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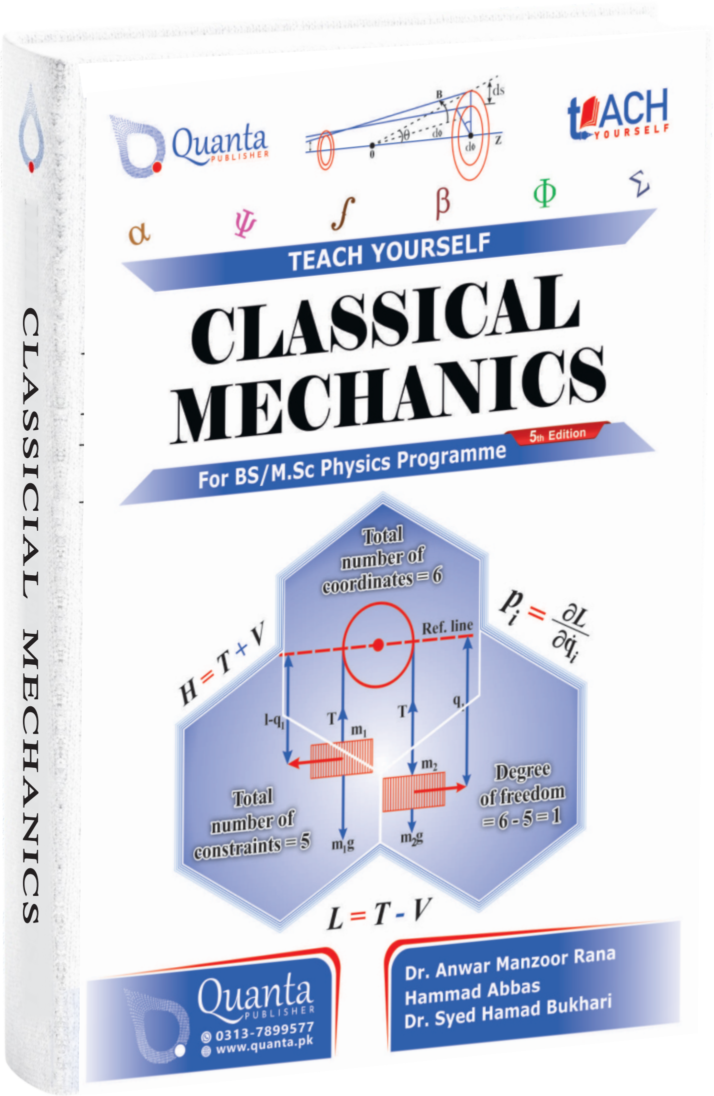

TEACH YOURSELF

CLASSICAL MECHANICS

6th Edition

For **BS/M.Sc Physics** students of all Pakistani Universities

Prof. Dr. Anwar Manzoor Rana
Department of Physics
Bahauddin Zakariya University, Multan

&

Hammad Abbas
Department of Physics
Emerson University, Multan

&

Dr. Syed Hamad Bukhari
Department of Physics
G.C. University Faisalabad, Sub-Campus, Layyah

Quanta Publisher, Raza Abad, Shah Shamas, Multan. 03137899577

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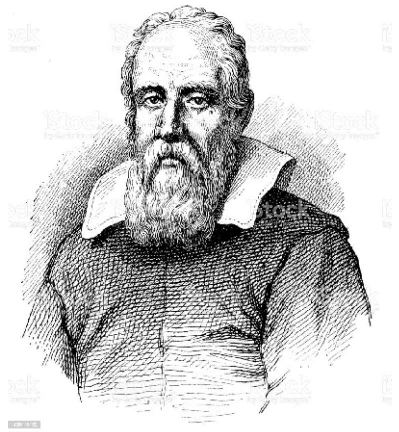

Galileo Galilei: 1564-1642

The study of the motion of bodies is an ancient one, making classical mechanics one of the oldest and largest subject in science, engineering, and technology.

Some Greek philosophers of antiquity, among them **Aristotle**, founder of Aristotelian physics, may have been the first to maintain the idea that everything happens for a reason and that theoretical principles can assist in the understanding of nature. While to a modern reader, many of these preserved ideas come forth as eminently reasonable, there is a conspicuous lack of both mathematical theory and controlled experiment, as we know it. These later became decisive factors in forming modern science, and their early application came to be known as classical mechanics. Classical mechanics is the mathematical science that studies the displacement of bodies under the action of forces.

Johannes Kepler: 1571-1630

The first published causal explanation of the motions of planets was **Johannes Keplers** Astronomia nova, published in 1609. He concluded, based on Tycho Brahes observations on the orbit of Mars, that the planets orbits were ellipses. This break with ancient thought was happening around the same time that Galileo was proposing abstract mathematical laws for the motion of objects. He may (or may not) have performed the famous experiment of dropping two cannonballs of different weights from the tower of Pisa, showing that they both hit the ground at the same

Chapter 2

Variational Principles

2.1 Hamiltons Principle

This principle states that If the configuration of a system is given at two instants t_1 and t_2 then the value of the time integral of kinetic-potential energies i.e. Lagrangian $(L = T \quad V)$ is stationary or extremum (minimum or maximum) for the path, actually described in the motion compared with any other infinitely near paths, which might be described in the same time between the same configurations.

Thus, according to Hamiltons principle, the motion of a system from its position in configuration space at time t_1 to time t_2 is such that the line integral

$$\text{Action Integral} \quad I = \int_{t_1}^{t_2} L dt \tag{2.1}$$

(Where $L = T \quad V$) is an extremum for the correct path of motion, the action integral I is stationary. That is, out of all possible paths along which the system point could travel from its position at time t_1 to its position at time t_2, it will actually travel along the path for which the integral in Eq.(2.1) is an extremum whether minimum or maximum (has stationary value). Hamiltons principle can be restated by saying the motion is such that the variation of the line integral I for fixed t_1 and t_2 is zero, i.e.

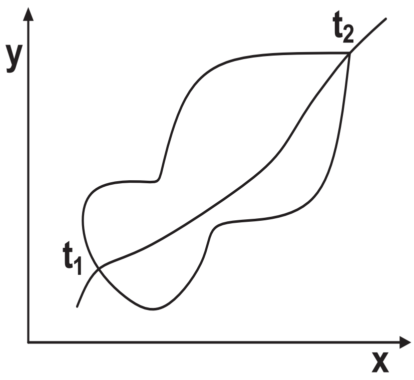

Fig. 2.1. The path of system points in configuration space.

$$I = \int_{t_1}^{t_2} L dt = 0$$

If $L = L(q_1 \ q_2 \qquad q_n; q_1 \ q_2 \qquad q_n; t)$. Then,

Chapter 3

Two Body Central Force Problems

3.1 Central Force

It is a force whose line of action passes through a single point or center (fixed or in motion with constant velocity) and whose magnitude depends only on the distance from the center.

Example

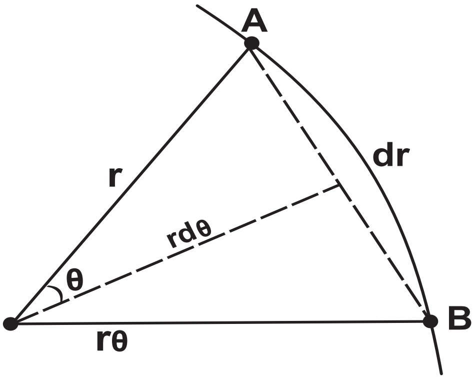

Fig. 3.1. The schematic picture which shows the motion of particle subjected to a central force.

Gravitational force, electrostatic force, motion of planets around sun. Bohr described model of hydrogen atom in term of classical body central force, scattering of -particles by nuclei. The motion of particle of mass m subjected to such force follows the equation of motion.

$$F(r) = m\frac{d^2 r}{dt^2}$$

$$\text{or} \quad F(r) = m\frac{d}{dt} \quad \frac{dr}{dt}$$

$$F(r) = m\frac{dV}{dt}$$

For 2 dimensional case, F is a function of r only as,

$$a_r = r \quad r^{\;2}$$

$$\text{and} \qquad a \; = a_T = \; r \; + 2r$$

Chapter 4

Kinematics of Rigid Body

4.1 Rigid Body Motion

Rigid Body

A rigid body is defined as a system of mass points subject to the holonomic constraints that the distances between all pairs of points remain constant throughout the motion. The position of a rigid body is fixed as soon as three non-collinear points in the body are fixed.

Any arbitrary point i in the body can be localized with reference to these three basic points as shown in Fig.(4.1). A rigid body with N particles can at most have $3N$ degrees of freedom. So to fix 3-points 1, 2 and 3 in the body, one needs 9 coordinates. But as r_{12} r_{23} and r_{13} are constants, so there are 3 constraints (holonomic). This reduces the degrees of freedom to 6 (9 - 3 = 6). A rigid body in space thus needs six independent generalized coordinates to specify its configuration. In case the body is constrained to more on a surface, or with one point fixed, this will further reduce the number of degrees of freedom and hence the number of independent coordinates.

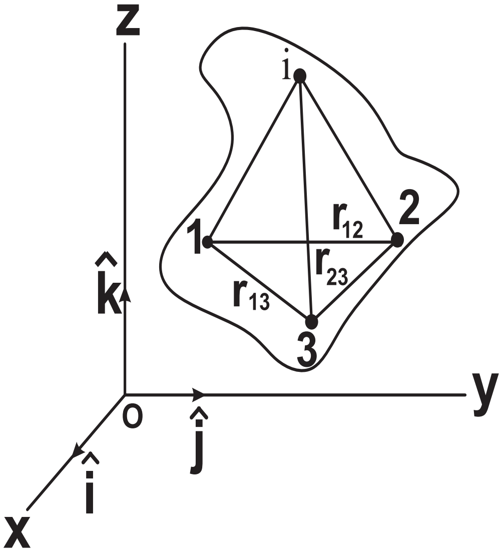

Fig. 4.1. The schematic picture which shows the location of a point in a rigid body by its distances from 3 reference points.

If the direction cosines of x y and z axes with respect to x y $z$may be designated as $_1$ $_2$ $_3$; $_1$ $_2$ $_3$ and $_1$ $_2$ $_3$ respectively in primed reference frame, then

$$i = {}_1 i + {}_2 j + {}_3 k \quad ; \quad j = {}_1 i + {}_2 j + {}_3 k \quad ; \quad k = {}_1 i + {}_2 j + {}_3 k$$

Chapter 5

The Rigid Body Equations of Motion

5.1 Angular Momentum

According to Chasles theorem *any general displacement of a rigid body can be represented by a translation plus a rotation* so the problem of a rigid body motion can be separated into two parts:

one part involves translational motion of the body only,

other is related with its rotational motion. By fixing one point of the body, only rotational motion is possible without any translation.

To specify the configuration of a rigid body in space, six independent generalized coordinates are required. These six coordinates can be divided into two sets: Three Cartesian coordinates of a point fixed in the rigid body to describe translational motion, and three angles (called Euler angles) for the motion about the point. If it is assumed that the origin of the body system is at its center of mass, then by using the mechanics of many particle system, angular momentum is given by

$$L = R \quad MV_{CM} + \sum_i r_i \quad p_i \tag{5.1}$$

Where R is the radius (position) vector of the center of mass of the body with mass M and velocity V_{CM} according to some fixed origin. Thus total angular momentum has naturally been divided into contributions from the translation of the center of mass and from the rotation about the center of mass. The rotational motion deals only with angles. In a similar way, total kinetic energy T, can also be written as:

$$T = \frac{1}{2} M V_{CM}^2 + T \left(\qquad \right) \tag{5.2}$$

i.e. the sum of kinetic energy of the entire body as if whole mass is centered at the center of mass, plus the kinetic energy of motion about the center of mass. The potential energy can often be divided in a similar fashion, i.e. the potential energy due to translation

Chapter 6

Hamiltons Equations of Motion

6.1 Generalized Momentum

The generalized momentum associated with the co-ordinate q_i is defined as;

$$p_i = \frac{L}{q_i} \qquad\qquad \left(\quad p_i = p_i(q_i \ q_i \ t) \right) \tag{6.1}$$

The terms canonical momentum or conjugate momentum are often also used for generalized momentum p_i. The following simple example will show that for certain simple cases, p_i as defined above cannot be obtained very easily. For a projectile, the Lagrangian L can be written as,

$$L = T \quad V = \frac{1}{2}\left(mx^2 + my^2 + mz^2\right) \quad mgz$$

Now, $p_1 = \frac{L}{q_i} = \frac{L}{x} = \frac{}{x}\left[\frac{1}{2}\left(mx^2 + my^2 + mz^2\right) \quad mgz\right]$

$$p_1 = \frac{L}{x} = \frac{1}{2}\left(m\frac{}{x}x^2 + m\frac{}{x}y^2 + m\frac{}{x}z^2\right) \quad mg\frac{}{x}z$$

$$p_1 = \frac{L}{x} = \frac{1}{2}\left(2mx + m(0) + m(0)\right) \quad mg(0)$$

$$p_1 = \frac{L}{x} = \frac{1}{2}\left(2mx + 0 + 0\right) \quad 0 = \frac{L}{x} = mx = \frac{L}{x} = p_x \qquad \left(\quad p_i = \frac{L}{q_i}\right)$$

This implies that;

$$p_1 = p_x = \frac{L}{x} = mx \quad ; \quad p_2 = p_y = \frac{L}{y} = my$$

$$\text{and} \quad p_3 = p_z = \frac{L}{z} = mz$$

Hence p_x p_y and p_y are the familiar components of linear momentum. However, if q_i is not a Cartesian co-ordinate, p_i does not necessarily have the dimension of linear momentum.

Chapter 7

Canonical Transformations

7.1 Examples of Canonical Transformations

In order to know about the nature of Canonical transformations and the importance of generating function, we consider some simple but important examples.

Generating Function F_2

Let us consider the second generating function F_2 as given below:

$$F_2 = q_j P_j \qquad \Big(\quad F_2 = F_2(q_j \;\; P_j \;\; t)\Big) \tag{7.1}$$

The transformation equations for F_2 are given as:

$$\underbrace{p_j = \frac{F_2}{q_j}}_{(7\ 2\text{-}\mathbf{a})} \qquad ; \qquad \underbrace{Q_j = \frac{F_2}{P_j}}_{(7\ 2\text{-}\mathbf{b})} \tag{7.2}$$

and

$$K = H + \frac{F_2}{t} \tag{7.3}$$

Now following Eq.(7.1) to obtain transformation equations. Taking partial derivative of Eq.(7.1) with respect to q_j, we get

$$\frac{F_2}{q_j} = P_j \frac{q_j}{q_j} \qquad ; \qquad p_j = P_j \tag{7.4}$$

$$\frac{F_2}{q_j} = P_j \qquad ; \qquad p_j = P_j \qquad \Big(\quad \text{Using Eq.(7.2-}\mathbf{a})\Big) \tag{7.5}$$

Similarly, partial derivative of Eq.(7.1) w.r.t. P_j gives;

$$\frac{F_2}{P_j} = q_i \frac{P_j}{P_j} = q_j$$

$$Q_j = q_j \qquad \Big(\quad \text{Using Eq.(7.2-}\mathbf{b})\Big) \tag{7.6}$$

And, now from Eq.(7.3), we get

$$K = H + 0 = H \qquad \left(\quad \frac{F_2}{t} = 0\right) \tag{7.7}$$


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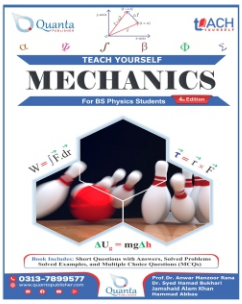

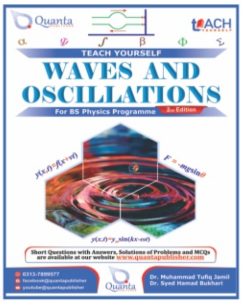

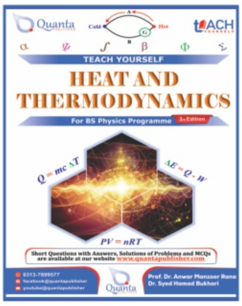

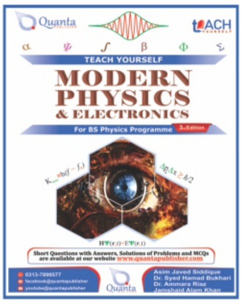

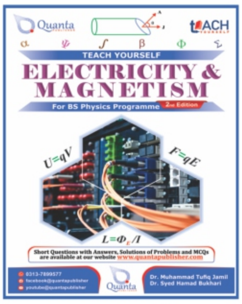

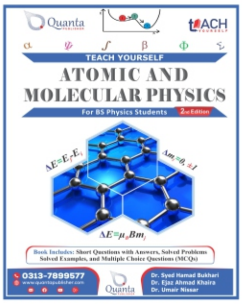

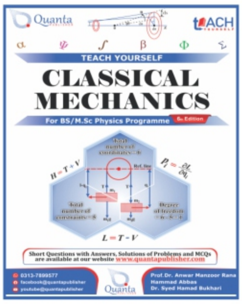

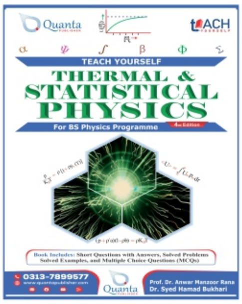

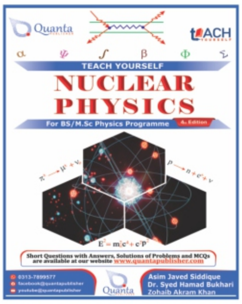

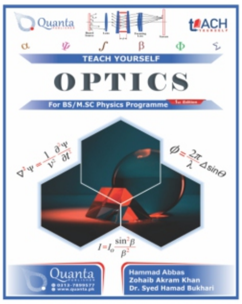

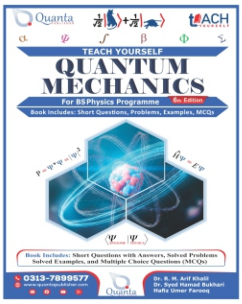

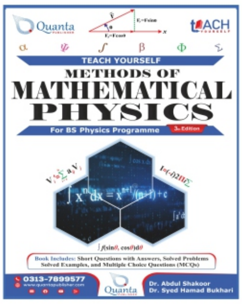

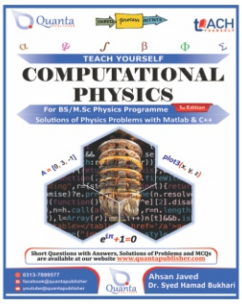

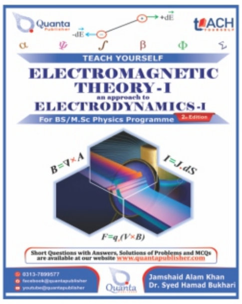

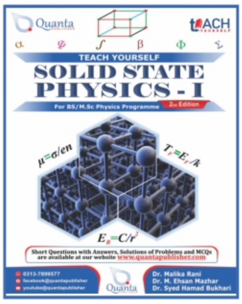

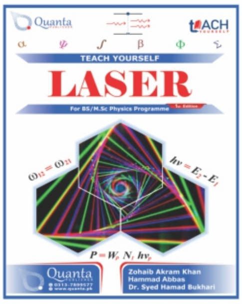

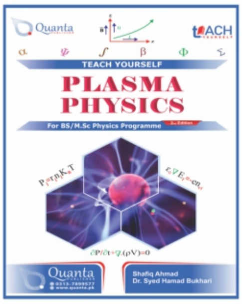

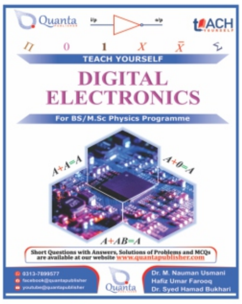

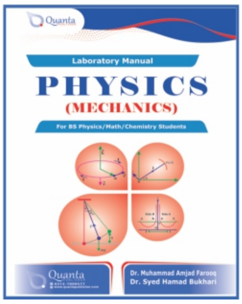

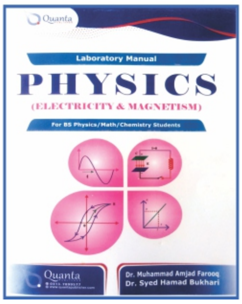

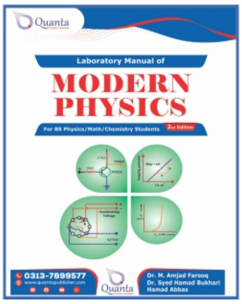

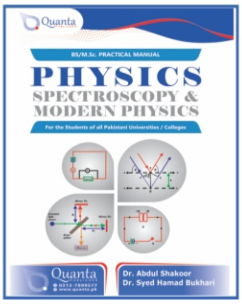

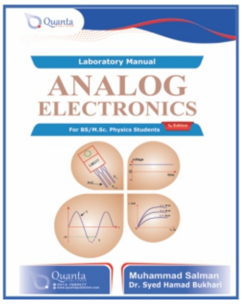

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