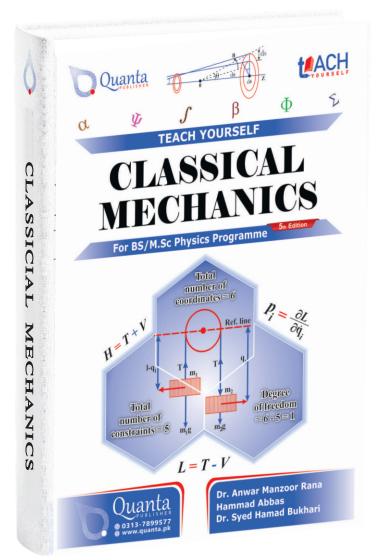
Introduction SAMPLE PAGES



SAMPLE PAGES



For Online Order

0 0313-7899577

www.quantapublisher.com



TEACH YOURSELF

CLASSICAL MECHANICS

______ 6th Edition

For BS/M.Sc Physics students of all Pakistani Universities

Prof. Dr. Anwar Manzoor Rana

Department of Physics Bahauddin Zakariya University, Multan

&

Hammad Abbas

Department of Physics Emerson University, Multan

&

Dr. Syed Hamad Bukhari

Department of Physics G.C. University Faisalabad, Sub-Campus, Layyah

Quanta Publisher, Raza Abad, Shah Shamas, Multan. 03137899577

Contents

1	Ele	ementary Particles	1
	1.1	Historical Perspective	1
	1.2	Introduction to Mechanics	4
	1.3	Brief Survey of Newtonian Mechanics	7
		1.3.1 Mechanics of a Single Particle	7
			12
	1.4		18
			19
			19
			20
			21
			22
			23
			23
	1.5		24
	1.6		25
	1.7		29
			30
		· ()	32
		1.7.3 Atwood Machine	36
			38
	1.8		40
2	Var	riational Principles	44
		-	44
		2.1.1 Calculus of Variations (Euler-Lagranges Equation)	46
	2.2	(49
			49
			51
	2.3		53
	2.4		55

TABLE OF CONTENTS

3	Tw	o Body Central Force Problems	
	3.1	Central Force	
		3.1.1 Motion of a Particle under the Influence of a Central Force	58
		3.1.2 Two Body Central Force Problems and their Reduction to the Equivalent One	
		Body Problem	60
		3.1.3 To Find an Orbit under an Inverse Square Law of Force	61
	3.2	Keplers Laws	64
	3.3	Laboratory and Center of Mass Coordinates and their Mutual Transformation	70
		3.3.1 The Equivalent One Dimensional Problem and Classification of Orbit	74
		3.3.2 Scattering in a Central Force Field (Rutherfords Scattering Formula)	78
	3.4	Definition of Scattering Cross-Section (Additional)	84
	3.5	Introduction to the General Theory of Relativity	
	3.6	Review Questions and Problems	
4	Kin	nematics of Rigid Body	92
4	4.1		
		Rigid Body Motion	
	4.2	Orthogonal Transformations	
	4.3	Eulerian Angles	
	4.4	Eulers Theorem	
	4.5	The Coriolis Force	
	4.6	Review Questions and Problems	107
5	$\operatorname{Th}\epsilon$	e Rigid Body Equations of Motion	
	5.1	Angular Momentum	
		5.1.1 Tensors and Dyadics	
	5.2	The Moment of Inertia	114
		5.2.1 Parallel Axis Theorem	115
	5.3	Rigid Body Problems and the Eulers Equations	116
	5.4	The Euler Angles	120
	5.5	Eulers Theorem on the Motion of a Rigid Body	125
	5.6	Review Questions and Problems	
6	Hai	niltons Equations of Motion	133
	6.1	Generalized Momentum	133
	6.2	Hamiltons Equations and Hamiltonian H	134
		6.2.1 Derivation of Hamilton Equations (from Hamiltons Principle)	
	6.3	Cyclic Coordinates and General Conservation Theorem	
	6.4	Legendre Transformation	141
	6.5	Canonical Transformations	143
	6.6	The Harmonic Oscillator	
	6.7	Review Questions and Problems	
7	Car	nonical Transformations	157
•	7.1	Examples of Canonical Transformations	157
	$7.1 \\ 7.2$	Lagrange and Poisson Brackets	
	7.2	Fundamental Poisson Brackets	
	7.3		172
		Poissons Theorem	
	7.5	Liouvilles Theorem	178
	7.6	Rouths Procedure	181
	7.7	Review Questions and Problems	184

CHAPTER # 1 SAMPLE PAGES

Chapter 1

Elementary Particles 1.1 Historical Perspective

The study of the motion of bodies is an ancient one, making classical mechanics one of the oldest and largest subject in science, engineering, and technology.

Some Greek philosophers of antiquity, among them **Aristotle**, founder of Aristotelian physics, may have been the first to maintain the idea that everything happens for a reason and that theoretical principles can assist in the understanding of nature. While to a modern reader, many of



Galileo Galilei: 1564-1642

these preserved ideas come forth as eminently reasonable, there is a conspicuous lack of both mathematical theory and controlled experiment, as we know it. These later became decisive factors in forming modern science, and their early application came to be known as classical mechanics. Classical mechanics is the mathematical science that studies the

displacement of bodies under the action of forces.

The first published causal explanation of the motions of planets was Johannes Keplers Astronomia nova, published in 1609. He concluded, based on Tycho Brahes observations on the orbit of Mars, that the planets orbits were ellipses. This break with ancient thought was happening around the same time that Galileo was proposing abstract mathematical laws for the motion of objects. He may (or may not) have performed the famous experiment of dropping two cannonballs of different weights from the tower of Pisa, showing that they both hit the ground at the same



Johannes Kepler: 1571-1630

CHAPTER # 2 SAMPLE PAGES

Chapter 2

Variational Principles

2.1 Hamiltons Principle

This principle states that If the configuration of a system is given at two instants t_1 and t_2 then the value of the time integral of kinetic-potential energies i.e. Lagrangian (L = T - V) is stationary or extremum (minimum or maximum) for the path, actually described in the motion compared with any other infinitely near paths, which might be described in the same time between the same configurations.

Thus, according to Hamiltons principle, the motion of a system from its position in configuration space at time t_1 to time t_2 is such that the line integral

Action Integral
$$I = \int_{t_1}^{t_2} L dt$$
 (2.1)

(Where L = T - V) is an extremum for the correct path of motion, the action integral I is stationary. That is, out of all possible paths along which the system point could travel from its position at time t_1 to its position at time t_2 , it will actually travel along the path for which the integral in

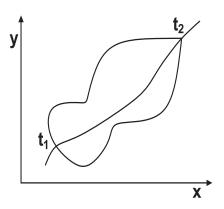


Fig. 2.1. The path of system points in configuration space.

Eq.(2.1) is an extremum whether minimum or maximum (has stationary value). Hamiltons principle can be restated by saying the motion is such that the variation of the line integral I for fixed t_1 and t_2 is zero, i.e.

$$I = \int_{t_1}^{t_2} Ldt = 0$$

$$q_n; q_1 \ q_2$$

$$q_n;t$$
). Then,

CHAPTER # 3 SAMPLE PAGES

Chapter 3

Two Body Central Force Problems

3.1 Central Force

It is a force whose line of action passes through a single point or center (fixed or in motion with constant velocity) and whose magnitude depends only on the distance from the center.

Example

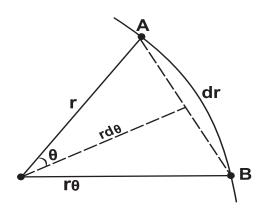


Fig. 3.1. The schematic picture which shows the motion of particle subjected to a central force.

Gravitational force, electrostatic force, motion of planets around sun. Bohr described model of hydrogen atom in term of classical body central force, scattering of $\,$ -particles by nuclei. The motion of particle of mass m subjected to such force follows the equation of motion.

$$F(r) = m\frac{d^{2} r}{dt^{2}}$$
or
$$F(r) = m\frac{d}{dt} \frac{d r}{dt}$$

$$F(r) = m\frac{dV}{dt}$$

For 2 dimensional case, F is a function of r only as,

$$a_r = r \quad r^{-2}$$
 and
$$a_r = a_T = r^{-2} + 2r$$

CHAPTER # 4 SAMPLE PAGES

Chapter 4

Kinematics of Rigid Body

4.1 Rigid Body Motion

Rigid Body

A rigid body is defined as a system of mass points subject to the holonomic constraints that the distances between all pairs of points remain constant throughout the motion. The position of a rigid body is fixed as soon as three non-collinear points in the body are fixed.

Any arbitrary point i in the body can be localized with reference to these three basic points as shown in Fig.(4.1). A rigid body with N particles can at most have 3N degrees of freedom. So to fix 3-points 1, 2 and 3 in the body, one needs 9 coordinates. But as r_{12} r_{23} and r_{13} are constants, so there are 3 constraints (holonomic). This reduces the degrees of freedom to 6 (9 - 3 = 6). A rigid body in space

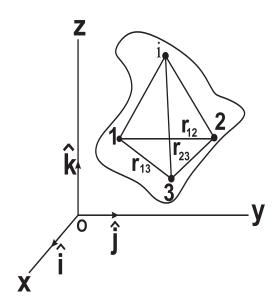


Fig. 4.1. The schematic picture which shows the location of a point in a rigid body by its distances from 3 reference points.

thus needs six independent generalized coordinates to specify its configuration. In case the body is constrained to more on a surface, or with one point fixed, this will further reduce the number of degrees of freedom and hence the number of independent coordinates.

If the direction cosines of x y and z axes with respect to x y zmay be designated as $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ respectively in primed reference frame, then

$$i = {}_{1}i + {}_{2}j + {}_{3}k$$
 ; $j = {}_{1}i + {}_{2}j + {}_{3}k$; $k = {}_{1}i + {}_{2}j + {}_{3}k$

CLASSICAL MECHANICS Quanta Publisher

CHAPTER # 5 SAMPLE PAGES

Chapter 5

The Rigid Body Equations of Motion 5.1 Angular Momentum

According to Chasles theorem any general displacement of a rigid body can be represented by a translation plus a rotation so the problem of a rigid body motion can be separated into two parts:

one part involves translational motion of the body only,

other is related with its rotational motion. By fixing one point of the body, only rotational motion is possible without any translation.

To specify the configuration of a rigid body in space, six independent generalized coordinates are required. These six coordinates can be divided into two sets: Three Cartesian coordinates of a point fixed in the rigid body to describe translational motion, and three angles (called Euler angles) for the motion about the point. If it is assumed that the origin of the body system is at its center of mass, then by using the mechanics of many particle system, angular momentum is given by

$$L = R \quad MV_{CM} + \sum_{i} r_{i} \quad p_{i} \tag{5.1}$$

Where R is the radius (position) vector of the center of mass of the body with mass M and velocity V_{CM} according to some fixed origin. Thus total angular momentum has naturally been divided into contributions from the translation of the center of mass and from the rotation about the center of mass. The rotational motion deals only with angles. In a similar way, total kinetic energy T, can also be written as:

$$T = \frac{1}{2}MV_{CM}^2 + T \, () \tag{5.2}$$

i.e. the sum of kinetic energy of the entire body as if whole mass is centered at the center of mass, plus the kinetic energy of motion about the center of mass. The potential energy can often be divided in a similar fashion, i.e. the potential energy due to translation

CLASSICAL MECHANICS Quanta Publisher

CHAPTER # 6 SAMPLE PAGES

Chapter 6

Hamiltons Equations of Motion

6.1 Generalized Momentum

The generalized momentum associated with the co-ordinate q_i is defined as;

$$p_i = \frac{L}{q_i} \qquad \left(p_i = p_i(q_i \ q_i \ t) \right) \tag{6.1}$$

The terms canonical momentum or conjugate momentum are often also used for generalized momentum p_i . The following simple example will show that for certain simple cases, p_i as defined above cannot be obtained very easily. For a projectile, the Lagrangian L can be written as,

$$L = T V = \frac{1}{2} \left(mx^2 + my^2 + mz^2 \right) mgz$$

$$\text{Now, } p_1 = \frac{L}{q_i} = \frac{L}{x} = \frac{1}{x} \left[\frac{1}{2} \left(mx^2 + my^2 + mz^2 \right) mgz \right]$$

$$p_1 = \frac{L}{x} = \frac{1}{2} \left(m - x^2 + m - y^2 + m - z^2 \right) mg - z$$

$$p_1 = \frac{L}{x} = \frac{1}{2} \left(2mx + m(0) + m(0) \right) mg(0)$$

$$p_1 = \frac{L}{x} = \frac{1}{2} \left(2mx + 0 + 0 \right) 0 = \frac{L}{x} = mx = \frac{L}{x} = p_x \left(p_i = \frac{L}{q_i} \right)$$

This implies that;

$$p_1 = p_x = \frac{L}{x} = mx \quad ; \quad p_2 = p_y = \frac{L}{y} = my$$
and
$$p_3 = p_z = \frac{L}{z} = mz$$

Hence p_x p_y and p_y are the familiar components of linear momentum. However, if q_i is not a Cartesian co-ordinate, p_i does not necessarily have the dimension of linear momentum.

CLASSICAL MECHANICS Quanta Publisher

CHAPTER # 6 SAMPLE PAGES

Chapter 7

Canonical Transformations

7.1 Examples of Canonical Transformations

In order to know about the nature of Canonical transformations and the importance of generating function, we consider some simple but important examples.

Generating Function F_2

Let us consider the second generating function F_2 as given below:

$$F_2 = q_j P_j \qquad (F_2 = F_2(q_j P_j t)) \tag{7.1}$$

The transformation equations for F_2 are given as:

$$\underbrace{p_{j} = \frac{F_{2}}{q_{j}}}_{(7 2-\mathbf{a})} ; \qquad \underbrace{Q_{j} = \frac{F_{2}}{P_{j}}}_{(7 2-\mathbf{b})} \tag{7.2}$$

and

$$K = H + \frac{F_2}{t} \tag{7.3}$$

Now following Eq.(7.1) to obtain transformation equations. Taking partial derivative of Eq.(7.1) with respect to q_j , we get

$$\frac{F_2}{q_j} = P_j \frac{q_j}{q_j} \qquad ; \qquad p_j = P_j \tag{7.4}$$

$$\frac{q_{j}}{q_{j}} = P_{j} \qquad ; \qquad p_{j} = P_{j} \qquad (\text{Using Eq.}(7.2-\mathbf{a})) \qquad (7.5)$$

Similarly, partial derivative of Eq.(7.1) w.r.t. P_j gives;

$$\frac{F_2}{P_j} = q_i \frac{P_j}{P_j} = q_j$$

$$Q_j = q_j \qquad \left(\text{Using Eq.}(7.2\text{-}\mathbf{b}) \right)$$
(7.6)

And, now from Eq.(7.3), we get

$$K = H + 0 = H \qquad \left(\frac{F_2}{t} = 0 \right) \tag{7.7}$$



	Liententary Farticles	. 01
	1.1 Historical Perspective	01
	1.2 Introduction to Mechanics	
	1.3 Brief Survey of Newtonian Mechanics	.07
	1.3.1 Mechanics of a Single Particle	. 07
	1.3.2 Mechanics of a System of Particles	.12
	1.4 Constraints	18
	1.4.1 Equations of Constraints	. 19
	1.4.2 Degrees of Freedom	19
	1.4.3 Generalized Coordinates	20
	1.4.4 Generalized Velocity	21
	1.4.5 Generalized Force	
	1.4.6 Actual and Virtual Displacements	
	1.4.7 Principle of Virtual Work	
	1.5 D'Alembert's Principle	
	1.6 Lagrange's Equations	
	1.7 Applications of Lagrange's Equations	
	1.7.1 Motion of a Particle in Space	
	1.7.2 Motion of a Particle in Polar	
	1.7.3 Atwood Machine	
	1.7.4 Lagrange's Equation for a Simple	
	1.8 Review Questions and Problems	
2	Variational Principles	
	2.1 Hamilton's Principle	44
	2.1.1 Calculus of Variations	
	2.2.1 Shortest Distance Between Two	
	2.2.2 Minimum Surface of Revolution	
	2.3 Derivation of Lagrange's Equation	
	2.4 Review Questions and Problems	
3	Two Body Central Force Problems	
,	3.1 Central Force	
	3.1.1 Motion of a Particle under the	
	3.1.2 Two Body Central Force Problems	
	J.L.Z IWO Dody Celitial Force Froblems	UC

	3.1.3 To Find an Orbit under an Inverse	
	3.2 Kepler's Laws	
	3.3 Laboratory and Center of Mass	
	3.3.1 The Equivalent One Dimensional	
	3.3.2 Scattering in a Central Force Field	
	3.4 Definition of Scattering Cross-Section	.84
	3.5 Introduction to the General Theory of	85
	3.6 Review Questions and Problems	89
4	Kinematics of Rigid Body	92
	4.1 Rigid Body Motion	92
	4.2 Orthogonal Transformations	93
	4.3 Eulerian Angles	96
	4.4 Euler's Theorem	100
	4.5 The Coriolis Force	104
	4.6 Review Questions and Problems	107
5	The Rigid Body Equations of Motion	109
	5.1 Angular Momentum	109
	5.1.1 Tensors and Dyadics	112
	5.2 The Moment of Inertia	114
	5.2.1 Parallel Axis Theorem	
	5.3 Rigid Body Problems and Euler's	
	5.4 The Euler Angles	
	5.5 Euler's Theorem on the Motion of	
	5.6 Review Questions and Problems	
6	Hamilton's Equations of Motion	
	6.1 Generalized Momentum	
	6.2 Hamilton's Equations and Hamiltonian H	
	6.2.1 Derivation of Hamilton Equations	
	6.3 Cyclic Coordinates and General	
	6.4 Legendre Transformation	
	6.5 Canonical Transformations	
	6.6 The Harmonic Oscillator	
_	6.7 Review Questions and Problems	
7	Canonical Transformations	
	7.1 Examples of Canonical Transformations	
	7.2 Lagrange and Poisson Brackets	
	7.3 Fundamental Poisson Brackets	
	7.4 Poisson's Theorem	
	7.5 Liouville's Theorem	
	7.6 Routh's Procedure	
	7.7 Review Questions and Problems	184

BOOKS BY QUANTA SAMPLE PAGES

Books by Quanta Publisher for BS Physics Students





BS Physics

- 9 0313-7899577
- www.quantapublisher.com
- **Quanta Publisher Physics**
- @Quanta Publisher
- Quanta Publisher