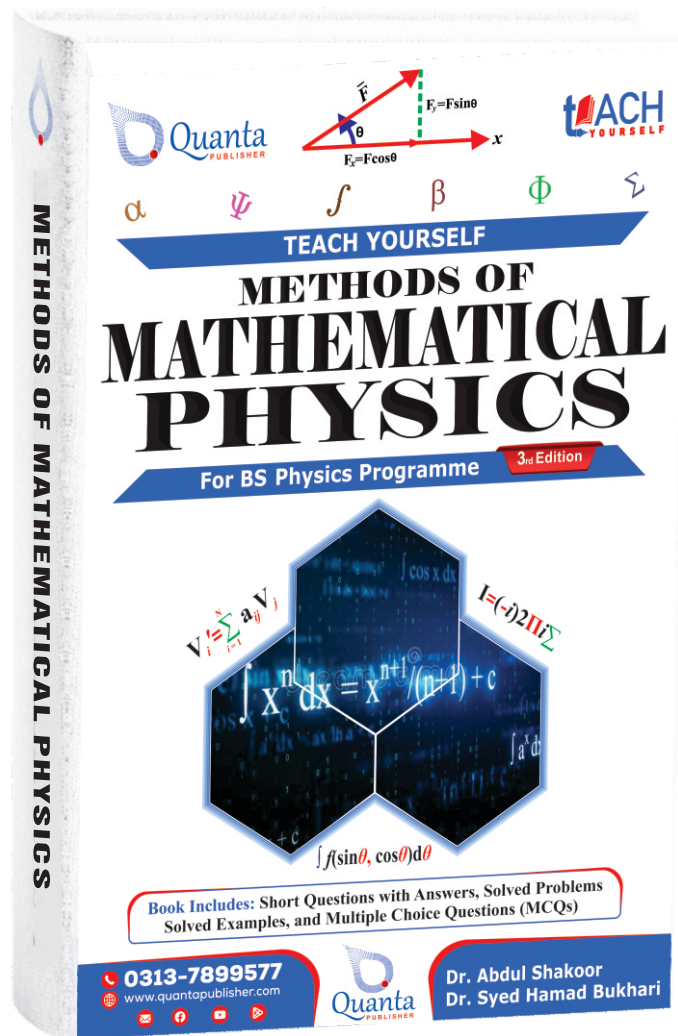


SAMPLE PAGES



For Online Order

0313-7899577

www.quantapublisher.com

BOOK PRICE
Rs: 650/-

TEACH YOURSELF
**METHODS OF
MATHEMATICAL
PHYSICS**

3rd Edition

For **BS Physics** Students of all Pakistani Universities/Colleges

•
Dr. Abdul Shakoor

Institute of Physics
Bahauddin Zakariya University, Multan

&

Dr. Syed Hamad Bukhari

Former HOD, Department of Physics
G. C. University Faisalabad, Sub-campus, Layyah

•
Assisted by

Fakhar Abbas

Department of Physics
University of the Punjab, Lahore

Quanta Publisher, 2660/6C Raza Abad, Shah Shamas, Multan. 03137899577

Contents

1	Vector Analysis	1
1.1	Scalar and Vector	1
1.2	Scalar Product	5
1.3	Vector Product	6
1.4	Triple Product	7
1.5	Vector Differentiation	8
1.6	Vector Integration	9
1.7	The Gradient	11
1.8	The Divergence	12
1.9	The Curl of a Vector	13
1.10	Curl and Divergence	14
1.11	Divergence Theorem	15
1.12	Stokes's Theorem	17
1.13	Green's Theorem in a Plane	19
1.14	Curvilinear Coordinates	20
	1.14.1 Cylindrical Coordinates	21
	1.14.2 Spherical Polar Coordinates	22
1.15	Short Questions with Answers	25
1.16	Solved Problems	29
1.17	Multiple Choice Questions (MCQs)	33
2	Matrix Algebra	35
2.1	Matrix	35
2.2	Jaccobi Identity	39
2.3	Successive Rotation	39
2.4	Symmetry Properties	40
2.5	Properties of Matrix	40
2.6	Orthogonal Transpose Matrix	40
2.7	Pauli and Dirac Matrix	41
2.8	Eigenvectors and Eigenvalues	44
2.9	Diagonalization of Matrix	46
2.10	Inertia Matrix	47
2.11	Short Questions with Answers	49
2.12	Solved Problems	51
2.13	Multiple Choice Questions (MCQs)	59
3	Complex Variables and Infinite Series	61
3.1	Complex Variables	61
3.2	De Moivre's Theorem	62
3.3	Analytic Function	63
3.4	Harmonic Functions	65
3.5	Elementary Function	65
3.6	Cauchy's Integral Theorem	67
3.7	Cauchy's Integral Formula	68
3.8	Taylor's Expansion	70
3.9	Laurent Series	71
3.10	Singularity	71
3.11	Residue	73
3.12	Cauchy Residue Theorem	74

3.13 Evaluation of Complex Integral	75
3.13.1 Integral of Type $\oint f(\theta)d\theta$	75
3.14 Multivalued Functions and Branch Points	76
3.15 Dispersion Relations	77
3.16 Method of Steepest Descents	82
3.17 Complex Logarithms and Powers	85
3.18 Contour Integrals	86
3.19 Short Questions with Answers	89
3.20 Solved Problems	91
3.21 Multiple Choice Questions(MCQs)	97
4 Differential Equations	99
4.1 Differential Equation	99
4.1.1 Classification of Differential Equations	100
4.2 First Order Linear Differential Equation	101
4.2.1 Separation of Variables	102
4.2.2 Homogeneous Differential Equations	103
4.2.3 Exact Differential Equation	103
4.3 Equations Reducible to the Linear Form (Bernoulli's Equation)	104
4.4 Equations Of First Order And Higher Degree	105
4.5 Equations Which do not Contain y Directly	106
4.6 Equation Which do not Contain x Directly	107
4.7 Linear Second Order Differential Equations	108
4.8 Fuchs Theorem	109
4.8.1 Ordinary and Singular Points	110
4.9 Power Series Solution (About Ordinary Points)	110
4.10 Frobenius Method (About Regular Singular Point)	111
4.11 Linear Classical Oscillator Equation	112
4.12 Linear Independence Solution	114
4.13 Wronskian	115
4.14 Partial Differential Equation in Physics	115
4.14.1 Laplace Equation in Different Coordinate Systems	116
4.14.2 Wave Equation in Different Coordinate Systems	117
4.15 Short Questions with Answers	119
4.16 Solved Problems	121
4.17 Multiple Choice Questions (MCQs)	125
5 Special Functions	127
5.1 Legendre's Differential Equation and its Solution	127
5.1.1 Generating Function for $P_n(x)$	130
5.1.2 Recurrence Formula for $P_n(x)$	131
5.1.3 Orthogonality of $P_n(x)$	134
5.1.4 Associated Legendre Polynomials	135
5.2 Bessel Function	136
5.2.1 Generating Function for the Bessel Function $J_n(x)$	138
5.2.2 Recurrence Relations for Bessel Function $J_n(x)$	138
5.2.3 Bessel Function of Second Type	140
5.2.4 Trigonometric Expansion Involving Bessel Functions	141
5.2.5 Integral Representation of Bessel Functions	141
5.2.6 Spherical Bessel Functions	143
5.3 Hermite Differential Equation	144
5.3.1 Generating Function for Hermite Function $H_n(x)$	145
5.3.2 Recurrence Relations for $H_n(x)$	146
5.3.3 Orthogonality of $H_n(x)$	146
5.4 Laguerre Function	148

5.4.1	Generating Function for $L_n(x)$	149
5.4.2	Recurrence Relations for $L_n(x)$	150
5.4.3	Orthogonality of $L_n(x)$	151
5.4.4	Associated Laguerre Polynomials	152
5.5	Gamma Function Γ	153
5.5.1	Di-Gamma and Poly-Gamma Functions	155
5.5.2	Transformation of Gamma Function	155
5.6	Stirling's Series	156
5.7	Beta Function	157
5.7.1	Transformation of Beta Function	158
5.8	Short Questions with Answers	161
5.9	Solved Problems	164
5.10	Multiple Choice Questions (MCQs)	169
6	Fourier Series and Integral Transformation.....	171
6.1	Fourier Series	171
6.2	Fourier Cosine Series and Fourier Sine Series	173
6.3	Exponential (Complex) Form of Fourier Series	174
6.4	Change of Interval of Fourier Series	175
6.5	Applications of Fourier Series	176
6.6	Fourier Integrals	179
6.7	Fourier Transformation	181
6.7.1	Shifting of Origin	182
6.8	Convolution Theorem	182
6.9	Fourier Cosine Transform	183
6.10	Fourier Sine Transform	184
6.11	Fourier Transform of Derivatives	185
6.12	Fourier Cosine Transform of Derivatives	186
6.13	Fourier Sine Transform of Derivatives	187
6.14	Dirac Delta Function	188
6.15	Laplace Transforms	188
6.15.1	Properties of Laplace Transform	190
6.16	Laplace Transform of Derivatives	192
6.17	Inverse Laplace Transform	193
6.18	Partial Fraction Expansion	195
6.19	Applications of Laplace Transform	195
6.19.1	Simple Harmonic Motion	195
6.19.2	Simultaneous Differential Equations	197
6.19.3	Motion of A Body	197
6.19.4	Step Function	198
6.20	Short Questions with Answers	199
6.21	Solved Problems	203
6.22	Multiple Choice Questions (MCQs)	209
7	Green's Functions and Boundary Value Problems	211
7.1	The Dirac Delta Function	211
7.2	Green's Function	213
7.3	Green's Functions for Initial/Boundary Value Problems	214
7.3.1	Green's Function Associated with Initial Value Problem	215
7.3.2	Green's Function for Boundary Value Problem	218
7.4	Solution of BVPs with Inhomogeneous B.Cs.....	219
7.4.1	Solution of SL System with Homogeneous B.Cs.	219
7.4.2	Non-homogeneous Boundary Conditions	221
7.5	Method of Eigenfunction Expansion for Green's Functions	222
7.6	Sturm-Liouville Theory	225

7.7	Short Questions with Answers	228
7.8	Solved Problems	230
7.9	Multiple Choice Questions (MCQs)	234
8	Tensor Analysis	236
8.1	Scalar, Vector and Dyadic	236
8.2	Coordinate Transformations	237
8.3	Summation Convention	238
8.4	Contravariant and Covariant Tensors	238
8.5	Order and Rank of a Tensor	239
8.6	Kronecker Delta	240
8.7	Symmetric and Anti-symmetric Tensor	241
8.8	Levi-Civita Symbol/Permutation Symbol/Epsilon Tensor	242
8.9	Relation between Levi-Civita and Kronecker Tensor	242
8.10	Fundamental Operation with Tensors	242
	8.10.1 Addition	243
	8.10.2 Subtraction	243
	8.10.3 Contraction	243
	8.10.4 Multiplication	244
	8.10.5 Direct Product	244
	8.10.6 Inner Product	244
	8.10.7 Quotient Law	244
8.11	Integral Theorems in Tensor Form	245
	8.11.1 Gauss's Divergence Theorem	245
	8.11.2 Stokes's Theorem	246
8.12	Short Questions with Answers	247
8.13	Solved Problems	249
8.14	Multiple Choice Questions (MCQs)	254
9	Group Theory	256
9.1	Groups and its Types	256
9.2	Lie Group	258
9.3	Isomorphism and Homomorphism	259
9.4	Representation of a Group	260
	9.4.1 Representation Through Similarity Transformations	260
	9.4.2 Equivalent Representations	261
	9.4.3 Reducible and Irreducible Representations	261
9.5	Cayley's Theorem	262
9.6	Cosets and Lagrange's Theorem	263
9.7	Special Unitary Group	265
	9.7.1 SU(2) Group	266
	9.7.2 SU(3) Group	267
9.8	Special Orthogonal Group	268
	9.8.1 SO(2) Group	268
	9.8.2 SO(3) Group	270
9.9	Continuous and Discrete Groups	271
9.10	Dihedral Group	273
9.11	Short Questions with Answers	275
9.12	Solved Problems	277
9.13	Multiple Choice Question (MCQs)	281

Chapter 1

Vector Analysis

In science and engineering, we frequently encounter quantities which have magnitude and direction only e.g., mass, time and temperature. We call these quantities as **scalars**. In contrast, many interesting physical quantities in addition to magnitude have direction as well. This group includes displacement, velocity, acceleration, force, momentum and angular momentum. Quantities with magnitude and direction are labeled as **vector quantities**. As an historical insight, it is interesting to note that the vector quantities listed are all taken from mechanics but that vector analysis was not used in the development of mechanics and, indeed, had not been created. The need for vector analysis become apparent only with the development of Maxwell's electromagnetic theory and in application of the inherent vector nature of quantities such as electric field and magnetic field.

1.1 Scalar and Vector

Question Define scalar and vector and terms associated with them. How we find the resultant of a vector? Discuss the transformation laws of coordinates.

Scalar

If a quantity does not depend on the orientation of the coordinate axes, then it is called **scalar**. If we assume that space is isotropic i.e. there is no preferred direction or all direction are equivalent, then the physical system being analyzed or the physical laws being expressed cannot and must not depend on our choice or orientation of coordinate axes. It is a geometrical object, independent of coordinate system. **These quantities remains invariant under rotation of system of coordinates.**

Vector

Quantities which changes under the rotation of system of coordinates according to transformation law are said to be **vectors**. Unit vector serves as basis.

Notation

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \quad (1.1)$$

A vector is indicated by a boldface letter, e.g. **A** or with an arrow over it (\vec{A}). The components of a vector is represented by a subscript, e.g., A_x is the x -component of vector \vec{A} , A_y is the y -component and A_z is the z -component of \vec{A} .

Chapter 2

Matrix Algebra

MATRICES are 2-D arrays of numbers or functions that obey the laws that define matrix algebra. The subject is important for physics because it facilitates the description of linear transformations such as changes of coordinate systems, provides a useful formulation of quantum mechanics, and facilitates a variety of analysis in classical and relativistic mechanics, particle theory, and other areas. Note also that the development of a mathematics of two-dimensionally ordered arrays is a natural and logical extension of concepts involving ordered pairs of numbers (complex numbers) or ordinary vectors (one-dimensional arrays). The most distinctive feature of matrix algebra is the rule for the multiplication of matrices.

2.1 Matrix

Question Define matrix and its types. Also discuss the algebraic operations on matrix.

2-D array of numbers or functions that obey the laws which governs the matrix algebra. It helps the transformations of system of coordinate. Provides useful formulation of quantum mechanics and facilitates a variety of analysis in classical and relativistic mechanics. Matrix of order ($m \times n$) is:

$$A_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}$$

Where, a_{nm} are the number of elements in a matrix and n is the number of rows and m is the number of columns.

Equal Matrix

Two matrix A and B are said to be equal if their orders and the number of elements are same i.e., $a_{ij} = b_{ij}$.

Example: $A = \begin{pmatrix} 3 & 4 \\ -4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2+1 & 5-1 \\ -5+1 & 0 \end{pmatrix}$, where A and B are equal matrix.

Chapter 3

Complex Variables and Infinite Series

WE turn now to a study of complex variable theory. In this area, we develop some of the most powerful and widely useful tools in all of analysis. To indicate, at least partly, why complex variables are important, we mention briefly several areas of application.

- In two dimensions, the electric potential, viewed as a solution of Laplace's equation.
- The time-dependent Schrödinger equation of quantum mechanics contains the imaginary unit i , and its solutions are complex.
- The change of a parameter k from real to imaginary, $k \rightarrow ik$, transforms the Helmholtz equation into the time-independent diffusion equation.
- Many physical quantities that were originally real become complex as a simple physical theory is made more general.

3.1 Complex Variables

Question Define and explain complex variables. Also, write the properties of complex numbers. Describe the polar form of complex number. What is Euler's formula.

A complex variable z may be expressed as $z = x + iy$ where $i = \sqrt{-1}$, and x, y are real values. In this form, x is known as **real part** of z whereas y is known as **imaginary part** of z . Usually, the real part x is denoted as R_z or $R(z)$ and the imaginary part y as I_z or $I(z)$.

Complex Function and Properties

Complex conjugate or conjugate of a complex number $z = x + iy$ is a complex number $\bar{z} = x - iy$.

1. When x, y are real numbers and $x + iy = 0$ then $x = 0, y = 0$.
2. For any three set of complex numbers z_1, z_2 and z_3 satisfies the commutative, associative and distributive laws.
 - a) $z_1 + z_2 = z_2 + z_1$ (Commutative law for addition).
 - b) $z_1 \cdot z_2 = z_2 \cdot z_1$ (Commutative law for multiplication).
 - c) $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ (Associative law for addition).
 - d) $(z_1 z_2) z_3 = z_1 (z_2 z_3)$ (Associative law for multiplication).
 - e) $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$ (Distributive law).
3. The sum of two conjugate complex is real.
4. The product of two conjugate complex is real.
5. When the sum of two complex numbers is real and the product of two complex numbers is also real then complex numbers are conjugate to each other.

Chapter 4

Differential Equations

In diverse scientific and technological domains, several challenges are framed in the language of differential equations, with partial differential equations (PDEs) incorporating two or more independent variables. Addressing these complex problems often involves a potent strategy—the method of separation of variables. This technique entails breaking down a PDE into (ordinary) differential equations, allowing for a systematic and more manageable solution. By decomposing the problem, researchers can analyze each component independently, simplifying the overall process. This approach is particularly valuable when dealing with intricate physical phenomena or complex systems, offering a powerful tool to resolve and realized the underlying mathematical complexities, ultimately facilitating the resolution of problems in physics and a many of other scientific and technological domains.

4.1 Differential Equation

Question Define differential equation and give examples. Also what is the order and degree of differential equation and give examples.

An equation containing the derivative or differential of one or more dependent variables w.r.t. one or more independent variables is called a **differential equation**. For example;

$$\frac{d^2y}{dx^2} + xy \left(\frac{dy}{dx} \right)^2 = 0 \quad \text{and} \quad \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = 0$$

Order of Differential Equation

The highest ordered derivative involved in a differential equation is called the **order of the differential equation**. For example;

$$\frac{d^2y}{dx^2} + xy \left(\frac{dy}{dx} \right)^2 = 0 \quad (\text{Order 2}), \quad \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = 0 \quad (\text{Order 1})$$

$$\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t \quad (\text{Order 4}), \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (\text{Order 2})$$

Chapter 5

Special Functions

SPECIAL functions, like Bessel, Legendre, Laguerre, and Hermite functions, are like superheroes in math for describing everyday things. Bessel helps with circular stuff, Legendre with things shaped like a ball, Laguerre tackles equations for things that expand, and Hermite takes on jobs in quantum physics and bouncing objects. These math heroes make hard problems simpler, helping scientists understand how waves, quantum stuff, and different things work. They're like a team of super-powered friends making math more friendly and helping us uncover the cool secrets of the world around us.

5.1 Legendre's Differential Equation and its Solution

Question What is Legendre's differential equation and write the solution of Legendre's differential equation. Also discuss Legendre polynomial of the first kind and second kind.

Legendre differential equation is:

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0 \quad (5.1)$$

Where, n is a positive integer. Solution of the Eq.(5.1 Legendre's Differential Equation Solution is known as **Legendre function**. In order to find out singular points and possibility of the series solution for the Eq.(5.1 Legendre's Differential Equation and its Solution equation.5.1.1), we have

$$P(x) = -\frac{2x}{1-x^2} \quad \text{and} \quad Q(x) = \frac{n(n+1)}{1-x^2}$$

Obviously, there are singular points at $x = -1$ and $x = 1$. However, both the singular points are regular as $(x+1)P(x)$ and $(x+1)^2Q(x)$ both are finite at the point $x = -1$, and $(x-1)P(x)$ and $(x-1)^2Q(x)$ both are finite at the point $x = 1$. Thus, as per Fuchs theorem, Legendre differential equation has a series solution. Let the series solution be:

$$y = \sum_{r=0}^{\infty} a_r x^{k-r} \quad (a_0 \neq 0) \quad (5.2)$$

(Solution can also be obtained as ascending powers of x , but the solution in descending powers of x is more convenient). On differentiating Eq.(5.2 Legendre's Differential Equation Solution w.r.t. x , we get

Chapter 6

Fourier Series and Integral Transformation

FOURIER series, a fundamental concept in mathematical analysis, analyze periodic functions into a sum of sine and cosine functions, discover their harmonic components. Fourier cosine and sine series provide specialized representations, while the exponential form extends the generality. Adaptable to varied contexts, changes in interval or origin shift refine applications, from signal processing to heat transfer. Fourier transformation develop this scope to a periodic functions, revealing frequency domain insights. Laplace transforms, powerful tools in solving linear differential equations, facilitate system analysis. Their properties, such as linearity, and applications, from modeling harmonic motion to solving simultaneous equations, underscore their crucial role in mathematics and engineering.

6.1 Fourier Series

Question Define and explain the Fourier series with examples.

A function $f(x)$ can be represented in th form of a series of sines and cosines:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad (6.1)$$

Where the coefficients a 's and b 's are to be determined. This series of representation of a function is known as **Fourier series of the function**. Fourier series representation of a function may be feasible even when the function has discontinuities, but its Taylor series expansion is not possible. Fourier series expansion of a function is possible provided it satisfies a set of conditions, known as the **Dirichlet conditions**, given as:

- (a) The function must be periodic.
- (b) Within one cycle (period), the function must be single-valued and differentiable, except at a finite number of discontinuities having finite magnitude.
- (c) The function must have finite number of maxima and minima (extremes) within a cycle.

These conditions are sufficient, but not necessary. In a large number of physical cases where Fourier series representation of a function is required, these conditions are generally satisfied. We know that sine and cosine functions are periodic with a period of 2π . Further, for Fourier series representation we account for the function over one complete cycle. The coefficients a 's and b 's are related to the given function $f(x)$, and can be calculated in the following manner. Here, we shall use the following orthogonality properties of sine and cosine functions:

Chapter 7

Green's Functions and Boundary Value Problems

GREEN'S function is a powerful mathematical tool widely employed in the realm of boundary value problems in physics and engineering. Deriving from the field of potential theory, Green's function provides a solution to partial differential equations subject to specific boundary conditions. Serving as a fundamental building block, it summarizes the influence of point sources on a system, offering a unique solution for various physical phenomena. In this context, the study of boundary value problems involving Green's function plays a pivotal role in understanding and solving complex mathematical models that arise in fields such as heat conduction, fluid dynamics, and electromagnetic theory.

7.1 The Dirac Delta Function

Question What is Dirac delta function? Define the properties of Dirac delta function and discuss the integral representation of the Dirac delta function.

The Dirac delta function can be regarded as the generalization of the Kronecker delta δ_{ij} , when the discrete integer variables i, j are replaced by the continuous variables x, x' . The **Kronecker delta** δ_{ij} has the following two well-known properties.

1. $\delta_{ij} = \begin{cases} 1 & ; \quad i = j \\ 0 & ; \quad i \neq j \end{cases}$
2. $\delta_{ij}A_i = A_j$

These properties are passed over to the **Dirac delta function** in the following form:

1. $\delta(x-t) = \begin{cases} 0 & ; \quad t \neq x \\ \infty & ; \quad t = x \end{cases}$
2. $\int \delta(x-t)f(t)dt = f(x)$

Where, the point x lies in the interval of integration and $f(x)$ is a continuous function. To understand the **significance of the Dirac delta function** in physical situations, we consider those situations in which a large effect lasting for a short duration or acting over a small stretch of length. The examples are an impulsive force or a load over a very small part of a beam. Also, we can define the Dirac delta function $\delta(x)$ in the following manner.

Chapter 8

Tensor Analysis

VENTURING into the realm of tensors and their applications, this exploration resolve the mathematical framework crucial to distinct fields. Tensors, multi-dimensional arrays, serve as powerful tools in physics, engineering, and computer science, capturing complex relationships within data. From the foundational concepts of tensor algebra to applications in general relativity, fluid dynamics, and machine learning, this chapter dig into their originality. Topics include the Levi-Civita tensor, Kronecker delta tensor, and their roles in expressing symmetry and antisymmetry. As, we navigate through tensors, their manipulation, and practical applications, we unveil the intense impact of these mathematical constructs on understanding the complexity of the physical world.

8.1 Scalar, Vector and Dyadic

Question Define scalar, vector and dyadic. Define the rank of each and give examples.

Scalar

Scalar is a tensor of rank zero. It is single real number or component. In three dimensional (3-D) space, the number of components of a scalar is $3^0 = 1$. Scalar is a quantity which do not change under rotation of coordinates i.e. it is invariant and transformation law for scalar is $A'_i = A$. **Example** of tensor of rank zero (i.e. scalar) are mass, charge, speed etc.

Vector

A tensor of rank one. In three dimensional space, the number of components of a vector is 3^1 (i.e.3). Components transform under rotation like those of the distance of a point from a chosen origin. The transformation law for vector's component is:

$$A'_i = \sum_{p=1}^3 a_{ip} A_p = \sum \frac{\partial x_p}{\partial x'_i} A_p \quad (8.1)$$

Examples: momentum, electric field, velocity.

Dyadic

Dyadic is a tensor of rank two. In three dimensional space, the number of components of a 2nd rank tensor is given by 3^2 (i.e. 9).

$$A'_{ij} = \sum_p \sum_q a_{ip} a_{jq} A_{pq}$$

Chapter 9

Group Theory

IN mathematics, a group is a fundamental concept that captures the idea of symmetry and structure. Groups come in various types, each with unique characteristics and applications. Lie groups are continuous groups often used in physics and geometry. Isomorphisms and homomorphisms describe relationships between groups, preserving their structure. Representations of groups help us understand their action in different contexts, with reducible and irreducible representations offering insights into their complexity. Cayley's theorem shows that every group is essentially a group of permutations. Cosets and Lagrange's theorem provide valuable tools in group theory. Special unitary groups like $SU(2)$ and $SU(3)$ and special orthogonal groups like $SO(2)$ and $SO(3)$ have significant roles in theoretical physics. The dihedral group and the distinction between continuous and discrete groups further enrich our understanding of symmetry and transformation.

9.1 Groups and its Types

Question Discuss the groups and its related terms.

A group G may be defined as a set of objects, operations, relations, transformations called elements of G ; that may be combined multiplied to form a well defined product in G denoted by “ $*$ ” and named as binary operation and satisfying the following properties:

- **Closure Property**

$$a, b \in G \text{ then } a * b \in G$$

$$(a, b) \simeq a * b$$

- **Unit Element Property**

$$1 * a = a * 1 = a$$

$$\text{Unit is unique } (1 = 1' * 1 = 1')$$

- **Associative Property**

$$a * (b * c) = (a * b) * c$$

- **Inverse Property**

Inverse or reciprocal of each element should exist.

$$a * a^{-1} = a^{-1} * a = 1$$

$$\Rightarrow a * b = ab$$

Subgroup

When we choose a set of some elements out of a group, and this set also a group, then this set is known as subgroup of the given group. $G' \subset G$ and G' is itself a group. The unit of G is subgroup of G .

Example 5.1.1 Prove that $P_n(1) = 1$.

Solution:

Since, we know that

$$(1 - 2xz + z^2)^{-1/2} = P_0(x) + zP_1(x) + z^2P_2(x) + \dots + z^nP_n(x) + \dots$$

Substituting 1 for x in the above equation, we get

$$\begin{aligned} (1 - 2z + z^2)^{-1/2} &= P_0(1) + zP_1(1) + z^2P_2(1) + \dots + z^nP_n(1) + \dots \\ [(1 - z)^2]^{-1/2} &= 1 + zP_1(1) + z^2P_2(1) + \dots + z^nP_n(1) + \dots && (\because P_0(1) = 1) \\ &= \sum_{n=0}^{\infty} z^n P_n(1) \Rightarrow (1 - z)^{-1} = \sum_{n=0}^{\infty} z^n P_n(1) \\ \text{or } \sum_{n=0}^{\infty} z^n P_n(1) &= (1 - z)^{-1} = 1 + z + z^2 + z^3 + \dots + z^n + \dots \end{aligned}$$

Equating the coefficients of z^n on both sides, we get

$$P_n(1) = 1$$

5.1.2 Recurrence Formula for $P_n(x)$

Question Derive the recurrence formulas for the Legendre polynomial $P_n(x)$.

Formula- I: $nP_n = (2n - 1)xP_{n-1} - (n - 1)P_{n-2}$

We know that $(1 - 2xz + z^2)^{-1/2} = \sum z^n P_n(x)$. Differentiating it w.r.t. z , we get

$$-\frac{1}{2}(1 - 2xz + z^2)^{-3/2}(-2x + 2z) = \sum n z^{n-1} P_n(x)$$

Multiplying both sides by $(1 - 2xz + z^2)$, we get

$$(1 - 2xz + z^2)^{-1/2}(x - z) = (x - z) \sum z^n P_n(x) = (1 - 2xz + z^2) \sum n z^{n-1} P_n(x)$$

Equating the coefficients of z^{n-1} from both sides, we get

$$\begin{aligned} xP_{n-1} - P_{n-2} &= nP_n - 2x(n - 1)P_{n-1} + (n - 2)P_{n-2} \\ \Rightarrow nP_n &= xP_{n-1} - P_{n-2} + 2x(n - 1)P_{n-1} - (n - 2)P_{n-2} \\ &= xP_{n-1} + 2x(n - 1)P_{n-1} - P_{n-2} - (n - 2)P_{n-2} = (2n - 2 + 1)xP_{n-1} - (1 + n - 2)P_{n-2} \\ \Rightarrow nP_n &= (2n - 1)xP_{n-1} - (n - 1)P_{n-2} \end{aligned}$$

Formula- II: $xP'_n - P'_{n-1} = nP_n$

We know that

$$(1 - 2xz + z^2)^{-1/2} = \sum z^n P_n(x) \tag{5.18}$$

Differentiating Eq.(5.18) Recurrence Formula for $P_n(x)$ equation.5.1.18) with respect to z , we get

$$-\frac{1}{2}(1 - 2xz + z^2)^{-3/2}(-2x + 2z) = (x - z)(1 - 2xz + z^2)^{-3/2} = \sum n z^{n-1} P_n(x) \tag{5.19}$$

Differentiating Eq.(5.18) Recurrence Formula for $P_n(x)$ equation.5.1.18) with respect to x , we get

$$-\frac{1}{2}(1 - 2xz + z^2)^{-3/2}(-2z) = z(1 - 2xz + z^2)^{-3/2} = \sum z^n P'_n(x) \tag{5.20}$$

1.15 Short Questions with Answers

1.1 If the coordinates of P be $(3, 4, 12)$, then find \overrightarrow{OP} , its magnitude and direction cosines.

Answer: First we calculate \overrightarrow{OP} and its magnitude as:

$$\overrightarrow{OP} = 3\hat{i} + 4\hat{j} + 12\hat{k} \Rightarrow |\overrightarrow{OP}| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144} = 13$$

Now, we calculate its direction cosines as:

$$\text{Direction cosines} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}} \text{ and } \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{3}{13}, \frac{4}{13} \text{ and } \frac{12}{13}$$

1.2 If for two vectors \vec{a} and \vec{b} , $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, find the angle between \vec{a} and \vec{b} .

Answer: Since, the given condition is

$$\begin{aligned} |\vec{a} + \vec{b}| &= |\vec{a} - \vec{b}| \Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2 \\ (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) &= (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) \Rightarrow (\vec{a})^2 + (\vec{b})^2 + 2\vec{a} \cdot \vec{b} = (\vec{a})^2 + (\vec{b})^2 - 2\vec{a} \cdot \vec{b} \\ 4\vec{a} \cdot \vec{b} &= 0 \Rightarrow \vec{a} \cdot \vec{b} = 0 \end{aligned}$$

This shows that \vec{a} and \vec{b} are perpendicular to each other and so the angle between them is 90° .

1.3 Determine λ and μ by using vectors, such that the points $(-1, 3, 2)$, $(-4, 2, -2)$ and $(5, \lambda, \mu)$ lie on a straight line.

Answer: Let A, B, C be three points whose coordinates are $(-1, 3, 2)$, $(-4, 2, -2)$ and $(5, \lambda, \mu)$. So,

$$\overrightarrow{AB} = \vec{B} - \vec{A} = (-4\hat{i} + 2\hat{j} - 2\hat{k}) - (-\hat{i} + 3\hat{j} + 2\hat{k}) = -3\hat{i} - \hat{j} - 4\hat{k}$$

$$\text{And, } \overrightarrow{BC} = \vec{C} - \vec{B} = (5\hat{i} + \lambda\hat{j} + \mu\hat{k}) - (-4\hat{i} + 2\hat{j} - 2\hat{k}) = 9\hat{i} + (\lambda - 2)\hat{j} + (\mu + 2)\hat{k}$$

$$\text{Now, } \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -1 & -4 \\ 9 & \lambda - 2 & \mu + 2 \end{vmatrix} = (-\mu + 4\lambda - 10)\hat{i} - (-3\mu + 30)\hat{j} + (-3\lambda + 15)\hat{k}$$

If \overrightarrow{AB} and \overrightarrow{BC} are in the same straight line, then $\overrightarrow{AB} \times \overrightarrow{BC} = 0$. Now,

$$(-\mu + 4\lambda - 10)\hat{i} - (-3\mu + 30)\hat{j} + (-3\lambda + 15)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

On comparing both sides, we get

$$-3\mu + 30 = 0 \Rightarrow \mu = 10 \quad \text{and} \quad -3\lambda + 15 = 0 \Rightarrow \lambda = 5$$

1.4 Show that the vectors $5\vec{a} + 6\vec{b} + 7\vec{c}$, $7\vec{a} - 8\vec{b} + 9\vec{c}$ and $3\vec{a} + 20\vec{b} + 5\vec{c}$ are co-planar. \vec{a} , \vec{b} , \vec{c} being three non-collinear vectors.

Answer: Let $\vec{\alpha} = 5\vec{a} + 6\vec{b} + 7\vec{c}$, $\vec{\beta} = 7\vec{a} - 8\vec{b} + 9\vec{c}$ and $\vec{\gamma} = 3\vec{a} + 20\vec{b} + 5\vec{c}$. Now,

$$\vec{\alpha} \cdot (\vec{\beta} \times \vec{\gamma}) = \begin{vmatrix} 5 & 6 & 7 \\ 7 & -8 & 9 \\ 3 & 20 & 5 \end{vmatrix} = 5(-40 - 180) - 6(35 - 27) + 7(140 + 24) = -1100 - 48 + 1148 = 0$$

Hence, $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$ are co-planar.

1.16 Solved Problems

Problem 1.1. If $|\vec{A} + \vec{B}| = 60$, $|\vec{A} - \vec{B}| = 40$ and $|\vec{B}| = 46$, find $|\vec{A}|$.

Solution

$$\begin{aligned} |\vec{A} + \vec{B}| = 60 &\Rightarrow |\vec{A} + \vec{B}|^2 = 3600 \Rightarrow (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = 3600 \\ (\vec{A})^2 + (\vec{B})^2 + 2(\vec{A}) \cdot (\vec{B}) &= 3600 \Rightarrow |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta = 3600 \end{aligned} \quad (1.37)$$

$$\begin{aligned} \text{Similarly, } |\vec{A} - \vec{B}| = 40 &\Rightarrow |\vec{A} - \vec{B}|^2 = 1600 \Rightarrow (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) = 1600 \\ (\vec{A})^2 + (\vec{B})^2 - 2(\vec{A}) \cdot (\vec{B}) &= 1600 \Rightarrow |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta = 1600 \end{aligned} \quad (1.38)$$

Adding Eq.(1.37) and Eq.(1.38), we get

$$\begin{aligned} 2|\vec{A}|^2 + 2|\vec{B}|^2 &= 5200 \Rightarrow |\vec{A}|^2 + |\vec{B}|^2 = 2600 \Rightarrow |\vec{A}|^2 + (46)^2 = 2600 \\ \Rightarrow |\vec{A}|^2 &= 484 \Rightarrow \sqrt{|\vec{A}|^2} = \sqrt{484} \Rightarrow |\vec{A}| = 22 \end{aligned}$$

Problem 1.2. Forces of magnitudes 5 and 3 units acting in the directions $(6\hat{i} + 2\hat{j} + 3\hat{k})$ and $(3\hat{i} - 2\hat{j} + 6\hat{k})$, respectively act on a particle which is displaced from the point $(2, 2, -1)$ to $(4, 3, 1)$. Find the work done by the forces.

Solution

First force of magnitude 5 units, acting in the direction $6\hat{i} + 2\hat{j} + 3\hat{k} = 5 \left(\frac{6\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{6^2 + 2^2 + 3^2}} \right) = \frac{5}{7}(6\hat{i} + 2\hat{j} + 3\hat{k})$

Second force of magnitude 3 units, acting in the direction $3\hat{i} - 2\hat{j} + 6\hat{k} = 3 \left(\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{3^2 + (-2)^2 + 6^2}} \right) = \frac{3}{7}(3\hat{i} - 2\hat{j} + 6\hat{k})$

Resultant force: $\vec{F} = \frac{5}{7}(6\hat{i} + 2\hat{j} + 3\hat{k}) + \frac{3}{7}(3\hat{i} - 2\hat{j} + 6\hat{k}) = \frac{1}{7}(39\hat{i} + 4\hat{j} + 33\hat{k})$

Displacement from the point $(2, 2, -1)$ to $(4, 3, 1)$

$$\vec{d} = (4\hat{i} + 3\hat{j} + \hat{k}) - (2\hat{i} + 2\hat{j} - \hat{k}) = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Work done} = \vec{F} \cdot \vec{d} = \frac{1}{7}(39\hat{i} + 4\hat{j} + 33\hat{k}) \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = \frac{1}{7}(78 + 4 + 66) = \frac{148}{7} \text{ units}$$

Problem 1.3. In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^2y^2z^4$ maximum? Find also the magnitude of this maximum.

Solution

$$\vec{\nabla}\phi = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) (x^2y^2z^4) = 2xy^2z^4\hat{i} + 2x^2yz^4\hat{j} + 4x^2y^2z^3\hat{k}$$

Directional derivative at $(3, 1, -2) = 96\hat{i} + 288\hat{j} - 288\hat{k}$. The directional derivative is maximum in the direction $96\hat{i} + 288\hat{j} - 288\hat{k}$ or $96(\hat{i} + 3\hat{j} - 3\hat{k})$. In any other direction it will have its component which is less than its maximum value.

$$\text{Maximum value} = 96\sqrt{1 + 9 + 9} = 96\sqrt{19}$$

Problem 1.4. Verify that $\vec{\nabla} \cdot \left[\frac{f(\vec{r})}{|\vec{r}|} \vec{r} \right] = \frac{1}{r^2} \frac{d}{dr} [r^2 f(\vec{r})]$.

Solution

First we take the left hand side,

$$\vec{\nabla} \cdot \left[\frac{f(\vec{r})}{|\vec{r}|} \vec{r} \right] = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \left[\frac{f(\vec{r})}{|\vec{r}|} \vec{r} \right] = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \left[\frac{f(\vec{r})}{|\vec{r}|} (x\hat{i} + y\hat{j} + z\hat{k}) \right]$$

1.17 Multiple Choice Questions (MCQs)

1: What is the magnitude of unit vector:

- (a) It has no magnitude (b) Zero (c) Constant but not zero (d) **Unity**

2: Flying a bird is the example of:

- (a) Collinear vector (b) Multiplication of vector (c) Addition of vector (d) **Composition of vector**

3: Cross product is the mathematical operation performed between:

- (a) 2 scaler numbers (b) A scaler and a vector (c) **2 vectors** (d) Any 2 numbers

4: The mathematical perception of the gradient is said to be:

- (a) Tangent (b) Chord (c) **Slope** (d) Arc

5: Gauss theorem uses which of the following operations:

- (a) Gradient (b) **Divergence** (c) Laplacian (d) Curl

6: Which of the following operation uses the curl operation:

- (a) Greens theorem (b) Gauss divergence theroem (c) **Stoke's theorem** (d) Maxwell equation

7: Find the curl of $\vec{A} = (y \cos ax)\hat{i} + (y + e^x)\hat{k}$

- (a) $2\hat{i} - ex\hat{j} - \cos ax\hat{k}$ (b) $m\hat{i} - ex\hat{j} - \cos ax\hat{k}$ (c) $2\hat{i} - ex\hat{j} + \cos ax\hat{k}$ (d) $\hat{i} - ex\hat{j} + \cos ax\hat{k}$

8: Mathematically the function in the green theorem will be:

- (a) Continuous derivative (b) Discrete derivative
(c) **Continuous partial derivatives** (d) Discrete partial derivatives

9: The first member in ordered pair (x,y) is considered as:

- (a) Mantissa (b) Cartesian coordinates (c) **Abscissa** (d) Ordinate

10: Example of spherical system in the following is:

- (a) **Charge in space** (b) Charge in box (c) Charge in dielectric (d) Uncharged system

11: The scalar triple product of three vectors \vec{a} , \vec{b} , and \vec{c} is defined as:

- (a) $\vec{a} \cdot \vec{b} \times \vec{c}$ (b) $\vec{a} \cdot (\vec{b} \times \vec{c})$ (c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ (d) $\vec{a} \cdot \vec{b} \cdot \vec{c}$

12: The vector triple product $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to:

- (a) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ (b) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$ (c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ (d) $(\vec{a} \cdot \vec{b})(\vec{a} \cdot \vec{c})$

13: For vectors $\vec{a} = (1,2,3)$, $\vec{b} = (4,5,6)$, and $\vec{c} = (7,8,9)$, the scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is:

- (a) **0** (b) 1 (c) 15 (d) -15

14: The vector triple product $(\vec{a} \times \vec{b}) \times \vec{c}$ is equivalent to:

- (a) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ (b) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$ (c) $(\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}$ (d) None of the above

15: The scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to:

- (a) **The volume of the parallelepiped formed by \vec{a} , \vec{b} , and \vec{c}** (b) The area of the parallelogram formed by \vec{a} and the cross product $\vec{b} \times \vec{c}$ (c) The dot product of \vec{a} with the cross product $\vec{b} \times \vec{c}$ (d) None of the above

16: The gradient of a scalar function $f(x,y,z)$ is a:

- (a) Scalar (b) **Vector** (c) Tensor (d) Matrix

17: The divergence of a vector field $\vec{F}(x,y,z)$ is a:

- (a) **Scalar** (b) Vector (c) Tensor (d) Matrix

18: The curl of a vector field $\vec{F}(x,y,z)$ is a:

- (a) Scalar (b) **Vector** (c) Tensor (d) Matrix

19: If $\vec{F}(x,y,z) = (x^2, 2y, z^3)$, then $\vec{\nabla} \cdot \vec{F}$ is:

- (a) $2x + 2 + 3z^2$ (b) $2x - 2y + 3z^2$ (c) $x^2 + 2y + z^3$ (d) $2x^2 + 2y + 3z^3$

Course outline of various universities are given below. The page numbers of topics are mentioned to follow your respective university outline.

Bahauddin Zakariya University (BZU), Multan

Course Title: Mathematical Methods of Physics-I **Course Code:** PHYS-301 **Semester-5**

Vector Analysis: Review of vectors Algebra (P#1), Vector differentiation and gradient (P#8,11), Divergence and Gauss's theorem (P#15), Vector integration (P#9), Green's theorem in the plane (P#19), Curl and Stoke's theorem (P#13, 17).

Curvilinear Coordinates and Tensors: Curvilinear coordinate system (P#20), Gradient, Divergence and Curl in the curvilinear coordinates system, Cartesian, Spherical and Cylindrical coordinate system (P#21,22), Covariant and contravariant tensors (P#238), Tensor algebra (P#242), Quotient rule (P#244).

Matrices: Linear vector spaces, Determinants (P#36), Matrices (P#35), Eigen values and eigenvectors of matrices (P#44), Orthogonal matrices (P#37), Hermitian matrices, Similarity transformations, Diagonalization of matrices (P#46).

Group Theory: Introduction to groups (P#256), group (P#256), Invariant subgroups (P#257), Discrete groups- Dihedral groups (P#271,273), Continuous groups-O groups (P#271), SU(2) groups (P#266), Lie groups (P#258)

Complex Variables: Functions of a complex variable (P#61), Cauchy Riemann conditions and analytic functions (P#64), Cauchy integral theorem and integral formula (P#67,68), Taylor and Laurent series (P#70,71), Calculus of residue (P#73), Complex integration (P#75).

Course Title: Mathematical Methods of Physics-II **Course Code:** PHYS-302 **Semester-6**

Differential Equations in Physics: First and second order linear differential equations (P#101,108), Partial differential equations of theoretical physics (P#115), Separation of variables (P#102), Homogeneous differential equations (P#103), Frobenius series solution of differential equations (P#111), Second solution, non-homogenous differential equations (P#108).

Special Functions: Bessel functions and Hankel functions (P#136), Spherical Bessel functions (P#143), Legendre polynomials (P#127), Associated Legendre polynomials (P#135), Spherical harmonics Laguerre polynomials (P#148), Hermite polynomials (P#144)

Fourier series: Definition and general properties (P#171), Fourier series of various physical functions (P#173), Uses and application of Fourier series (P#176)

Integral Transforms: Integral transform (P#179), Fourier transform (P#181), Convolution theorem (P#182), Elementary Laplace transform and its application (P#188,195).

Boundary Value Problems and Green's Functions: Boundary value problems in Physics (P#214), Non-homogeneous boundary value problems and Green's functions (P#221), Green's functions for one dimensional

problem (P#223), Eigen function expansion of Green's function (P#222), Construction of Green's functions in higher dimensions (P#224).

Government College University, Faisalabad (GCUF)

Course Title: Mathematical Methods of Physics-I **Course Code:** PHYS-501 **Semester-5**

Vector operations (P#2), Physical significance of DEL operator (P#12), Gauss's divergence theorem (P#15), Green's theorem (P#19), Stokes's theorem (P#17), Orthogonal curvilinear coordinates system (P#20), Gradient (P#11), Divergence (P#12), Curl (P#13) and Laplacian in orthogonal curvilinear coordinates, Spherical polar and cylindrical coordinates systems (P#21,22). Complex numbers (P#62), Euler's formula (P#69), De Moivre's theorem (P#62), elementary functions (P#65), analytic functions of complex variables (P#63), Cauchy- Riemann equation (P#64), harmonic functions (P#65), complex integration (P#75), Cauchy's theorem (P#67), Cauchy's integral formula (P#68), Taylor and Laurent series (P#70,71), Contour integrals (P#86), singularities and residues (P#71,73), residue theorem (P#74), branch points and integrals of multivalued functions (P#76). Tensors Analysis and applications (P#257).

Course Title: Mathematical Methods of Physics-II **Course Code:** PHYS-502 **Semester-6**

Fourier Analysis (P#171), Fourier cosine and sine series (P#173), change of interval (P#175), Fourier integral (P#179), complex form of Fourier series (P#174), Fourier transform (P#181), Fourier transform of derivatives (P#185), Laplace transform (P#188), Inverse Laplace Transform (P#193), Convolution theorem (P#182), Initial boundary value problem, Laplace transform of derivatives (P#192). Physical significance along with examples of Fourier and Laplace transforms (P#176, 195). Special functions (P#127), Hermite (P#144), Laguerre (P#148), Legendre and associate Legendre polynomial (P#127,135). Bessel function (P#136), Neumann function (P#140), and spherical Bessel function (P#143), Gamma function (P#153). Nonhomogeneous equations- Green's function (P#213), Green's function in terms of Eigen-function (P#222), the Sturm-Liouville problem (P#225), Green's function for Dirac Delta functions (P#222).

University of the Punjab, Lahore

Course Title: Mathematical Methods of Physics-I **Course Code:** Phys-3501 **Semester-5**

Series solutions about an ordinary point and regular singular point (P#110,111), Sturm-Liouville theory (P#225), self-adjoint ODEs, orthogonal functions, Hermitian operators, eigenvalue problems, completeness of eigenfunctions, Green's Functions (P#213), Green's function for one-dimensional problem (P#223), eigenfunction expansion of Green's function (P#222), special functions (P#127), Gamma Function (P#153), digamma and polygamma functions (P#155), Stirling's series (P#156), Beta function (P#177), Bessel functions of first kind (P#138), orthogonality, Neumann functions (P#140), Bessel functions of the second kind (P#140), Hankel functions, modified Bessel functions (P#138), asymptotic expansions, spherical Bessel functions (P#143), Legendre functions (P#127), Legendre

polynomials (P#128), orthogonality (P#134), generating function (P#130), recurrence relation (P#131), associated Legendre equation (P#135), spherical harmonics, orbital angular momentum operator, addition theorem for spherical harmonics, Legendre functions of the second kind (P#128), Hermite functions (P#145), Hermite equation as Schrodinger equation of quantum harmonic oscillator, Laguerre functions and associated Laguerre functions (P#148,152), Fourier series (P#171), properties of Fourier series, Fourier transform (P#181), properties of Fourier transforms, Fourier convolution theorem (P#182), discrete Fourier transform, Laplace transforms (P#188), properties of Laplace transforms (P#190), Laplace transform of derivatives (P#192), Laplace Convolution theorem, inverse Laplace transform (P#193)

Course Title: Mathematical Methods of Physics-II **Course Code:** Phys-3503 **Semester-6**

Tensor analysis (P#237, some notations, Cartesian tensors, First- and zero-order Cartesian tensors, second- and higher-order Cartesian tensors, the algebra of tensors, the quotient law (P#244), Kronecker delta δ_{ij} and Levi Civita tensor ϵ_{ijk} (P#240,242), Isotropic tensors, improper rotations and pseudo tensors, dual tensors, physical applications of tensors, integral theorems for tensors (P#245), non-Cartesian coordinates, the metric tensor, General coordinate transformations and tensors (P#237), relative tensors, derivatives of basis vectors and Christoffel symbols, covariant differentiation, vector operators in tensor form, absolute derivatives along curves, Riemann curvature tensor, Complex Analysis Complex numbers, powers and roots, Sets in the Complex planes, Functions of a complex variables (P#61), Cauchy–Riemann equations (P#64), Exponential and Logarithmic functions (P#62), Contour Integrals (P#86), Cauchy-Goursat theorem, Independence of path, Cauchy’s Integral formulas (P#68), Sequences and Series, Taylor series (P#70), Laurent Expansion (P#71), Zeros and Poles (P#72), Singularities (P#71), Residues and Residues Theorem (P#73,74), Evaluation of real Integrals, Groups Theory, Review of groups (P#256), subgroup (P#256), cyclic groups (P#257), and permutation groups, isomorphism (P#259), Cayley’s theorem (P#262), properties of isomorphism, automorphism, cosets (P#263), properties of cosets (P#265), Lagrange’s theorem (P#265), an application of cosets to permutation groups, the rotation groups of a cube and soccer ball, conjugate classes and invariant subgroups, group representations (P#260), some special groups (P#265), the symmetry group D_2, D_3 , one-dimensional unitary group $U(1)$, orthogonal groups $SO(2)$ and $SO(3)$ (P#268,270), the $SU(n)$ groups, Homogeneous Lorentz group.

Ghazi University, D.G. Khan

Course Title: Mathematical Methods of Physics-I **Course Code:** PHY-501 **Semester-5**

Special Functions: Bessel Functions (P#136), Neumann Functions (P#140), Hankel Functions, Spherical Bessel Functions (P#143), Legendre Functions (P#127), Associated Legendre Functions (P#135), Spherical Harmonics, Hermite Polynomials (P#145).

Partial Differential Equations: Introduction to important PDEs in Physics (P#115) (wave equation, diffusion equation, Poisson’s equation, Schrodinger’s equation), general form of solution (P#106, 107), general and particular solutions (first order, inhomogeneous, second order)(P#108), characteristics and existence of solutions, unique-

ness of solutions, separation of variables in Cartesian coordinates (P#102), superposition of separated solutions, separation of variables in curvilinear coordinates, special functions (P#127), integral transform methods, Green's functions (P#213).

Complex Analysis: Review (polar form of complex numbers and de Moivre's theorem, complex logarithms and powers) (P#85), functions of a complex variable (P#62), Cauchy-Riemann conditions (P#64), power series in a complex variable and analytic continuation with examples (P#70), multi-valued functions and branch cuts (P#76), singularities and zeroes of complex functions (P#71), complex integration (P#75), Cauchy's theorem (P#67), Cauchy's integral formula (P#68), Laurent series and residues (P#71,73), residue integration theorem (P#74), definite integrals using contour integration (P#86).

Course Title: Mathematical Methods of Physics-I **Course Code:** PHY-502 **Semester-6**

Fourier Series and Integral Transforms: Definition and general properties (P#171), Fourier Series of Various Physical Functions (P#173), Uses and Applications of Fourier Series (P#176), Fourier Transforms (P#181), Convolution Theorems (P#182), Laplace transforms and applications (P#188).

Tensor Analysis: Vector calculus (differentiation, integration, space curves, multi-variable vectors, surfaces, scalar and vector fields, gradient, divergence and curl, cylindrical and spherical coordinates, general curvilinear coordinates), change of basis, Cartesian tensor as a geometrical object, order/rank of a tensor (P#239), tensor algebra (P#242), quotient law (P#244), pseudo-tensors, Kronecker delta and Levi-Civita (P#240,242), dual tensors, physical applications, integral theorems for tensors (P#245), non-Cartesian tensors, general coordinate transformations and tensors (P#237).

Group Theory and Representations for finite groups: Transformations, groups – definitions and examples (P#256), subgroups and Cayley's theorem (P#256,262), cosets and Lagrange's theorem (P#263, 265), conjugate classes, invariant subgroups (P#257), factor groups, homomorphism (P#259), direct products, mappings, linear operators, matrix representations, similarity transformation and equivalent matrix representations, group representations (P#260), equivalent representations and characters (P#261), construction of representations and addition of representations, invariance of functions and operators, unitary spaces and Hermitian matrices, operators: adjoint, self-adjoint, unitary, Hilbert space, reducibility of representations (P#261), Schur's lemmas, orthogonality relations, group algebra, expansion of functions in basis of irreducible representations (P#261), Kronecker product, symmetrized and anti-symmetrized representations, adjoint and complex-conjugate representations, real representations.

University of Education, Lahore

Course Title: Mathematical Methods of Physics-I **Course Code:** Phys-3111 **Semester-5**

Vector Analysis: Divergence theorem (P#15), Green's Theorem (P#19), Stock's theorem (P#17), Cylindrical, spherical and curvilinear coordinates (P#21). Orthogonal curvilinear coordinates (P#20). Gradient, Divergence,

Curl and Laplacian in Spherical and Cylindrical Coordinates.

Special Functions-I: Helmholtz Equation. Legendre's Differential Equation and its Solution (P#127), Legendre's Polynomials (P#127), Associated Legendre functions and Spherical harmonics (P#135).

Functions of Complex Variable: Complex functions (P#61), Analyticity (P#63), Cauchy-Riemann equations (P#64), Harmonic Function (P#65), Multi-valued Functions (P#76), Complex Integration (P#75), Cauchy's integral formula and its problems (P#68), Taylor and Laurent series (P#70,71), Contour integrals (P#86), Singularities and Residue theorem and its applications (P#71, 74).

Boundary Value Problem: Boundary value problems in Physics (P#214), The Sturm-Liouville Problems (P#225).

Group Theory: Introduction to group (P#256), Invariant Subgroup (P#257), Discrete groups (P#271), Continuous group (P#271), $GL(n)$, $SU(2)$ (P#266), $SU(3)$ (P#267), O-group's $O(2)$ group (P#268).

Course Title: Mathematical Methods of Physics-II **Course Code:** Phys-3116 **Semester-5**

Fourier series and Transforms: Fourier series and its complex form (P#171, 174), Applications of Fourier series (P#176), Representations of a function, Fourier integral theorem (P#179), Fourier transforms (P#181), Fourier Sine and Cosine transforms (P#183, 184), Applications of Fourier transform, Laplace transforms (P#188), Application of Laplace Transform (P#195).

Tensor Analysis: Cartesian Tensors, Coordinate Transformation (P#237), Rank of a Tensor (P#239), Tensor Algebra (P#242), Quotient Theorem (P#244), Tensor Density, Covariant and contravariant tensor (P#238).

Green's Function: Definition of Green's functions (P#213), Problems of Green's Function (P#214), Green's Functions in Electrodynamics.

Special Functions-II: Bessel's Differential Equation (P#136), Solution of Bessel's Differential Equation (P#136), Bessel's Functions (P#136), Neumann functions (P#140), Hermite Differential Equation (P#144), Solution of Hermite differential Equation (P#145), Hermite Polynomials (P#145).

Higher Education Commission (HEC), Islamabad

Course Title: Mathematical Methods of Physics-I **Course Code:** Phys-352 **Semester-5**

Partial Differential Equations: Introduction to important PDEs in Physics (P#115) (wave equation (P#117), diffusion equation (P#116), Poisson's equation (P#116), Schrodinger's equation (P#116)), General form of solution (P#101), General and particular solutions (first order (P#101), Inhomogeneous second order (P#108)), Characteristics and existence of solutions, Uniqueness of solutions, Separation of variables in Cartesian coordinates (P#102), Superposition of separated solutions, Separation of variables in curvilinear coordinates (P#102), Integral transform methods, Green's functions (P#213).

Complex Analysis: Review (polar form of complex numbers and de Moivre's theorem (P#61), Complex logarithms and powers) (P#85), Functions of a complex variable (P#62), Cauchy-Riemann conditions (P#64), Power series in a complex variable and analytic continuation with examples (P#70), Multi-valued functions and branch cuts (P#76), Singularities and zeroes of complex functions (P#71, 72), Complex integration (P#75), Cauchy's theorem (P#67),

Cauchy's integral formula (P#68), Laurent series and residues (P#71, 73), Residue integration theorem (P#74), Definite integrals using contour integration (P#86).

Course Title: Mathematical Methods of Physics-II **Course Code:** Phys-352 **Semester-6**

Group Theory and Representations for Finite Groups: Transformations, Groups definitions and examples (P#256), subgroups and Cayley's theorem (P#256, 262), Cosets and Lagrange's theorem (P# 263, 265), Conjugate classes, Invariant subgroups (P# 257), factor groups, Homomorphism (P#259), direct products (P#244), Mappings, Linear operators, matrix representations (P#35), Similarity transformation and equivalent matrix representations, Group representations (P#260), Equivalent representations and characters (P#261), Construction of representations and addition of representations, Invariance of functions and operators, unitary spaces and Hermitian matrices, Operators: Adjoint, Self-adjoint (P#37), Unitary (P#39), Hilbert space, Reducibility of representations, Schur's lemmas, Orthogonality relations, Group algebra, Expansion of functions in basis of irreducible representations (P#261), Kronecker product, Symmetrized and anti-symmetrized representations, Adjoint and complex-conjugate representations, real representations, Clebsch-Gordan series and coefficients, Applications of these ideas to classification of spectral terms, Perturbation theory and coupled systems.

Tensor Analysis: Vector calculus (differentiation (P#8), integration (P#9), Space curves, Multi-variable vectors, Surfaces, Scalar and vector fields (P#1), Gradient (P#11), Divergence and curl (P#12, 13), Cylindrical and spherical coordinates (P#21, 22), General curvilinear coordinates) (P#20), Change of basis, Cartesian tensor as a geometrical object, Order/rank of a tensor (P#239), Tensor algebra (P#242), Quotient law (P# 244), pseudotensors, Kronecker delta and Levi cevita (P#240, 242), Dual tensors, Physical applications, Integral theorems for tensors (P#245), non-Cartesian tensors, General coordinate transformations and tensors (P#237), relative tensors, Christoffel symbols, Covariant differentiation, Vector operators in tensor form, Absolute derivatives along curves, Geodesics.



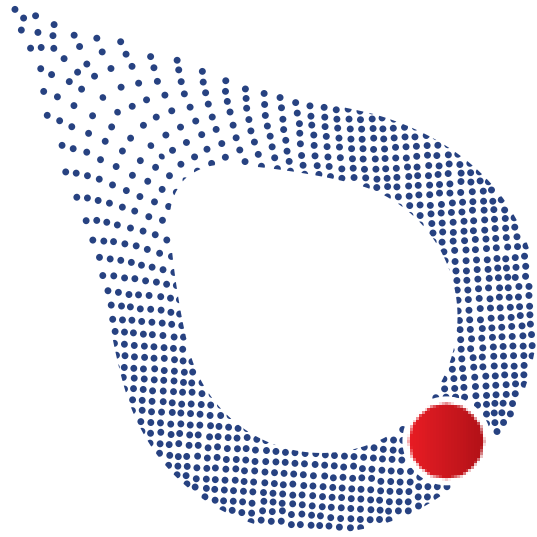
1	Vector Analysis	01
1.1	Scalar and Vector	01
1.2	Scalar Product	05
1.3	Vector Product	06
1.4	Triple Product	07
1.5	Vector Differentiation.....	08
1.6	Vector Integration.....	09
1.7	The Gradient	11
1.8	The Divergence	12
1.9	The Curl of a Vector.....	13
1.10	Curl and Divergence.....	14
1.11	Divergence Theorem.....	15
1.12	Stokes's Theorem.....	17
1.13	Green's Theorem in a Plane.....	19
1.14	Curvilinear Coordinates	20
1.14.1	Cylindrical Coordinates	21
1.14.2	Spherical Polar Coordinates.....	22
1.15	Short Questions with Answers.....	25
1.16	Solved Problems.....	29
1.17	Multiple Choice Questions (MCQs).....	33
2	Matrix Algebra	35
2.1	Matrix	35
2.2	Jacobi Identity	39
2.3	Successive Rotation	39
2.4	Symmetry Properties	40
2.5	Properties of Matrix	40
2.6	Orthogonal Transpose Matrix.....	40
2.7	Pauli and Dirac Matrix.....	41
2.8	Eigenvectors and Eigenvalues.....	44
2.9	Diagonalization of Matrix	46
2.10	Inertia Matrix.....	47
2.11	Short Questions with Answers.....	49
2.12	Solved Problems.....	51
2.13	Multiple Choice Questions (MCQs)	59
3	Complex Variables and Infinite Series	61
2.1	Complex Variables.....	61
2.2	De Moivre's Theorem	62
2.3	Analytic Function.....	63

3.4	Harmonic Functions.....	65
3.5	Elementary Function	65
3.6	Cauchy's Integral Theorem.....	67
3.7	Cauchy's Integral Formula.....	68
3.8	Taylor's Expansion.....	70
3.9	Laurent Series.....	71
3.10	Singularity.....	71
3.11	Residue.....	77
3.12	Cauchy Residue Theorem	74
3.12	Evaluation of Complex Integral.....	75
3.13	Integral of Type $\int \int f(z) dz$	75
3.14	Multivalued Functions and Branch Points.....	76
3.15	Dispersion Relations.....	77
3.16	Method of Steepest Descents.....	82
3.17	Complex Logarithms and Powers.....	85
3.18	Contour Integrals.....	86
3.19	Short Questions with Answers.....	89
3.20	Solved Problems.....	91
3.21	Multiple Choice Questions(MCQs).....	97
4	Differential Equations.....	99
4.1	Differential Equation.....	99
4.1.1	Classification of Differential Equations	100
4.2	First Order Linear Differential Equation.....	101
4.2.1	Separation of Variables.....	102
4.2.2	Homogeneous Differential Equations.....	103
4.2.3	Exact Differential Equation	103
4.3	Equations Reducible to the Linear Form (Bernoulli's Equation).....	104
4.4	Equations Of First Order And Higher Degree	105
4.5	Equations Which do not Contain y Directly	106
4.6	Equation Which do not Contain x Directly.....	107
4.7	Linear Second Order Differential Equations	108
4.8	Fuchs Theorem	109
4.8.1	Ordinary and Singular Points	110
4.9	Power Series Solution (About Ordinary Points) ...	110
4.10	Frobenius Method (About Regular Singular Point)	111
4.11	Linear Classical Oscillator Equation.....	112
4.12	Linear Independence Solution.....	114
4.13	Wronskian.....	115
4.14	Partial Differential Equation in Physics	115
4.14.1	Laplace Equation in Different Coordinate Systems	116
4.14.2	Wave Equation in Different Coordinate Systems ..	117
4.15	Short Questions with Answers.....	119
4.16	Solved Problems	121
4.17	Multiple Choice Questions (MCQs).....	125
5	Special Functions.....	127
5.1	Legendre's Differential Equation and its Solution.....	127
5.1.1	Generating Function for $P_n(x)$	130
5.1.2	Recurrence Formula for $P_n(x)$	131
5.1.3	Orthogonality of $P_n(x)$	134
5.1.4	Associated Legendre Polynomials	135
5.2	Bessel Function.....	136
5.2.1	Generating Function for the Bessel Function $J_n(x)$	138
5.2.2	Recurrence Relations for Bessel Function $J_n(x)$..	138
5.2.3	Bessel Function of Second Type.....	140
5.2.4	Trigonometric Expansion Involving Bessel Functions.....	141
5.2.5	Integral Representation of Bessel Functions..	141
5.2.6	Spherical Bessel Functions.....	143
5.3	Hermite Differential Equation.....	144
5.3.1	Generating Function for Hermite Function $H_n(x)$	145

5.3.2	Recurrence Relations for $H_n(x)$	146	7.7	Short Questions with Answers	228
5.3.3	Orthogonality of $H_n(x)$	146	7.8	Solved Problems.....	230
5.4	Laguerre Function	148	7.9	Multiple Choice Questions (MCQs).....	234
5.4.1	Generating Function for $L_n(x)$	149	8	Tensor Analysis	236
5.4.2	Recurrence Relations for $L_n(x)$	150	8.1	Scalar, Vector and Dyadic	236
5.4.3	Orthogonality of $L_n(x)$	151	8.2	Coordinate Transformations.....	237
5.4.4	Associated Laguerre Polynomials	152	8.3	Summation Convention.....	238
5.5	Gamma Function (Γ).....	153	8.4	Contravariant and Covariant Tensors	238
5.5.1	Di-Gamma and Poly-Gamma Functions.....	155	8.5	Order and Rank of a Tensor.....	239
5.5.2	Transformation of Gamma Function	155	8.6	Kronecker Delta	240
5.6	Stirling's Series.....	156	8.7	Symmetric and Anti-symmetric Tensor.....	241
5.7	Beta Function.....	157	8.8	Levi-Civita Symbol/Permutation Symbol/Epsilon Tensor	242
5.7.1	Transformation of Beta Function	158	8.9	Relation between Levi-Civita and Kronecker Tensor	242
5.8	Short Questions with Answers	161	8.10	Fundamental Operation with Tensors.....	242
5.9	Solved Problems	164	8.10.1	Addition	243
5.10	Multiple Choice Questions (MCQs)	169	8.10.2	Subtraction.....	243
6	Fourier Series and Integral Transformation.....	171	8.10.3	Contraction.....	243
6.1	Fourier Series.....	171	8.10.4	Multiplication.....	244
6.2	Fourier Cosine Series and Fourier Sine Series ...	173	8.10.5	Direct Product.....	244
6.3	Exponential (Complex) Form of Fourier Series ...	174	8.10.6	Inner Product	244
6.4	Change of Interval of Fourier Series.....	175	8.10.7	Quotient Law	244
6.5	Applications of Fourier Series.....	176	8.11	Integral Theorems in Tensor Form.....	245
6.6	Fourier Integrals	179	8.11.1	Gauss's Divergence Theorem	245
6.7	Fourier Transformation.....	181	8.11.2	Stokes's Theorem.....	246
6.7.1	Shifting of Origin.....	182	8.12	Short Questions with Answers.....	247
6.8	Convolution Theorem.....	182	8.13	Solved Problems.....	254
6.9	Fourier Cosine Transform.....	183	8.14	Multiple Choice Questions (MCQs).....	256
6.10	Fourier Sine Transform.....	184	9	Group Theory.....	256
6.11	Fourier Transform of Derivatives	185	9.1	Groups and its Types	256
6.12	Fourier Cosine Transform of Derivatives.....	186	9.2	Lie Group.....	258
6.13	Fourier Sine Transform of Derivatives	187	9.3	Isomorphism and Homomorphism	259
6.14	Dirac Delta Function.....	188	9.4	Representation of a Group.....	260
6.15	Laplace Transforms.....	188	9.4.1	Representation Through Similarity Transformations	260
6.15.1	Properties of Laplace Transform.....	190	9.4.2	Equivalent Representations.....	261
6.16	Laplace Transform of Derivatives.....	292	9.4.3	Reducible and Irreducible Representations ...	261
6.17	Inverse Laplace Transform	193	9.5	Cayley's Theorem.....	262
6.18	Partial Fraction Expansion.....	195	9.6	Cosets and Lagrange's Theorem	236
6.19	Applications of Laplace Transform	195	9.7	Special Unitary Group.....	265
6.19.1	Simple Harmonic Motion.....	195	9.7.1	SU(2) Group.....	266
6.19.2	Simultaneous Differential Equations.....	197	9.7.2	SU(3) Group.....	267
6.19.3	Motion of A Body.....	197	9.8	Special Orthogonal Group	268
6.19.4	Step Function.....	198	9.8.1	SO(2) Group.....	268
6.20	Short Questions with Answers	199	9.8.2	SO(3) Group.....	270
6.21	Solved Problems.....	203	9.9	Continuous and Discrete Groups.....	271
6.21	Multiple Choice Questions (MCQs)	209	9.10	Dihedral Group.....	273
7	Green's Functions and Initial/Boundary Value Problems 211		9.11	Short Questions with Answers.....	275
7.1	The Dirac Delta Function.....	211	9.12	Solved Problems	277
7.2	Green's Function	213	9.13	Multiple Choice Question (MCQs)	281
7.3	Green's Functions for Initial/Boundary Value Problems	214			
7.3.1	Green's Function Associated with Initial Value Problem	215			
7.3.2	Green's Function for Boundary Value Problem	218			
7.4	Solution of BVPs with Inhomogeneous B.Cs.....	219			
7.4.1	Solution of SL System with Homogeneous B.Cs.	219			
7.4.2	Non-homogeneous Boundary Conditions.....	221			
7.5	Method of Eigenfunction Expansion for Green's Functions.....	222			
7.6	Sturm-Liouville Theory.....	225			

Books by Quanta Publisher for BS Physics Students

<p>MECHANICS For BS Physics Students 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems, Subst Examples, and Multiple Choice Questions (MCQs)</p> <p>0313-7899577 Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>WAVES AND OSCILLATIONS For BS Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Muhammad Tahir Jamil Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>HEAT AND THERMODYNAMICS For BS Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Anwar Masood Bano Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>MODERN PHYSICS & ELECTRONICS For BS Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Anwar Masood Bano Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>ELECTRICITY & MAGNETISM For BS Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Muhammad Tahir Jamil Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>
<p>ATOMIC AND MOLECULAR PHYSICS For BS Physics Students 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems, Subst Examples, and Multiple Choice Questions (MCQs)</p> <p>0313-7899577 Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari Dr. Umar Nisar</p>	<p>CLASSICAL MECHANICS For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Anwar Masood Bano Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>THERMAL & STATISTICAL PHYSICS For BS Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems, Subst Examples, and Multiple Choice Questions (MCQs)</p> <p>0313-7899577 Dr. Anwar Masood Bano Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>NUCLEAR PHYSICS For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Anwar Masood Bano Dr. Syed Hamid Bukhari Zohab Akram Khan</p>	<p>OPTICS For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Muhammad Tahir Jamil Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>
<p>QUANTUM MECHANICS For BS Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems, Subst Examples, and Multiple Choice Questions (MCQs)</p> <p>0313-7899577 Dr. M. Arif Khalil Dr. Syed Hamid Bukhari Hafiz Umar Farooq</p>	<p>METHODS OF MATHEMATICAL PHYSICS For BS Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems, Subst Examples, and Multiple Choice Questions (MCQs)</p> <p>0313-7899577 Dr. Abdul Shakoor Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>COMPUTATIONAL PHYSICS For BS/M.Sc. Physics Programme Solutions of Physics Problems with Matlab & C++</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Anwar Masood Bano Dr. Syed Hamid Bukhari Anwar Masood Bano</p>	<p>ELECTROMAGNETIC THEORY-I ELECTRODYNAMICS-I For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>ELECTROMAGNETIC THEORY-II ELECTRODYNAMICS-II For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>
<p>SOLID STATE PHYSICS-I For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Malik Razi Dr. M. Ehsan Raheer Dr. Syed Hamid Bukhari</p>	<p>LASER For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Zohab Akram Khan Muhammad Abbas Dr. Syed Hamid Bukhari</p>	<p>PLASMA PHYSICS For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Shahid Ahmad Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>APPLIED PHYSICS For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari Dr. Syed Hamid Bukhari</p>	<p>DIGITAL ELECTRONICS For BS/M.Sc. Physics Programme 2nd Edition</p> <p>Book Includes: Short Questions with Answers, Subst Problems and MCQs are available at our website www.quanta-publisher.com</p> <p>0313-7899577 Dr. M. Noman Usman Hafiz Umar Farooq Dr. Syed Hamid Bukhari</p>
<p>PHYSICS (MECHANICS) Laboratory Manual For BS Physics/Math/Chemistry Students</p> <p>Dr. Muhammad Anjar Feroq Dr. Syed Hamid Bukhari</p>	<p>PHYSICS (ELECTRICITY & MAGNETISM) Laboratory Manual For BS Physics/Math/Chemistry Students</p> <p>Dr. Muhammad Anjar Feroq Dr. Syed Hamid Bukhari</p>	<p>MODERN PHYSICS Laboratory Manual For BS Physics/Math/Chemistry Students</p> <p>0313-7899577 Dr. M. Anjar Feroq Dr. Syed Hamid Bukhari Muhammad Abbas</p>	<p>PHYSICS SPECTROSCOPY & MODERN PHYSICS Laboratory Manual For BS/M.Sc. Physics Programme</p> <p>Dr. Abdul Shakoor Dr. Syed Hamid Bukhari</p>	<p>ANALOG ELECTRONICS Laboratory Manual For BS/M.Sc. Physics Students</p> <p>Muhammad Salman Dr. Syed Hamid Bukhari</p>



Quanta
PUBLISHER

BS Physics



0313-7899577



www.quantapublisher.com



Quanta Publisher - Physics



@Quanta Publisher



@Quanta Publisher