate the frequency of cyclotron that the electric field is applied between the does in which **1:-** Proton, **2:-** Deutron, **3:-** Alpha particles are accelerated. The applied magnetic flux is 5.2 wb/m^2.

Solution

1. For proton

$$\text{Frequency} = Bq/2\pi m = \frac{5.2 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 1.67 \times 10^{-27}} = 79.2 \text{ MHz}$$

2. For deutron

$$\text{Frequency} = Bq/2\pi m = \frac{79.2}{2} = 39.6 \text{ MHz}$$

3. For Alpha

$$\text{Frequency} = Bq/2\pi m = \frac{39.6}{2} = 19.8 \text{ MHz}$$

Problem: 7.4- A 20 MV Van de Graff generator is equipped to accelerate protons, alpha and deuterons. What is the kinetic energy of different beams as they hit the target ? Is it possible.

Solution

The energy gained by an ion having q charge as it passes through a potential of V volt is qV, eV. Therefore for Protons= 5MeV, for alpha = 10 MeV, and for deuteron = 5 MeV

The possible reaction to produce ${}_8O^{15}$ isotope at these energies are;

$$P +_7 N^{17} \rightarrow {}_8O^{15} + \gamma$$

$$d +_7 N^{17} \rightarrow {}_8O^{15} + n$$

$${}_2He^4 +_6 C^{12} \rightarrow {}_8O^{15} + n$$

Problem: 7.5- In a cyclotron, if the potential applied across the does is 30 kV and a magnetic field of 2 tesla and accelerated protons are extracted from the dees at radius of 30 cm from the center of the dees then find;

1. Maximum energy acquired by the protons
2. The oscillator frequency

Solution

$$V = 30 \times 20^3 V$$

$$B = 2 \text{ tesla}$$

$$R = 0.3 \text{ m}$$

1. Maximum energy;

$$E_{max} = \frac{1}{2(B^2q^2R^2/m)}$$

$$= \frac{(2)^2 \times (1.6 \times 10^{-19})^2}{2 \times 1.67 \times 10^{-27}}$$

$$= \frac{1.024 \times 10^{-37}}{3.34 \times 10^{-27}}$$

$$= 3.06 \times 10^{-11}$$

$$= 191 \text{ MeV}$$

2. b) Frequency

$$= \text{Bq}/2\pi\text{m}$$

$$= 2 \times 1.6 \times 10^{-19}/2 \times 3.14 \times 1.67 \times 10^{-27}$$

$$= 50 \text{ MHz}$$

Chapter 8

Neutron Physics and Nuclear Fission

SOLVED PROBLEMS

Problem: 8.1- If an alpha particle were released with zero velocity near the surface of a thorium $_{90}Th^{228}$ nucleus, what would its kinetic energy be when far away ?

Solution

When an alpha particle is on the surface of the thorium, the distance between the two r is equal to the sum of their radii i.e.

$$R = R_0 A^{\frac{1}{3}}$$
$$R = 1.2 \times 10^{15} \left(4^{\frac{1}{3}} + (228)^{\frac{1}{3}}\right)$$
$$R = 9.23 \times 10^{-15} m$$

When the alpha particle is one the surface with zero velocity, it will have only potential energy:

$$\begin{aligned} P.E &= \frac{kq_1q_2}{r^2} \\ &= \frac{9 \times 10^9 \times 2 \times 90 \times (1.6 \times 10^{-19})^2}{9.23 \times 10^{-15}} \\ &= 4.49 \times 10^{12}\ J \end{aligned}$$

$$= \frac{4.49 \times 10^{-12}}{1.6 \times 10^{-13}}$$
$$= 28.1\ MeV$$

When the alpha particle is far away, the potential energy is zero and the kinetic energy will be 28.1 MeV.

Problem: 8.2- If an alpha particle were released with zero velocity near the surface of a ${}_{90}Th^{228}$ nucleus, what would be kinetic energy when it goes far away ?

Solution

When an alpha particle is on the surface of the ${}_{90}Th^{228}$, the distance between the two r is equal to the sum of their radii i.e. When the alpha particle is one the surface with zero velocity, it will have only potential energy:

$$P.E = \frac{kq_1q_2}{r^2}$$
$$= \frac{9 \times 10^9 \times 2 \times 90 \times (1.6 \times 10^{-19})^2}{9.23 \times 10^{-15}}$$
$$= 4.49 \times 10^{12}\ J$$
$$= \frac{4.49 \times 10^{-12}}{1.6 \times 10^{-13}}$$
$$= 28.1\ MeV$$
$$= 4.49 \times 10^{-12}\ J$$

When the alpha particle is far away, the potential energy is zero and the kinetic energy will be 28.1 MeV.

Problem: 8.3- ${}^{235}U$ loses about 0.1% of its mass when it undergoes fission.

1. How much energy is released when $1kg$ of ${}^{235}U$ undergoes fission?
2. One ton of TNT releases about 4 GJ when it is detonated. How many tons of TNT are equivalent in destructive power to a bomb that contains $1kg$ of ${}^{235}U$?

Solution

(1) 0.1% of $1kg = 0.001kg$

Applying the relation $E = mc^2$, we get

$$E = 0.001 \times (3 \times 10^8)^2 = 9 \times 10^{13} J$$

(2) $4GJ = 4 \times 10^9 J$ of energy is produced by $1t$ of TNT. $9 \times 10^{13} J$ of energy requires $TNT = \frac{1}{4 \times 10^9} \times 910^{13} = 22500t$.
Therefore, the destructive power of bomb using $1kg$ of $^{235}U = 22,500t$ of $TNT = 22.5$ kt of TNT.

Problem: 8.4- The nuclide ^{252}Cf undergoes spontaneous fission with a half-life of 2.62 years. Energy released in this process is about 210 MeV. If $100mg$ of this isotope is taken in a space- ship, calculate the power produced by this isotope.

Solution

We have the fission rate

$$\frac{dN}{dt} = Nl$$

Here, N is the number of nuclei present and l is the decay constant. Number of nuclei of ^{252}Cf present in $100mg$ of sample is

$$= \frac{6.023 \times 10^{23}}{252} \times 0.1 = \ 2.39 \times 10^{20}$$

$$l = \ \frac{ln2}{t_{\frac{1}{2}}} = \frac{ln2}{2.62 \times 365 \times 3600} = \ 8.39 \times 10^{-9} s^{-1}$$

and decay constant
Therefore,

$$fission\ rate = 2.39 \times 10^{20} \times 8.39 \times 10^{-9}$$
$$= 2.01 \times 10^{12} fission/s$$

Energy released in fission of one nucleus $= 210MeV$

$$= 210 \times 1.602 \times 10^{-13} J$$
$$= 3.36 \times 10^{-11} J$$

Energy released in fission of 2.01×10^{12} $nuclei = \ 3.36 \times 10^{-11} \times 2.01 \times 10^{12}$

$$67.6W$$

Problem: 8.5- The energy released during the nuclear explosion at Hiroshima has been estimated as equivalent to that released by 20,000 tons of TNT. Assume that $200MeV$ is released when a ^{235}U nucleus absorbs a neutron and fissions and that $4.18 \times 109J$ is released during detonation of 1 *ton* of TNT. How many nuclear fissions occurred at Hiroshima, and what was the total decrease

Solution

Energy released in detonation of 1 *ton* of $TNT = 4.18 \times 109J$ Energy released in detonation of 20,000 tons of TNT

$$= 4.18 \times 10^9 \times 20,000J$$
$$= 8.36 \times 10^{13}J$$

6 1013 J Energy released in fission of 1 nucleus of $^{235}U = 200\ MeV$

$$= 200 \times 1.603 \times 10^{-13}J$$
$$= 3.206 \times 10^{-11}J$$

$3.206 \times 10^{-11}J$ of energy requires 1 fission $8.36 \times 10^{13}J$ of energy will required $\frac{8.36\times10^{13}}{3.206\times10^{-11}} = 2.61 \times 10^{24} fisson$
Total energy released in 2.61×10^{24} fissions

$$2.61 \times 10^{24} \times 200\ MeV$$
$$= 5.22 \times 10^{26}\ MeV$$
$$= 8.362 \times 10^{13}J$$

Using the relation $E = mc^2$, we get

$$m = \frac{E}{c^2} = \frac{8.36 \times 10^{13}J}{(3 \times 10^8)^2} = 9.29 \times 10^{-4}kg = 0.929g$$
$$= 929\ mg$$

Chapter 9

Thermonuclear Reactions

SOLVED PROBLEMS

Problem: 9.1- Find the closest approach of a $2MeV$ proton to a gold nucleus. How does this distance compare with those for a deuteron and alpha particle of the same energy ?

Solution

The distance of closes approaches r is that distance from the nucleus at which the total energy of incident particle is potential and is given by

$$\frac{1}{2}Mv^2 = \frac{Zze^2}{4\pi\epsilon_0 r}$$

$$E = 2MeV = 2 \times 1.6 \times 10^{-13} joule$$

$$R = \frac{Zze^2}{4}\pi\epsilon_0 r$$

$$ER = 5.688 \times 10^{-14} meter$$

This distance is same for deuteron of the same energy as charge ze on the deuteron is same as that od proton. Since the charge on the alpha particle is double that of one the proton. Hence $r = 1.376 \times 10^{-14} m$.

Problem: 9.2- If the energy of alpha particle emitted by Am^{241} is $5.48MeV$, find the closest distance it can approach to a Au nucleus.

Solution

The distance of closest approach is given by $D = \ 2Ze^2/4\pi\in_\circ E$. As

$$\begin{aligned} E &= 5.48\text{MeV} \\ &= 5.48 \times 1.6 \times 10^{-13} \end{aligned}$$

$$\begin{aligned} D &= \frac{9 \times 10^9 \times 2 \times 79 \times \left(1.6 \times 10^{-19}\right)^2}{5.48 \times 1.6 \times 10^{-13}} \\ D &= 4.14 \times 10^{-14} m \end{aligned}$$

Problem: 9.3- Consider a gas of atoms undergoing fusion. Calculate the temperature required to overcome the Coulomb barrier and the released if the gas consists of

1. 10B
2. 24Mg.

Solution

(1) Let us estimate the height of the Coulomb barrier. It is given by the relation

$$V_{coul} = \frac{Z_1 Z_2 e^2}{r}$$

Here, r is the separation between two nuclei at the point of closest approach. It is given by the sum of radii of two ^{10}B nuclei. The radius of each of the nucleus can be estimated using $r = 1.2A^{\frac{1}{3}}f$. Therefore,

$$r = 1.2 \times 10^{\frac{1}{3}} + 1.2 \times 10^{\frac{1}{3}} = 5.17f$$

Coulomb barrier can be written

$$\begin{aligned} V_{coul} &= \frac{Z_1 Z_2 \times \hbar c}{r} \times \frac{e^2}{\hbar c} \\ &= \frac{e^2}{\hbar c} = \ \frac{1}{137} = a \end{aligned}$$

Substituting various values, we get

$$
\begin{aligned}
V_{coul} &= \frac{1}{137} \times \frac{5 \times 5 \times 197.5 MeV\ f}{5.17\ f} \\
&= 6.97\ MeV \\
&= 1.12 \times 10^{-12} J
\end{aligned}
$$

In order to calculate the temperature required to overcome the Coulomb barrier, we equate this energy to thermal energy as

$$
\frac{3}{2} KT = E = \ V_{coul}
$$

where K is Boltzmann's constzant and T is absolute temperature. or

$$
\begin{aligned}
\frac{3}{2} \times 1.38 \times 10^{-23} T &= 1.12 \times 10^{-12} \\
T &= 5.4 \times 10^{10} K
\end{aligned}
$$

Similar calculations are performed for the case of ^{24}Mg fusing with ^{24}Mg as under.
(2) Let us estimate the height of the Coulomb barrier. It is given by the relation

$$
V_{coul} = \frac{Z_1 Z_2 e^2}{r}
$$

Here, r is the separation between two nuclei at the point of closest approach. It is given by the sum of radii of two ^{24}Mg nuclei. The radius of each of the nucleus can be estimated using $r = 1.2 A^{\frac{1}{3}} f$. Therefore,

$$
r = 1.2 \times 24 \frac{1}{3} + 1.2 \times 24^{\frac{1}{3}} = 6.92 f
$$

Coulomb barrier can be written

$$
\begin{aligned}
V_{coul} &= \frac{Z_1 Z_2 \times \hbar c}{r} \times \frac{e^2}{\hbar c} \\
&= \frac{e^2}{\hbar c} = \ \frac{1}{137} = a
\end{aligned}
$$

Substituting various values, we get

$$V_{coul} = \frac{1}{137} \times \frac{12 \times 12 \times 197.5 MeV\ f}{6.92\ f}$$
$$= 29.99\ MeV$$
$$= 4.80 \times 10^{-12} J$$

In order to calculate the temperature required to overcome the Coulomb barrier, we equate this energy to thermal energy as

$$\frac{3}{2} KT = E = V_{coul}$$

where K is Boltzmann's constzant and T is absolute temperature. or

$$\frac{3}{2} \times 1.38 \times 10^{-23} T = 4.80 \times 10^{-12}$$
$$T = 23.2 \times 10^{10} K$$

Problem: 9.4- Calculate the mass defect and Q-values for the fusion

$$d + d \longrightarrow {}^3He + n$$
$$d + d \longrightarrow {}^3H + p$$

Assuming these occur with the deuterons at rest, find the kinetic energies of the outgoing particles in each case. Given

$$m_p = 1.007825\ amu$$
$$m_n = 1.008665\ amu$$
$$m(^2H) = 2.014102\ amu$$
$$m(^3H) = 3.016049\ amu$$
$$m(^3He) = 3.016029\ amu$$

Solution

We have

$$d + d \longrightarrow {}^3He + n$$

Mass defect is given by the relation

$$Mass\ defect = 2 \times m_d - m(^3He) - m_n$$

Substituting various given masses, we get

$$\begin{aligned} Mass\ defect &= 2 \times 2.014102 - 3.016029 - 1.008665 \\ &= 0.00351\ amu \end{aligned}$$

and

$$\begin{aligned} Q - value &= mass\ defect(amu) \times 931.47\ Mev \\ &= 0.00351 \times 931.47\ MeV \\ &= 3.27\ MeV \end{aligned}$$

$$d + d \longrightarrow {}^3H + p$$

Mass defect is given by the relation

$$Mass\ defect = 2 \times m_d - m(^3H) - m_p$$

Substituting various given masses, we get

$$\begin{aligned} Mass\ defect &= 2 \times 2.014102 - 3.016049 - 1.007825 \\ &= 0.00433\ amu \end{aligned}$$

and

$$\begin{aligned} Q - value &= mass\ defect(amu) \times 931.47\ Mev \\ &= 0.00433 \times 931.47\ MeV \\ &= 4.03\ MeV \end{aligned}$$

4.03 Mev Assuming the initial state deuterons are essentially at rest then the final state kinetic energy is equal to Q. By applying the conservation of momentum it can be seen that the share of the kinetic energy that each particle has is inversely proportional to its mass. Thus, for these reactions, the heavier particle takes one quarter while the lighter particle takes three quarters of the total kinetic energy (Q)

Note: Review questions, problems, and MCQ's are also available at then end of each chapter.