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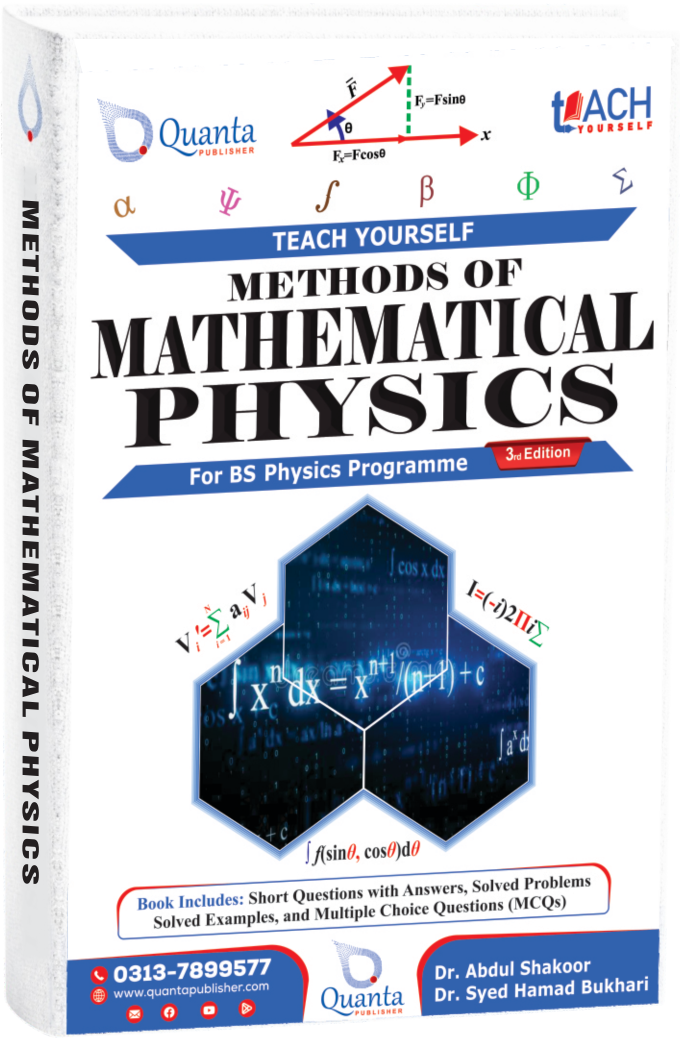

TEACH YOURSELF

METHODS OF MATHEMATICAL PHYSICS

3rd Edition

For **BS Physics** Students of all Pakistani Universities/Colleges

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Chapter 1

Vector Analysis

In science and engineering, we frequently encounter quantities which have magnitude and direction only e.g., mass, time and temperature. We call these quantities as **scalars**. In contrast, many interesting physical quantities in addition to magnitude have direction as well. This group includes displacement, velocity, acceleration, force, momentum and angular momentum. Quantities with magnitude and direction are labeled as **vector quantities**. As an historical insight, it is interesting to note that the vector quantities listed are all taken from mechanics but that vector analysis was not used in the development of mechanics and, indeed, had not been created. The need for vector analysis become apparent only with the development of Maxwell's electromagnetic theory and in application of the inherent vector nature of quantities such as electric field and magnetic field.

1.1 Scalar and Vector

Question **Define scalar and vector and terms associated with them. How we find the resultant of a vector? Discuss the transformation laws of coordinates.**

Scalar

If a quantity does not depend on the orientation of the coordinate axes, then it is called **scalar**. If we assume that space is isotropic i.e. there is no preferred direction or all direction are equivalent, then the physical system being analyzed or the physical laws being expressed cannot and must not depend on our choice or orientation of coordinate axes. It is a geometrical object, independent of coordinate system. **These quantities remains invariant under rotation of system of coordinates.**

Vector

Quantities which changes under the rotation of system of coordinates according to transformation law are said to be **vectors**. Unit vector serves as basis.

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \tag{1.1}$$

Notation

A vector is indicated by a boldface letter, e.g. $\mathbf{A}$ or with an arrow over it $(\vec{A})$. The components of a vector is represented by a subscript, e.g., A_x is the x-component of vector $\vec{A}$, A_y is the y-component and A_z is the z-component of $\vec{A}$.

Chapter 2

Matrix Algebra

MATRICES are 2-D arrays of numbers or functions that obey the laws that define matrix algebra. The subject is important for physics because it facilitates the description of linear transformations such as changes of coordinate systems, provides a useful formulation of quantum mechanics, and facilitates a variety of analysis in classical and relativistic mechanics, particle theory, and other areas. Note also that the development of a mathematics of two-dimensionally ordered arrays is a natural and logical extension of concepts involving ordered pairs of numbers (complex numbers) or ordinary vectors (one-dimensional arrays). The most distinctive feature of matrix algebra is the rule for the multiplication of matrices.

2.1 Matrix

Question **Define matrix and its types. Also discuss the algebraic operations on matrix.**

2-D array of numbers or functions that obey the laws which governs the matrix algebra. It helps the transformations of system of coordinate. Provides useful formulation of quantum mechanics and facilitates a variety of analysis in classical and relativistic mechanics. Matrix of order $(m \times n)$ is:

$$A_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}$$

Where, a_{nm} are the number of elements in a matrix and n is the number of rows and m is the number of columns.

Equal Matrix

Two matrix A and B are said to be equal if their orders and the number of elements are same i.e., $a_{ij} = b_{ij}$.

Example: $A = \begin{pmatrix} 3 & 4 \\ -4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2+1 & 5-1 \\ -5+1 & 0 \end{pmatrix}$, where A and B are equal matrix.

Chapter 3

Complex Variables and Infinite Series

WE turn now to a study of complex variable theory. In this area, we develop some of the most powerful and widely useful tools in all of analysis. To indicate, at least partly, why complex variables are important, we mention briefly several areas of application.

- In two dimensions, the electric potential, viewed as a solution of Laplace's equation.
- The time-dependent Schrödinger equation of quantum mechanics contains the imaginary unit i, and its solutions are complex.
- The change of a parameter k from real to imaginary, $k \to ik$, transforms the Helmholtz equation into the time-independent diffusion equation.
- Many physical quantities that were originally real become complex as a simple physical theory is made more general.

3.1 Complex Variables

Question **Define and explain complex variables. Also, write the properties of complex numbers. Describe the polar form of complex number. What is Euler's formula.**

A complex variable z may be expressed as $z = x + iy$ where $i = \sqrt{-1}$, and $x,\ y$ are real values. In this form, x is known as **real part** of z whereas y is known as **imaginary part** of z. Usually, the real part x is denoted as R_z or $R(z)$ and the imaginary part y as I_z or $I(z)$.

Complex Function and Properties

Complex conjugate or conjugate of a complex number $z = x + iy$ is a complex number $\bar{z} = x - iy$.

1. When $x,\ y$ are real numbers and $x + iy = 0$ then $x = 0,\ y = 0$.
2. For any three set of complex numbers $z_1,\ z_2$ and z_3 satisfies the commutative, associative and distributive laws.
   a) $z_1 + z_2 = z_2 + z_1$ (Commutative law for addition).
   b) $z_1 \cdot z_2 = z_2 \cdot z_1$ (Commutative law for multiplication).
   c) $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ (Associative law for addition).
   d) $(z_1 z_2) z_3 = z_1 (z_2 z_3)$ (Associative law for multiplication).
   e) $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$ (Distributive law).
3. The sum of two conjugate complex is real.
4. The product of two conjugate complex is real.
5. When the sum of two complex numbers is real and the product of two complex numbers is also real then complex numbers are conjugate to each other.

Chapter 4

Differential Equations

IN diverse scientific and technological domains, several challenges are framed in the language of differential equations, with partial differential equations (PDEs) incorporating two or more independent variables. Addressing these complex problems often involves a potent strategy-the method of separation of variables. This technique entails breaking down a PDE into (ordinary) differential equations, allowing for a systematic and more manageable solution. By decomposing the problem, researchers can analyze each component independently, simplifying the overall process. This approach is particularly valuable when dealing with intricate physical phenomena or complex systems, offering a powerful tool to resolve and realized the underlying mathematical complexities, ultimately facilitating the resolution of problems in physics and a many of other scientific and technological domains.

4.1 Differential Equation

Question **Define differential equation and give examples. Also what is the order and degree of differential equation and give examples.**

An equation containing the derivative or differential of one or more dependent variables w.r.t. one or more independent variables is called a **differential equation**. For example;

$$\frac{d^2y}{dx^2}+xy\left(\frac{dy}{dx}\right)^2 = 0 \quad \text{and} \quad \frac{\partial V}{\partial s}+\frac{\partial V}{\partial t} = 0$$

Order of Differential Equation

The highest ordered derivative involved in a differential equation is called the **order of the differential equation**. For example;

$$\frac{d^2y}{dx^2}+xy\left(\frac{dy}{dx}\right)^2 = 0 \quad \text{(Order 2)}, \qquad \frac{\partial V}{\partial s}+\frac{\partial V}{\partial t} = 0 \quad \text{(Order 1)}$$

$$\frac{d^4x}{dt^4}+5\frac{d^2x}{dt^2}+3x = \sin t \quad \text{(Order 4)}, \qquad \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = 0 \quad \text{(Order 2)}$$

Chapter 5

Special Functions

SPECIAL functions, like Bessel, Legendre, Laguerre, and Hermite functions, are like superheroes in math for describing everyday things. Bessel helps with circular stuff, Legendre with things shaped like a ball, Laguerre tackles equations for things that expand, and Hermite takes on jobs in quantum physics and bouncing objects. These math heroes make hard problems simpler, helping scientists understand how waves, quantum stuff, and different things work. They're like a team of super-powered friends making math more friendly and helping us uncover the cool secrets of the world around us.

5.1 Legendre's Differential Equation and its Solution

Question **What is Legendre's differential equation and write the solution of Legendre's differential equation. Also discuss Legendre polynomial of the first kind and second kind.**

Legendre differential equation is:

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y \;=\; 0 \tag{5.1}$$

Where, n is a positive integer. Solution of the Eq.(5.1Legendre's Differential Equation Solution is known as **Legendre function**. In order to find out singular points and possibility of the series solution for the Eq.(5.1Legendre's Differential Equation and its Solutionequation.5.1.1), we have

$$P(x) \;=\; -\frac{2x}{1-x^2} \qquad \text{and} \qquad Q(x) \;=\; \frac{n(n+1)}{1-x^2}$$

Obviously, there are singular points at $x=-1$ and $x=1$. However, both the singular points are regular as $(x+1)P(x)$ and $(x+1)^2Q(x)$ both are finite at the point $x=-1$, and $(x-1)P(x)$ and $(x-1)^2Q(x)$ both are finite at the point $x=1$. Thus, as per Fuchs theorem, Legendre differential equation has a series solution. Let the series solution be:

$$y \;=\sum_{r=0}^{\infty} a_r x^{k-r} \qquad\qquad (a_0 \neq 0) \tag{5.2}$$

(Solution can also be obtained as ascending powers of x, but the solution in descending powers of x is more convenient). On differentiating Eq.(5.2Legendre's Differential Equation Solution w.r.t. x, we get

Chapter 6

Fourier Series and Integral Transformation

FOURIER series, a fundamental concept in mathematical analysis, analyze periodic functions into a sum of sine and cosine functions, discover their harmonic components. Fourier cosine and sine series provide specialized representations, while the exponential form extends the generality. Adaptable to varied contexts, changes in interval or origin shift refine applications, from signal processing to heat transfer. Fourier transformation develop this scope to a periodic functions, revealing frequency domain insights. Laplace transforms, powerful tools in solving linear differential equations, facilitate system analysis. Their properties, such as linearity, and applications, from modeling harmonic motion to solving simultaneous equations, underscore their crucial role in mathematics and engineering.

6.1 Fourier Series

Question **Define and explain the Fourier series with examples.**

A function $f(x)$ can be represented in th form of a series of sines and cosines:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \qquad (6.1)$$

Where the coefficients a's and b's are to be determined. This series of representation of a function is known as **Fourier series of the function**. Fourier series representation of a function may be feasible even when the function has discontinuities, but its Taylor series expansion is not possible. Fourier series expansion of a function is possible provided it satisfies a set of conditions, known as the **Dirichlet conditions**, given as:

(a) The function must be periodic.

(b) Within one cycle (period), the function must be single-valued and differentiable, except at a finite number of discontinuities having finite magnitude.

(c) The function must have finite number of maxima and minima (extremes) within a cycle.

These conditions are sufficient, but not necessary. In a large number of physical cases where Fourier series representation of a function is required, these conditions are generally satisfied. We know that sine and cosine functions are periodic with a period of 2π. Further, for Fourier series representation we account for the function over one complete cycle. The coefficients a's and b's are related to the given function $f(x)$, and can be calculated in the following manner. Here, we shall use the following orthogonality properties of sine and cosine functions:

Chapter 7

Green's Functions and Boundary Value Problems

GREEN'S function is a powerful mathematical tool widely employed in the realm of boundary value problems in physics and engineering. Deriving from the field of potential theory, Green's function provides a solution to partial differential equations subject to specific boundary conditions. Serving as a fundamental building block, it summarized the influence of point sources on a system, offering a unique solution for various physical phenomena. In this context, the study of boundary value problems involving Green's function plays a pivotal role in understanding and solving complex mathematical models that arise in fields such as heat conduction, fluid dynamics, and electromagnetic theory.

7.1 The Dirac Delta Function

Question **What is Dirac delta function? Define the properties of Dirac delta function and discuss the integral representation of the Dirac delta function.**

The Dirac delta function can be regarded as the generalization of the Kronecker delta δ_{ij}, when the discrete integer variables $i,\ j$ are replaced by the continuous variables $x,\ x'$. The **Kronecker delta** δ_{ij} has the following two well-known properties.

1. $\delta_{ij} = \begin{cases} 1 & ; \quad i = j \\ 0 & ; \quad i \neq j \end{cases}$
2. $\delta_{ij}A_i = A_j$

These properties are passed over to the **Dirac delta function** in the following form:

1. $\delta(x-t) = \begin{cases} 0 & ; \quad t \neq x \\ \infty & ; \quad t = x \end{cases}$
2. $\int \delta(x-t) f(t) dt = f(x)$

Where, the point x lies in the interval of integration and $f(x)$ is a continuous function. To understand the **significance of the Dirac delta function** in physical situations, we consider those situations in which a large effect lasting for a short duration or acting over a small stretch of length. The examples are an impulsive force or a load over a very small part of a beam. Also, we can define the Dirac delta function $\delta(x)$ in the following manner.

Chapter 8

Tensor Analysis

VENTURING into the realm of tensors and their applications, this exploration resolve the mathematical framework crucial to distinct fields. Tensors, multi-dimensional arrays, serve as powerful tools in physics, engineering, and computer science, capturing complex relationships within data. From the foundational concepts of tensor algebra to applications in general relativity, fluid dynamics, and machine learning, this chapter dig into their originality. Topics include the Levi-Civita tensor, Kronecker delta tensor, and their roles in expressing symmetry and antisymmetry. As, we navigate through tensors, their manipulation, and practical applications, we unveil the intense impact of these mathematical constructs on understanding the complexity of the physical world.

8.1 Scalar, Vector and Dyadic

Question **Define scalar, vector and dyadic. Define the rank of each and give examples.**

Scalar

Scalar is a tensor of rank zero. It is single real number or component. In three dimensional (3-D) space, the number of components of a scalar is $3^{\circ} = 1$. Scalar is a quantity which do not change under rotation of coordinates i.e. it is invariant and transformation law for scalar is $A_i^{'} = A$. **Example** of tensor of rank zero (i.e. scalar) are mass, charge, speed etc.

Vector

A tensor of rank one. In three dimensional space, the number of components of a vector is $3^{'}$(i.e.3). Components transform under rotation like those of the distance of a point from a chosen origin. The transformation law for vector's component is:

$$A_i^{'} = \sum_{p=1}^{3} a_{ip}A_p = \sum \frac{\partial x_p}{\partial x_i^{'}} A_p \tag{8.1}$$

Examples: momentum, electric field, velocity.

Dyadic

Dyadic is a tensor of rank two. In three dimensional space, the number of components of a 2nd rank tensor is given by 3^2 (i.e. 9).

$$A_{ij}^{'} = \sum_p \Sigma_q a_{ip} a_{jq} A_{pq}$$

Chapter 9

Group Theory

IN mathematics, a group is a fundamental concept that captures the idea of symmetry and structure. Groups come in various types, each with unique characteristics and applications. Lie groups are continuous groups often used in physics and geometry. Isomorphisms and homomorphisms describe relationships between groups, preserving their structure. Representations of groups help us understand their action in different contexts, with reducible and irreducible representations offering insights into their complexity. Cayley's theorem shows that every group is essentially a group of permutations. Cosets and Lagrange's theorem provide valuable tools in group theory. Special unitary groups like SU(2) and SU(3) and special orthogonal groups like SO(2) and SO(3) have significant roles in theoretical physics. The dihedral group and the distinction between continuous and discrete groups further enrich our understanding of symmetry and transformation.

9.1 Groups and its Types

Question **Discuss the groups and its related terms.**

A group G may be defined as a set of objects, operations, relations, transformations called elements of G; that may be combined multiplied to form a well defined product in G denoted by "$*$" and named as binary operation and satisfying the following properties:

- **Closure Property**

$a, b \in G$ then $a * b \in G$

$(a, b) \backsimeq a * b$

- **Associative Property**

$a * (b * c) = (a * b) * c$

- **Unit Element Property**

$1 * a = a * 1 = a$

Unit is unique $(1 = 1' * 1 = 1')$

- **Inverse Property**

Inverse or reciprocal of each element should exist.

$a * a^{-1} = a^{-1} * a = 1$

$\Rightarrow \ a * b = ab$

Subgroup

When we choose a set of some elements out of a group, and this set also a group, then this set is known as subgroup of the given group. $G' \subset G$ and G' is itself a group. The unit of G is subgroup of G.

Example 5.1.1 Prove that $P_n(1)=1$.
Solution:
Since, we know that

$$(1-2xz+z^2)^{-1/2}=P_0(x)+zP_1(x)+z^2P_2(x)+\cdots+z^nP_n(x)+\cdots$$

Substituting 1 for x in the above equation, we get

$$\begin{aligned}(1-2z+z^2)^{-1/2} &= P_0(1)+zP_1(1)+z^2P_2(1)+\cdots+z^nP_n(1)+\cdots\\ [(1-z)^2]^{-1/2} &= 1+zP_1(1)+z^2P_2(1)+\cdots+z^nP_n(1)+\cdots \qquad (\because\ P_0(1)=1)\\ &= \sum_{n=0}^{\infty} z^nP_n(1) \Rightarrow (1-z)^{-1}=\sum z^nP_n(1)\\ \text{or}\quad \sum z^nP_n(1) &= (1-z)^{-1}=1+z+z^2+z^3+\cdots+z^n+\cdots\end{aligned}$$

Equating the coefficients of z^n on both sides, we get

$$P_n(1)=1$$

5.1.2 Recurrence Formula for $P_n(x)$

Question Derive the recurrence formulas for the Legendre polynomial $P_n(x)$.

Formula- I: $nP_n=(2n-1)xP_{n-1}-(n-1)P_{n-2}$

We know that $(1-2xz+z^2)^{-1/2} = \sum z^nP_n(x)$. Differentiating it w.r.t. z, we get

$$-\frac{1}{2}(1-2xz+z^2)^{-3/2}(-2x+2z) = \sum nz^{n-1}P_n(x)$$

Multiplying both sides by $(1-2xz+z^2)$, we get

$$(1-2xz+z^2)^{-1/2}(x-z) = (x-z)\sum z^nP_n(x) = (1-2xz+z^2)\sum nz^{n-1}P_n(x)$$

Equating the coefficients of z^{n-1} from both sides, we get

$$\begin{aligned}xP_{n-1}-P_{n-2} &= nP_n-2x(n-1)P_{n-1}+(n-2)P_{n-2}\\ \Rightarrow\quad nP_n &= xP_{n-1}-P_{n-2}+2x(n-1)P_{n-1}-(n-2)P_{n-2}\\ &= xp_{n-1}+2x(n-1)P_{n-1}-P_{n-2}-(n-2)P_{n-2} = (2n-2+1)xP_{n-1}-(1+n-2)P_{n-2}\\ \Rightarrow\quad nP_n &= (2n-1)xP_{n-1}-(n-1)P_{n-2}\end{aligned}$$

Formula- II: $xP_n'-P_{n-1}' = nP_n$

We know that

$$(1-2xz+z^2)^{-1/2} = \sum z^nP_n(x) \tag{5.18}$$

Differentiating Eq.(5.18Recurrence Formula for $P_n(x)$equation.5.1.18) with respect to z, we get

$$-\frac{1}{2}(1-2xz+z^2)^{-3/2}(-2x+2z) = (x-z)(1-2xz+z^2)^{-3/2} = \sum nz^{n-1}P_n(x) \tag{5.19}$$

Differentiating Eq.(5.18Recurrence Formula for $P_n(x)$equation.5.1.18) with respect to x, we get

$$-\frac{1}{2}(1-2xz+z^2)^{-3/2}(-2z) = z(1-2xz+z^2)^{-3/2} = \sum z^nP_n'(x) \tag{5.20}$$

1.15 Short Questions with Answers

1.1 If the coordinates of P be $(3,\ 4,\ 12)$, then find $\overrightarrow{OP}$, its magnitude and direction cosines.

Answer: First we calculate $\overrightarrow{OP}$ and its magnitude as:

$$\overrightarrow{OP} = 3\hat{i}+4\hat{j}+12\hat{k} \quad \Rightarrow \quad |\overrightarrow{OP}| = \sqrt{3^2+4^2+12^2} = \sqrt{9+16+144} = 13$$

Now, we calculate its direction cosines as:

$$\text{Direction cosines} = \frac{x}{\sqrt{x^2+y^2+z^2}}, \quad \frac{y}{\sqrt{x^2+y^2+z^2}} \quad \text{and} \quad \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{3}{13}, \quad \frac{4}{13} \quad \text{and} \quad \frac{12}{13}$$

1.2 If for two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, find the angle between $\vec{a}$ and $\vec{b}$.

Answer: Since, the given condition is

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| \quad \Rightarrow \quad |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) \quad \Rightarrow \quad (\vec{a})^2+(\vec{b})^2+2\vec{a}\cdot\vec{b} = (\vec{a})^2+(\vec{b})^2-2\vec{a}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} = 0 \quad \Rightarrow \quad \vec{a}\cdot\vec{b} = 0$$

This shows that $\vec{a}$ and $\vec{b}$ are perpendicular to each other and so the angle between them is 90°.

1.3 Determine λ and μ by using vectors, such that the points $(-1,\ 3,\ 2)$, $(-4,\ 2,\ -2)$ and $(5,\ \lambda,\ \mu)$ lie on a straight line.

Answer: Let $A,\ B,\ C$ be three points whose coordinates are $(-1,\ 3,\ 2)$, $(-4,\ 2,\ -2)$ and $(5,\ \lambda,\ \mu)$. So,

$$\overrightarrow{AB} = \vec{B}-\vec{A} = (-4\hat{i}+2\hat{j}-2\hat{k})-(-\hat{i}+3\hat{j}+2\hat{k}) = -3\hat{i}-\hat{j}-4\hat{k}$$

$$\text{And,} \quad \overrightarrow{BC} = \vec{C}-\vec{B} = (5\hat{i}+\lambda\hat{j}+\mu\hat{k})-(-4\hat{i}+2\hat{j}-2\hat{k}) = 9\hat{i}+(\lambda-2)\hat{j}+(\mu+2)\hat{k}$$

$$\text{Now,} \quad \overrightarrow{AB}\times\overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -1 & -4 \\ 9 & \lambda-2 & \mu+2 \end{vmatrix} = (-\mu+4\lambda-10)\hat{i}-(-3\mu+30)\hat{j}+(-3\lambda+15)\hat{k}$$

If $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are in the same straight line, then $\overrightarrow{AB}\times\overrightarrow{BC} = 0$. Now,

$$(-\mu+4\lambda-10)\hat{i}-(-3\mu+30)\hat{j}+(-3\lambda+15)\hat{k} = 0\hat{i}+0\hat{j}+0\hat{k}$$

On comparing both sides, we get

$$-3\mu+30 = 0 \quad \Rightarrow \quad \mu = 10 \quad \text{and} \quad -3\lambda+15 = 0 \quad \Rightarrow \quad \lambda = 5$$

1.4 Show that the vectors $5\vec{a}+6\vec{b}+7\vec{c}$, $7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$ are co-planar. $\vec{a},\ \vec{b},\ \vec{c}$ being three non-collinear vectors.

Answer: Let $\vec{\alpha} = 5\vec{a}+6\vec{b}+7\vec{c}$, $\vec{\beta} = 7\vec{a}-8\vec{b}+9\vec{c}$ and $\vec{\gamma} = 3\vec{a}+20\vec{b}+5\vec{c}$. Now,

$$\vec{\alpha}\cdot(\vec{\beta}\times\vec{\gamma}) = \begin{vmatrix} 5 & 6 & 7 \\ 7 & -8 & 9 \\ 3 & 20 & 5 \end{vmatrix} = 5(-40-180)-6(35-27)+7(140+24) = -1100-48+1148 = 0$$

Hence, $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$ are co-planar.

1.16 Solved Problems

Problem 1.1. If $|\vec{A}+\vec{B}|=60,\ |\vec{A}-\vec{B}|=40$ and $|\vec{B}|=46$, find $|\vec{A}|$.

Solution

$$|\vec{A}+\vec{B}|=60 \quad\Rightarrow\quad |\vec{A}+\vec{B}|^2=3600 \quad\Rightarrow\quad (\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B})=3600$$

$$(\vec{A})^2+(\vec{B})^2+2(\vec{A})\cdot(\vec{B})=3600 \quad\Rightarrow\quad |\vec{A}|^2+|\vec{B}|^2+2|\vec{A}||\vec{B}|\cos\theta=3600 \tag{1.37}$$

Similarly, $|\vec{A}-\vec{B}|=40 \quad\Rightarrow\quad |\vec{A}-\vec{B}|^2=1600 \quad\Rightarrow\quad (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})=1600$

$$(\vec{A})^2+(\vec{B})^2-2(\vec{A})\cdot(\vec{B})=1600 \quad\Rightarrow\quad |\vec{A}|^2+|\vec{B}|^2-2|\vec{A}||\vec{B}|\cos\theta=1600 \tag{1.38}$$

Adding Eq.(1.37Solved Problemsequation.1.16.37) and Eq.(1.38Solved Problemsequation.1.16.38), we get

$$2|\vec{A}|^2+2|\vec{B}|^2=5200 \quad\Rightarrow\quad |\vec{A}|^2+|\vec{B}|^2=2600 \quad\Rightarrow\quad |\vec{A}|^2+(46)^2=2600$$

$$\Rightarrow\quad |\vec{A}|^2=484 \quad\Rightarrow\quad \sqrt{|\vec{A}|^2}=\sqrt{484} \quad\Rightarrow\quad |\vec{A}|=22$$

Problem 1.2. Forces of magnitudes 5 and 3 units acting in the directions $(6\hat{i}+2\hat{j}+3\hat{k})$ and $(3\hat{i}-2\hat{j}+6\hat{k})$, respectively act on a particle which is displaced from the point $(2,\ 2,\ -1)$ to $(4,\ 3,\ 1)$. Find the work done by the forces.

Solution

First force of magnitude 5 units, acting in the direction $6\hat{i}+2\hat{j}+3\hat{k} = 5\left(\dfrac{6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{6^2+2^2+3^2}}\right) = \dfrac{5}{7}(6\hat{i}+2\hat{j}+3\hat{k})$

Second force of magnitude 3 units, acting in the direction $3\hat{i}-2\hat{j}+6\hat{k} = 3\left(\dfrac{3\hat{i}-2\hat{j}+6\hat{k}}{\sqrt{3^2+(-2)^2+6^2}}\right) = \dfrac{3}{7}(3\hat{i}-2\hat{j}+6\hat{k})$

Resultant force: $\vec{F} = \dfrac{5}{7}(6\hat{i}+2\hat{j}+3\hat{k})+\dfrac{3}{7}(3\hat{i}-2\hat{j}+6\hat{k})=\dfrac{1}{7}(39\hat{i}+4\hat{j}+33\hat{k})$

Displacement from the point $(2,\ 2,\ -1)$ to $(4,\ 3,\ 1)$

$$\vec{d}=(4\hat{i}+3\hat{j}+\hat{k})-(2\hat{i}+2\hat{j}-\hat{k})=2\hat{i}+\hat{j}+2\hat{k}$$

Work done $=\vec{F}\cdot\vec{d}=\dfrac{1}{7}(39\hat{i}+4\hat{j}+33\hat{k})\cdot(2\hat{i}+\hat{j}+2\hat{k})=\dfrac{1}{7}(78+4+66)=\dfrac{148}{7}$ units

Problem 1.3. In what direction from $(3,\ 1,\ -2)$ is the directional derivative of $\phi=x^2y^2z^4$ maximum? Find also the magnitude of this maximum.

Solution

$$\vec{\nabla}\phi=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)(x^2y^2z^4)=2xy^2z^4\hat{i}+2x^2yz^4\hat{j}+4x^2y^2z^3\hat{k}$$

Directional derivative at $(3,\ 1,\ -2)=96\hat{i}+288\hat{j}-288\hat{k}$. The directional derivative is maximum in the direction $96\hat{i}+288\hat{j}-288\hat{k}$ or $96(\hat{i}+3\hat{j}-3\hat{k})$. In any other direction it will have its component which is less than its maximum value.

Maximum value $=96\sqrt{1+9+9}=96\sqrt{19}$

Problem 1.4. Verify that $\vec{\nabla}\cdot\left[\frac{f(\vec{r})}{|\vec{r}|}\vec{r}\right]=\frac{1}{r^2}\frac{d}{d\vec{r}}[r^2f(\vec{r})]$.

Solution

First we take the left hand side,

$$\vec{\nabla}\cdot\left[\frac{f(\vec{r})}{|\vec{r}|}\vec{r}\right]=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\cdot\left[\frac{f(\vec{r})}{|\vec{r}|}\vec{r}\right]=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\cdot\left[\frac{f(\vec{r})}{|\vec{r}|}(x\hat{i}+y\hat{j}+z\hat{k})\right]$$

1.17 Multiple Choice Questions (MCQs)

1: What is the magnitude of unit vector:
(a) It has no magnitude (b) Zero (c) Constant but not zero (d) **Unity**

2: Flying a bird is the example of:
(a) Collinear vector (b) Multiplication of vector (c) Addition of vector (d) **Composition of vector**

3: Cross product is the mathematical operation performed between:
(a) 2 scaler numbers (b) A scaler and a vector (c) **2 vectors** (d) Any 2 numbers

4: The mathematical perception of the gradient is said to be:
(a) Tangent (b) Chord (c) **Slope** (d) Arc

5: Gauss theorem uses which of the following operations:
(a) Gradient (b) **Divergence** (c) Laplacian (d) Curl

6: Which of the following operation uses the curl operation:
(a) Greens theorem (b) Gauss divergence theroem (c) **Stoke's theorem** (d) Maxwell equation

7: Find the curl of $\vec{A} = (y\cos ax)\hat{i} + (y + e^x)\hat{k}$
(a) $2\hat{i} - ex\hat{j} - \cos ax\hat{k}$ (b) $m\hat{i} - ex\hat{j} - \cos ax\hat{k}$ (c) $2\hat{i} - ex\hat{j} + \cos ax\hat{k}$ **(d)** $\hat{i} - ex\hat{j} + \cos ax\hat{k}$

8: Mathematically the function in the green theorem will be:
(a) Continuous derivative (b) Discrete derivative
(c) **Continuous partial derivatives** (d) Discrete partial derivatives

9: The first member in ordered pair (x, y) is considered as:
(a) Mantissa (b) Cartesian coordinates (c) **Abscissa** (d) Ordinate

10: Example of spherical system in the following is:
(a) **Charge in space** (b) Charge in box (c) Charge in dielectric (d) Uncharged system

11: The scalar triple product of three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ is defined as:
(a) $\vec{a} \cdot \vec{b} \times \vec{c}$ **(b)** $\vec{a} \cdot (\vec{b} \times \vec{c})$ (c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ (d) $\vec{a} \cdot \vec{b} \cdot \vec{c}$

12: The vector triple product $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to:
(a) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ (b) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$ (c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ (d) $(\vec{a} \cdot \vec{b})(\vec{a} \cdot \vec{c})$

13: For vectors $\vec{a} = (1, 2, 3)$, $\vec{b} = (4, 5, 6)$, and $\vec{c} = (7, 8, 9)$, the scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is:
(a) **0** (b) 1 (c) 15 (d) -15

14: The vector triple product $(\vec{a} \times \vec{b}) \times \vec{c}$ is equivalent to:
(a) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ (b) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$ **(c)** $(\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}$ (d) None of the above

15: The scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to:
(a) **The volume of the parallelepiped formed by $\vec{a}$, $\vec{b}$, and $\vec{c}$** (b) The area of the parallelogram formed by $\vec{a}$ and the cross product $\vec{b} \times \vec{c}$ (c) The dot product of $\vec{a}$ with the cross product $\vec{b} \times \vec{c}$ (d) None of the above

16: The gradient of a scalar function $f(x, y, z)$ is a:
(a) Scalar (b) **Vector** (c) Tensor (d) Matrix

17: The divergence of a vector field $\vec{F}(x, y, z)$ is a:
(a) **Scalar** (b) Vector (c) Tensor (d) Matrix

18: The curl of a vector field $\vec{F}(x, y, z)$ is a:
(a) Scalar (b) **Vector** (c) Tensor (d) Matrix

19: If $\vec{F}(x, y, z) = (x^2, 2y, z^3)$, then $\vec{\nabla} \cdot \vec{F}$ is:
(a) $2x + 2 + 3z^2$ (b) $2x - 2y + 3z^2$ (c) $x^2 + 2y + z^3$ (d) $2x^2 + 2y + 3z^3$

Course outline of various universities are given below. The page numbers of topics are mentioned to follow your respective university outline.

Bahauddin Zakariya University (BZU), Multan

Course Title: Mathematical Methods of Physics-I **Course Code:** PHYS-301 **Semester**-5

Vector Analysis: Review of vectors Algebra **(P#1)**, Vector differentiation and gradient **(P#8,11)**, Divergence and Gauss's theorem **(P#15)**, Vector integration **(P#9)**, Green's theorem in the plane **(P#19)**, Curl and Stoke's theorem **(P#13, 17)**.

Curvilinear Coordinates and Tensors: Curvilinear coordinate system **(P#20)**, Gradient, Divergence and Curl in the curvilinear coordinates system, Cartesian, Spherical and Cylindrical coordinate system **(P#21,22)**, Covariant and contravariant tensors **(P#238)**, Tensor algebra **(P#242)**, Quotient rule **(P#244)**.

Matrices: Linear vector spaces, Determinants **(P#36)**, Matrices **(P#35)**, Eigen values and eigenvectors of matrices **(P#44)**, Orthogonal matrices **(P#37)**, Hermitian matrices, Similarity transformations , Diagonalization of matrices **(P#46)**.

Group Theory: Introduction to groups **(P#256)**, group **(P#256)**, Invariant subgroups **(P#257)**, Discrete groups-Dihedral groups **(P#271,273)**, Continuous groups-O groups **(P#271)**, SU(2) groups **(P#266)**, Lie groups **(P#258)**

Complex Variables: Functions of a complex variable **(P#61)**, Cauchy Riemann conditions and analytic functions **(P#64)**, Cauchy integral theorem and integral formula **(P#67,68)**, Taylor and Laurent series **(P#70,71)**, Calculus of residue **(P#73)**, Complex integration **(P#75)**.

Course Title: Mathematical Methods of Physics-II **Course Code:** PHYS-302 **Semester**-6

Differential Equations in Physics: First and second order linear differential equations **(P#101,108)**, Partial differential equations of theoretical physics **(P#115)**, Separation of variables **(P#102)**, Homogeneous differential equations **(P#103)**, Frobenius series solution of differential equations **(P#111)**, Second solution, non-homogenous differential equations **(P#108)**.

Special Functions: Bessel functions and Hankel functions **(P#136)**, Spherical Bessel functions **(P#143)**, Legendre polynomials **(P#127)**, Associated Legendre polynomials **(P#135)**, Spherical harmonics Laguerre polynomials **(P#148)**, Hermite polynomials **(P#144)**

Fourier series: Definition and general properties **(P#171)**, Fourier series of various physical functions **(P#173)**, Uses and application of Fourier series **(P#176)**

Integral Transforms: Integral transform **(P#179)**, Fourier transform **(P#181)**, Convolution theorem **(P#182)**, Elementary Laplace transform and its application **(P#188,195)**.

Boundary Value Problems and Green's Functions: Boundary value problems in Physics **(P#214)**, Non-homogeneous boundary value problems and Green's functions **(P#221)**, Green's functions for one dimensional problem **(P#223)**, Eigen function expansion of Green's function **(P#222)**, Construction of Green's functions in higher dimensions **(P#224)**.

Government College University, Faisalabad (GCUF)

Course Title: Mathematical Methods of Physics-I **Course Code:** PHYS-501 **Semester**-5

Vector operations **(P#2)**, Physical significance of DEL operator **(P#12)**, Gauss's divergence theorem **(P#15)**, Green's theorem **(P#19)**, Stokes's theorem **(P#17)**, Orthogonal curvilinear coordinates system **(P#20)**, Gradient **(P#11)**, Divergence **(P#12)**, Curl **(P#13)**and Laplacian in orthogonal curvilinear coordinates, Spherical polar and cylindrical coordinates systems **(P#21,22)**. Complex numbers **(P#62)**, Euler's formula **(P#69)**, De Moivre's theorem **(P#62)**, elementary functions **(P#65)**, analytic functions of complex variables **(P#63)**, Cauchy- Riemann equation **(P#64)**, harmonic functions **(P#65)**, complex integration **(P#75)**, Cauchy's theorem **(P#67)**, Cauchy's integral formula **(P#68)**, Taylor and Laurent series **(P#70,71)**, Contour integrals **(P#86)**, singularities and residues **(P#71,73)**, residue theorem **(P#74)**, branch points and integrals of multivalued functions **(P#76)**. Tensors Analysis and applications **(P#257)**.

Course Title: Mathematical Methods of Physics-II **Course Code:** PHYS-502 **Semester**-6

Fourier Analysis **(P#171)**, Fourier cosine and sine series **(P#173)**, change of interval **(P#175)**, Fourier integral **(P#179)**, complex form of Fourier series **(P#174)**, Fourier transform **(P#181)**, Fourier transform of derivatives **(P#185)**, Laplace transform **(P#188)**, Inverse Laplace Transform **(P#193)**, Convolution theorem **(P#182)**, Initial boundary value problem, Laplace transform of derivatives **(P#192)**. Physical significance along with examples of Fourier and Laplace transforms **(P#176, 195)**. Special functions **(P#127)**, Hermite **(P#144)**, Laguerre **(P#148)**, Legendre and associate Legendre polynomial **(P#127,135)**. Bessel function **(P#136)**, Neumann function **(P#140)**, and spherical Bessel function **(P#143)**, Gamma function **(P#153)**. Nonhomogeneous equations- Green's function **(P#213)**, Green's function in terms of Eigen-function **(P#222)**, the Sturm-Liouville problem **(P#225)**, Green's function for Dirac Delta functions **(P#222)**.

University of the Punjab, Lahore

Course Title: Mathematical Methods of Physics-I **Course Code:** Phys-3501 **Semester**-5

Series solutions about an ordinary point and regular singular point **(P#110,111)**, Sturm-Liouville theory **(P#225)**, self-adjoint ODEs, orthogonal functions, Hermitian operators, eigenvalue problems, completeness of eigenfunctions, Green's Functions **(P#213)**, Green's function for one-dimensional problem **(P#223)**, eigenfunction expansion of Green's function **(P#222)**, special functions **(P#127)**, Gamma Function **(P#153)**, digamma and polygama functions **(P#155)**, Stirling's series **(P#156)**, Beta function **(P#177)**, Bessel functions of first kind **(P#138)**, orthogonality, Neumann functions **(P#140)**, Bessel functions of the second kind **(P#140)**, Hankel functions, modified Bessel functions **(P#138)**, asymptotic expansions, spherical Bessel functions **(P#143)**, Legendre functions **(P#127)**, Legendre polynomials **(P#128)**, orthogonality **(P#134)**, generating function **(P#130)**, recurrence relation **(P#131)**, associated Legendre equation **(P#135)**, spherical harmonics, orbital angular momentum operator, addition theorem for spherical harmonics, Legendre functions of the second kind **(P#128)**, Hermite functions **(P#145)**, Hermite equation as Schrodinger equation of quantum harmonic oscillator, Laguerre functions and associated Laguerre functions **(P#148,152)**, Fourier series **(P#171)**, properties of Fourier series , Fourier transform **(P#181)**, properties of Fourier transforms, Fourier convolution theorem **(P#182)**, discrete Fourier transform, Laplace transforms **(P#188)**, properties of Laplace transforms **(P#190)**, Laplace transform of derivatives **(P#192)**, Laplace Convolution theorem, inverse Laplace transform **(P#193)**

Course Title: Mathematical Methods of Physics-II **Course Code:** Phys-3503 **Semester**-6

Tensor analysis **(P#237**, some notations, Cartesian tensors, First- and zero-order Cartesian tensors, second- and higher-order Cartesian tensors, the algebra of tensors, the quotient law **(P#244)**, Kronecker delta δ_{ij} and Levi Civita tensor ε_{ijk} **(P#240,242)**, Isotropic tensors, improper rotations and pseudo tensors, dual tensors, physical applications of tensors, integral theorems for tensors **(P#245)**, non-Cartesian coordinates, the metric tensor, General coordinate transformations and tensors **(P#237)**, relative tensors, derivatives of basis vectors and Christoffel symbols, covariant differentiation, vector operators in tensor form, absolute derivatives along curves, Riemann curvature tensor, Complex Analysis Complex numbers, powers and roots, Sets in the Complex planes, Functions of a complex variables **(P#61)**, Cauchy–Riemann equations **(P#64)**, Exponential and Logarithmic functions **(P#62)**, Contour Integrals **(P#86)**, Cauchy-Goursat theorem, Independence of path, Cauchy's Integral formulas **(P#68)**, Sequences and Series, Taylor series **(P#70)**, Laurent Expansion **(P#71)**, Zeros and Poles **(P#72)**, Singularities **(P#71)**, Residues and Residues Theorem **(P#73,74)**, Evaluation of real Integrals, Groups Theory, Review of groups **(P#256)**, subgroup **(P#256)**, cyclic groups **(P#257)**, and permutation groups, isomorphism **(P#259)**, Cayley's theorem **(P#262)**, properties of isomorphism, automorphism, cosets **(P#263)**, properties of cosets **(P#265)**, Lagrange's theorem **(P#265)**, an application of cosets to permutation groups, the rotation groups of a cube and soccer ball, conjugate classes and invariant subgroups, group representations **(P#260)**, some special groups **(P#265)**, the symmetry group D2,D3, one-dimensional unitary group U(1), orthogonal groups SO(2) and SO(3) **(P#268,270)**, the SU(n) groups, Homogeneous Lorentz group.

Ghazi University, D.G. Khan

Course Title: Mathematical Methods of Physics-I **Course Code:** PHY-501 **Semester**-5

Special Functions: Bessel Functions **(P#136)**, Neumann Functions **(P#140)**, Hankel Functions, Spherical Bessel Functions **(P#143)**, Legendre Functions **(P#127)**, Associated Legendre Functions **(P#135)**, Spherical Harmonics, Hermite Polynomials **(P#145)**.

Partial Differential Equations: Introduction to important PDEs in Physics **(P#115)** (wave equation, diffusion equation, Poisson's equation, Schrodinger's equation), general form of solution **(P#106, 107)**, general and particular solutions (first order, inhomogeneous, second order)**(P#108)**, characteristics and existence of solutions, uniqueness of solutions, separation of variables in Cartesian coordinates **(P#102)**, superposition of separated solutions, separation of variables in curvilinear coordinates, special functions **(P#127)**, integral transform methods, Green's functions **(P#213)**.

Complex Analysis: Review (polar form of complex numbers and de Moivre's theorem, complex logarithms and powers) **(P#85)**, functions of a complex variable **(P#62)**, Cauchy-Riemann conditions **(P#64)**, power series in a complex variable and analytic continuation with examples **(P#70)**, multi-valued functions and branch cuts **(P#76)**, singularities and zeroes of complex functions **(P#71)**, complex integration **(P#75)**, Cauchy's theorem **(P#67)**, Cauchy's integral formula **(P#68)**, Laurent series and residues **(P#71,73)**, residue integration theorem **(P#74)**, definite integrals using contour integration **(P#86).**

Course Title: Mathematical Methods of Physics-I **Course Code:** PHY-502 **Semester**-6

Fourier Series and Integral Transforms: Definition and general properties **(P#171)**, Fourier Series of Various Physical Functions **(P#173)**, Uses and Applications of Fourier Series **(P#176)**, Fourier Transforms **(P#181)**, Convolution Theorems **(P#182)**, Laplace transforms and applications **(P#188).**

Tensor Analysis: Vector calculus (differentiation, integration, space curves, multi-variable vectors, surfaces, scalar and vector fields, gradient, divergence and curl, cylindrical and spherical coordinates, general curvilinear coordinates), change of basis, Cartesian tensor as a geometrical object, order/rank of a tensor **(P#239)**, tensor algebra **(P#242)**, quotient law **(P#244)**, pseudo-tensors, Kronecker delta and Levi-Civita **(P#240,242)**, dual tensors, physical applications, integral theorems for tensors **(P#245)**, non-Cartesian tensors, general coordinate transformations and tensors **(P#237).**

Group Theory and Representations for finite groups: Transformations, groups – definitions and examples **(P#256)**, subgroups and Cayley's theorem **(P#256,262)**, cosets and Lagrange's theorem **(P#263, 265)**, conjugate classes, invariant subgroups **(P#257)**, factor groups, homomorphism **(P#259)**, direct products, mappings, linear operators, matrix representations, similarity transformation and equivalent matrix representations , group representations **(P#260)**, equivalent representations and characters**(P#261)**, construction of representations and addition of representations, invariance of functions and operators, unitary spaces and Hermitian matrices, operators: adjoint, self-adjoint, unitary, Hilbert space, reducibility of representations**(P#261)**, Schur's lemmas, orthogonality relations, group algebra, expansion of functions in basis of irreducible representations**(P#261)**, Kronecker product, symmetrized and anti-symmetrized representations, adjoint and complex-conjugate representations, real representations.

University of Education, Lahore

Course Title: Mathematical Methods of Physics-I **Course Code:** Phys-3111 **Semester**-5

Vector Analysis: Divergence theorem **(P#15)**, Green's Theorem **(P#19)**, Stock's theorem **(P#17)**, Cylindrical, spherical and curvilinear coordinates **(P#21)**. Orthogonal curvilinear coordinates **(P#20)**. Gradient, Divergence, Curl and Laplacian in Spherical and Cylindrical Coordinates.

Special Functions-I: Helmholtz Equation. Legendre's Differential Equation and its Solution **(P#127)**, Legendre's Polynomials **(P#127)**, Associated Legendre functions and Spherical harmonics **(P#135).**

Functions of Complex Variable: Complex functions **(P#61)**, Analyticity **(P#63)**, Cauchy-Riemann equations **(P#64)**, Harmonic Function **(P#65)**, Multi-valued Functions **(P#76)**, Complex Integration **(P#75)**, Cauchy's integral formula and its problems **(P#68)**, Taylor and Laurent series **(P#70,71)**, Contour integrals **(P#86)**, Singularities and Residue theorem and its applications **(P#71, 74).**

Boundary Value Problem: Boundary value problems in Physics **(P#214)**, The Sturm-Liouville Problems **(P#225).**

Group Theory: Introduction to group **(P#256)**, Invariant Subgroup **(P#257)**, Discrete groups **(P#271)**, Continuous group **(P#271)**, GL(n), SU(2) **(P#266)**, SU(3) **(P#267)**, O-group's O(2) group **(P#268).**

Course Title: Mathematical Methods of Physics-II **Course Code:** Phys-3116 **Semester**-5

Fourier series and Transforms: Fourier series and its complex form **(P#171, 174)**, Applications of Fourier series **(P#176)**, Representations of a function, Fourier integral theorem **(P#179)**, Fourier transforms **(P#181)**, Fourier Sine and Cosine transforms **(P#183, 184)**, Applications of Fourier transform, Laplace transforms **(P#188)**, Application of Laplace Transform **(P#195).**

Tensor Analysis: Cartesian Tensors, Coordinate Transformation **(P#237)**, Rank of a Tensor **(P#239)**, Tensor Algebra **(P#242)**, Quotient Theorem **(P#244)**, Tensor Density, Covariant and contravariant tensor **(P#238).**

Green's Function: Definition of Green's functions **(P#213)**, Problems of Green's Function **(P#214)**, Green's Functions in Electrodynamics.

Special Functions-II: Bessel's Differential Equation **(P#136)**, Solution of Bessel's Differential Equation **(P#136)**, Bessel's Functions **(P#136)**, Neumann functions **(P#140)**, Hermite Differential Equation **(P#144)**, Solution of Hermite differential Equation **(P#145)**, Hermite Polynomials **(P#145).**

Higher Education Commission (HEC), Islamabad

Course Title: Mathematical Methods of Physics-I **Course Code:** Phys-352 **Semester**-5

Partial Differential Equations: Introduction to important PDEs in Physics **(P#115)** (wave equation **(P#117)**, diffusion equation **(P#116)**, Poisson's equation **(P#116)**, Schrodinger's equation **(P#116)**), General form of solution **(P#101)**, General and particular solutions (first order **(P#101)**, Inhomogeneous second order **(P#108)**), Characteristics and existence of solutions, Uniqueness of solutions, Separation of variables in Cartesian coordinates **(P#102)**, Superposition of separated solutions, Separation of variables in curvilinear coordinates **(P#102)**, Integral transform methods, Green's functions **(P#213).**

Complex Analysis: Review (polar form of complex numbers and de Moivre's theorem **(P#61)**, Complex logarithms and powers) **(P#85)**, Functions of a complex variable **(P#62)**, Cauchy-Riemann conditions **(P#64)**, Power series in a complex variable and analytic continuation with examples **(P#70)**, Multi-valued functions and branch cuts **(P#76)**, Singularities and zeroes of complex functions **(P#71, 72)**, Complex integration **(P#75)**, Cauchy's theorem **(P#67)**, Cauchy's integral formula **(P#68)**, Laurent series and residues **(P#71, 73)**, Residue integration theorem **(P#74)**, Definite integrals using contour integration **(P#86)**.

Course Title: Mathematical Methods of Physics-II **Course Code:** Phys-352 **Semester**-6

Group Theory and Representations for Finite Groups: Transformations, Groups definitions and examples **(P#256)**, subgroups and Cayley's theorem **(P#256, 262)**, Cosets and Lagrange's theorem **(P# 263, 265)**, Conjugate classes, Invariant subgroups **(P# 257)**, factor groups, Homomorphism **(P#259)**, direct products **(P#244)**, Mappings, Linear operators, matrix representations **(P#35)**, Similarity transformation and equivalent matrix representations, Group representations **(P#260)**, Equivalent representations and characters **(P#261)**, Construction of representations and addition of representations, Invariance of functions and operators, unitary spaces and Hermitian matrices, Operators: Adjoint, Self-adjoint **(P#37)**, Unitary **(P#39)**, Hilbert space, Reducibility of representations, Schur's lemmas, Orthogonality relations, Group algebra, Expansion of functions in basis of irreducible representations **(P#261)**, Kronecker product, Symmetrized and anti-symmetrized representations, Adjoint and complex-conjugate representations, real representations, Clebsch-Gordan series and coefficients, Applications of these ideas to classification of spectral terms, Perturbation theory and coupled systems.

Tensor Analysis: Vector calculus (differentiation **(P#8)**, integration **(P#9)**, Space curves, Multi-variable vectors, Surfaces, Scalar and vector fields **(P#1)**, Gradient **(P#11)**, Divergence and curl **(P#12, 13)**, Cylindrical and spherical corrdinates **(P#21, 22)**, General curvilinear coordinates) **(P#20)**, Change of basis, Cartesian tensor as a geometrical object, Order/rank of a tensor **(P#239)**, Tensor algebra **(P#242)**, Quotient law **(P# 244)**, pseudotensors, Kronecker delta and Levi cevita **(P#240, 242)**, Dual tensors, Physical applications, Integral theorems for tensors **(P#245)**, non-Cartesian tensors, General coordinate transformations and tensors **(P#237)**, relative tensors, Christoffel symbols, Covariant differentiation, Vector operators in tensor form, Absolute derivatives along curves, Geodesics.


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