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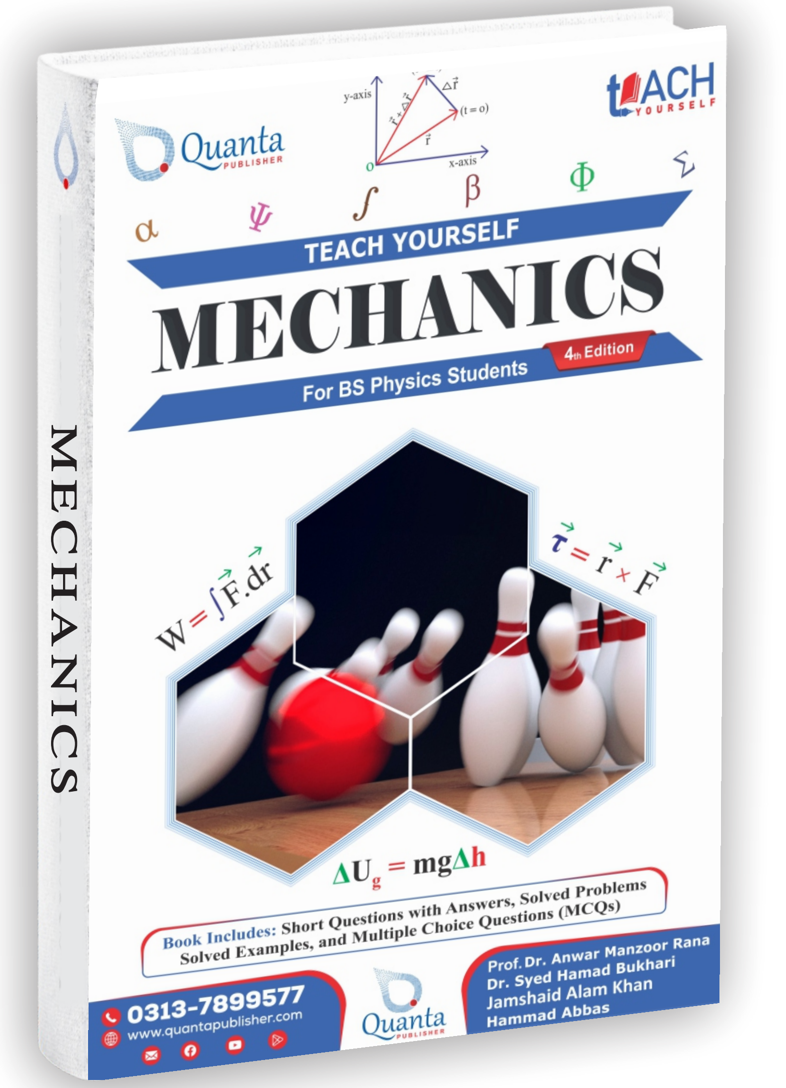

TEACH YOURSELF

MECHANICS

4th Edition

For **BS/ADS, Physics/Mathematics & Engineering** Students
Book followed by Halliday, Resnick, Krane (HRK)

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Chapter 1

Vector Analysis

IN physics, engineering and medical science, precise measurement relies on understanding units and dimensions. The international system of units (SI) provides a standardized framework, and scientific notation aids in handling vast or microscopic values. Changing units involves conversion techniques. Scalars have magnitude, while vectors possess both magnitude and direction. Vector and scalar triple products follow vector algebra rules. Spherical and cylindrical polar coordinates offer alternative systems for spatial representation. Circular motion involves rotating objects. Integrals (line, surface, volume) and operators like del are fundamental in calculus. Gauss's divergence theorem and Stoke's theorem link flux and circulation, while curl and vector identities further extend vector calculus applications.

1.1 Introduction to Mechanics

Mechanics

Mechanics is the branch of physics which deals with the effect of forces on the motion of a body. **It is the study of rest/motion of an object, its causes and effects**. Historically, mechanics is the first branch of physics, which was developed as an exact science.

Types of Mechanics: Mechanics can be divided into three main classes:

i - Classical Mechanics

Applied to those bodies whose speed remains small in comparison with the speed of light. It is developed on the basis of work done by Aristotle (322-384 BC), Galileo (1564-1642), Kepler (1571-1630), Newton (1642-1727), Lagrange (1736-1813), Hamilton (1805-1865) and Mach (1839-1916).

ii - Quantum Mechanics

Quantum mechanics is applied to the physical systems of molecular size or even smaller i.e. for the description of small state phenomena of atomic and nuclear physics. It is based upon the work of Heisenberg and Schrödinger in 1925-1926.

iii - Relativistic Mechanics

Applied to those bodies whose speeds are comparable to the speed of light. It begins with the work of Einstein in 1905, the theory of relativity.

Chapter 2

Particle Dynamics

WELCOME to the fascinating realm of mechanics, where we explore the fundamental principles governing motion and forces. In this comprehensive review, we delve into the dynamics of motion, ranging from the various types of forces acting upon objects to the complexities of frictional forces. Resolve the complications of uniform circular motion and the pivotal role of centripetal force, followed by an exploration of the rotor, banked curves, and the doubtful conical pendulum. Engage with equations characterizing motion under constant force, analyze time-dependent forces, and examine the profound impact of drag forces on motion. Journey through projectile motion, navigate non-inertial frames, and hold with the concept of pseudo forces. Join us in this enlightening journey through the physics of motion and forces. Mechanics of single particle is described by different types of motion like linear, circular, rotational and vibrational motion. In this chapter, we will study the Newton's laws and their applications. We will study frictional forces and their consequences. For variable forces we will solve the equations of motion for such forces. Finally, we will show how using inertial and non-inertial reference frames produce effects that can be analyzed by introducing inertial forces or pseudo-forces that are not caused by specific objects in the mechanical system.

2.1 Review of Motion

Question **Define and explain the position, displacement, velocity and acceleration.**

Position

The position of a particle at a particular time on an x-axis is the location of the particle with respect to some reference point called the **origin**, or zero point of the axis.

Displacement

The change in position of an object from initial to final points or the shortest distance between two points. The displacement $\Delta\vec{r}$ of a particle is the change in its position along a line.

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1} \quad (2.1)$$

Displacement is a vector quantity and can be either positive or negative, depending on the choice of positive direction. Whereas the magnitude of the displacement is always positive.

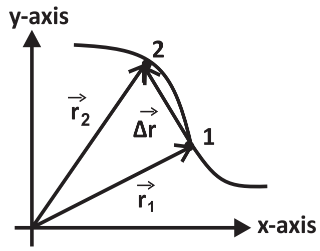

Fig. 2.1. Schematic picture showing displacement between two points.

Chapter 3

Work, Power and Energy

IN the realm of mechanics, several fundamental concepts govern the interactions and dynamics of physical systems. The concept of work, both by constant and variable forces, is pivotal in understanding how energy is transferred or transformed within a system. The work-energy theorem establishes a connection between work and the resulting kinetic energy. Power, a measure of the rate at which work is done, further illuminates the dynamics of energy transfer. Conservation of energy is a fundamental principle, asserting that the total energy within an isolated system remains constant. Specific applications, such as the force of gravity, potential energy, and the gravitational force, enrich our understanding of energy dynamics in diverse scenarios. The conservation of energy in systems of particles extends these principles to complex interactions within multi-particle systems. In particle dynamics, we solve for particle motion under varying forces using integration techniques. Forces depending on particle position, such as gravitational force and spring force, lead to the work-energy theorem. Work done by conservative forces depends only on the initial and final positions, not the path. The law of conservation of energy ensures total energy in mechanical systems remains constant.

3.1 Work Done by the Constant Force

Question **Define and explain work done by the constant force, discuss its cases.**

Consider, a constant force $\vec{F}$ acts on a body and displaces it through a distance x in its own direction. Then, the work done is defined as the product of magnitude of force and displacement as:

$$\text{Work } = W = |\vec{F}||\Delta\vec{x}| = F\Delta x$$

However, if the force makes an angle θ with the direction of motion of the body, then work is defined as **the product of component of force along the line of motion and the magnitude of displacement**.

$$W = (F\cos\theta)\Delta x = F\Delta x\cos\theta = \vec{F}\cdot\Delta\vec{x}$$

In this case, work is also defined as the product of magnitude of force and the component of displacement along the direction of force. Thus,

$$W = F(\Delta x\cos\theta) = F\Delta x\cos\theta = \vec{F}\cdot\Delta\vec{x}$$

So, work is defined as the scalar product of force and displacement. The work done is a scalar quantity and its unit is Joule. **Work is one Joule when force of one Newton acts on a body and displaces it through a distance of** 1 **meter in its own direction (1J=1Nm)**.

Chapter 4

Systems of Particles

EMBARK on a fascinating exploration of particle systems and the interesting concept of the center of mass in solid objects. This comprehensive review reveal the dynamics of momentum changes within systems of variable mass, smoothing the principles that govern their behavior. As we examine deeper, witness the practical application of these principles in the fascinating realm of rocket motion. From understanding particle systems to unraveling the secret of momentum in variable mass scenarios, join us on a journey through the complexity of physics, where theoretical knowledge seamlessly converges with real-world applications.

A continuous system of particles is that in which the separation of particles is very small such that it approaches zero. An extended object is a continuous system of particles. When an object rotates as it moves or when its parts vibrate relative to one another, it would not be valid to treat the entire object as a single particle. There is one point of object whose motion under the influence of external forces can be analyzed as that of single particle, is called center of mass. The center of mass is a position defined relative to an object or system of objects. It is the average position of all the parts of the system, weighted according to their masses. For simple rigid objects with uniform density, the center of mass is located at the centroid. In this chapter, we describe how to find center of mass of different objects. Moreover, we discuss about linear momentum, conservation ofl inear momentum and its applications including rocket motion.

4.1 Systems of Particles

Question **Define and explain particle systems. Also, explain the two particle and many particle system. Calculate the velocity and acceleration for that systems.**

Two Particle System

Consider, a system of two particles of masses m_1 and m_2 are separated by distance x. The center of mass is a point where all the mass of a system is concentrated and total applied external forces act at this point. It moves with constant velocity $v_{c.m}$ in the absence of net external force. From Fig.(4.1), x_1 and x_2 are the positions of m_1 and m_2 relative to origin and if $x_{c.m}$ is position of center of mass relative to origin then;

Distance or moment arm of m_1 relative to c.m $= \vec{x}_{c.m} - \vec{x}_1$

Distance or moment arm of m_2 relative to c.m $= \vec{x}_2 - \vec{x}_{c.m}$

Chapter 5

Collisions

IMPULSE and momentum are fundamental concepts in physics that describe the motion of objects. Momentum, the product of an object's mass and velocity, is conserved in isolated systems. Collisions, interactions between objects, are classified into various types, including elastic collisions in one and two dimensions, where kinetic energy is conserved. Oblique collisions involve two objects moving in different directions. Inelastic collisions result in kinetic energy loss, and two-dimensional inelastic collisions exhibit complex dynamics. Analyzing collisions in the center of mass reference frame simplifies calculations, providing a valuable perspective in understanding the outcomes of physical interactions. In this chapter, we develop and define another conserved quantity, called linear momentum, and another relationship (the impulse-momentum theorem), which will put an additional constraint on how a system evolves in time. Conservation of momentum is useful for understanding collisions.

5.1 Impulse and Momentum

Question **Give the basic idea related to momentum and impulse of the force.**

Impulse

Impulse in physics is a term that is used to describe or quantify the effect of force acting over time to change the momentum of an object. It is represented by the symbol I and usually expressed in Newton-seconds or kg ms^{-1}. Impulse is often stated to be the product of the average net force F that acts on an object for a certain duration Δt. The equation for impulse is given as

$$\vec{I} = \vec{F}\Delta t$$

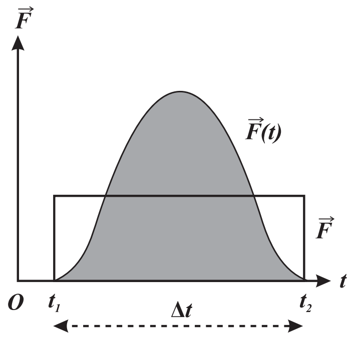

Fig. 5.1. Impulsive force varying with time.

Note: We assume that force is constant over time. Impulse is a vector quantity like force and it also has direction.

Impulsive Forces

An impulsive force is a force that acts for a short duration of time on an object. An impulsive force is mainly generated in a collision that results in a change in velocity or momentum of one or all objects involved in the collision.

Consider, a collision in which the impulsive force $\vec{F}(t)$ is applied on the body. The magnitude of the forces is shown in the Fig.(5.1). The collision begins at time t_i and ends at time t_f. The force

Chapter 6

Gravitation

GRAVITATION, a force everywhere in the universe, underlies the attraction between masses, shaping celestial dynamics. The universal gravitational law, formulated by Newton, quantifies this force's universality and strength. Henry Cavendish's groundbreaking experiment measured the gravitational constant (G), pivotal for understanding gravitational interactions. Exploring the gravitational effect of a spherical mass distribution unveils insights into celestial bodies' structural influence. Gravitational potential energy describes the energy associated with an object's position in a gravitational field. Calculating escape velocity reveals the minimum speed required for an object to overcome gravitational pull, a crucial concept in space exploration. Understanding the gravitational field and potential further refines our comprehension of gravitational interactions. Distinguishing radial and transversal components of velocity and acceleration explain the complex nature of celestial motion. This insight is particularly relevant in analyzing the trajectories of satellites, whose motion is governed by gravitational forces and Kepler's laws. Similar principles extend to planetary motion, where energy considerations play a key role in understanding the intricate dance of celestial bodies within our solar system. The application of the universal law extends even beyond, shaping the dynamics of galaxies on a grand scale. In this vast cosmic ballet, gravitational forces dictate the motion of celestial objects, providing a cohesive framework to comprehend the wonders of the universe.

6.1 Gravitation and Universal Gravitation Law

Question **Define gravitation. Also, define and explain the universal law of gravitation by using the Newton's idea.**

Gravitation

The property of a massive particle or big body to attract every other particle is known as gravitation Gravitational property of earth is known as gravity. Every body in universe possesses two properties inherently due to its mass:

- Inertia
- Gravitation

Newton got the idea of gravitation by fall of an apple and applied this property of gravitation to the motion of earth and heavenly bodies. There are three overlapping realm into which we can discuss gravitation:

- The gravitational attraction between two bowling balls.

Chapter 7

Rotational Dynamics

EMBARK on a captivating journey through the principles of rotational dynamics, where we explore the intricate relationship between linear and angular variables. Unveil the secrets of kinetic energy within rigid bodies and delve into the parallel axis theorem and perpendicular axis, unraveling their profound implications. Navigate the complexities of rotational dynamics for rigid bodies and witness the seamless integration of translational and rotational motion. Finally, immerse yourself in the fascinating world of rolling without slipping. Join us in this exploration, as we unravel the dynamic interplay between motion, energy, and the fascinating mechanics of rotating systems.

In this chapter, we shall consider the causes of rotation which forms a subject called **rotational dynamics**. When a force is applied at a certain location to a rigid body, it rotates about any particular axis, the resulting motion depends upon the magnitude and location of the application of the force. From experiments, we know that a given force applied to a body at one location may produce a different rotation at some other location.

The quantity which takes into account both the magnitude of the force applied, location and the direction in which it is applied is called torque. **It is also defined as the turning effect of force produced in a body about an axis**. Just as we regard force as push or pull, we can regard torque as twist.

7.1 Relationship Between Linear and Angular Variables

Question **What is torque? Discuss the relationship between linear and angular variables in scalar and vector forms. Also, discuss about rotational inertia / moment of inertia.**

We also know from experiments that the effect required to put a body into rotation depends on the distribution of mass in the body. The closer the mass to the axis of rotation, the easier it is for the force to rotate the body about that axis. This inertial quantity that takes into account the distribution of mass of the body around the axis of rotation is called rotational inertia and is an intrinsic property of everything that has mass and defined body or shape like mass. The equation for rotational dynamics is:

$$\text{Torque} = \tau = \text{Moment of inertia} \times \text{Angular acceleration} = I\alpha$$

Chapter 8

Angular Momentum

ANGULAR momentum is a fundamental concept in physics, describing the rotational motion of an object. It is defined as the product of an object's moment of inertia and its angular velocity. The relationship between torque and angular momentum is crucial; torque acting on an object causes changes in its angular momentum. The law of conservation of angular momentum states that in the absence of external torques, the total angular momentum of a system remains constant. This principle underlies the stability of spinning objects and phenomena like precessional motion, as seen in the mesmerizing motion of a spinning top.

8.1 Angular Momentum of a Particle

Question **Differentiate between angular velocity and angular acceleration. Write the equations of angular motion. Also, discuss the angular momentum of a particle.**

Angular momentum is a quantity that plays an important role in rotational dynamics. Like the law of conservation of linear momentum, the law of conservation of angular momentum is a fundamental law of physics, equally valid for relativistic and quantum systems. Just as the idea of linear momentum helps us to analyze translational motion, angular momentum helps us to analyze the rotational motion. The resultant torque acting on a system about an axis through the center of mass equals the time rate of change of angular momentum of the system regardless of the motion of the center of mass. This fact applies even if the center of mass is accelerating, provided (τ) and (L) are evaluated, relative to the center of mass. The net external torque acting on a rigid object, rotating about a fixed axis equals the moment of inertia about the rotational axis multiplied by the object's angular acceleration relative to that axis.

In general, the expression $L = I\omega$ is not always valid. If a rigid object rotates about an arbitrary axis L and ω may points in different directions. In this case, the moment of inertia can't be treated as a scalar. Strictly speaking, $L = I\omega$ applies only to rigid objects of any shape that rotate about one of three mutually perpendicular axes (principal axes) through the center of mass.

Angular Displacement

Angular displacement is defined as **the angle in radians (degrees, revolutions) through which a point or line has been rotated in a specified sense about a specified axis.**

Chapter 9

Bulk Properties of Matters

THE exploration of matter involves delving into its various character, including bulk properties and elastic behavior. Bulk properties encircle characteristics like density and volume, crucial for understanding material interactions. Elastic properties, which include tension, compression, and shearing, delve into how substances deform under stress. Elastic modules like Young's modulus and Poisson ratio establish vital relationships, shaping our comprehension of material behavior. Surface tension, a force at liquid interfaces, plays a pivotal role in forming droplets and bubbles. Additionally, fluid flow through cylindrical pipes, described by Poiseuille's law, contributes to our knowledge offl uid dynamics in practical applications.
Matter is divided into three states: solid, liquid and gas. A solid has a definite volume and shape, a liquid has definite volume but no definite shape and a gas has neither definite volume nor definite shape. A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and forces exerted by the walls of a container. Both liquids and gases are fluids. Fluid dynamics deals with the aspects offl uid flow. Note that the extent to which fluids yield to shearing forces depends on a quantity called the viscosity which is the main topic of this chapter. We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up the matter in three phases. The physics offl uids is the basis of hydraulic engineering, a branch of engineering that is applied in many fields.

9.1 Elastic Properties of Matter

Question **Define and explain the elastic properties of matter and discuss its types. Also, write the physical basis of elasticity.**

When an external force acts on a body, relative displacement ofi ts various parts takes place. The displaced particles tend to come to their original position to restore the original length, volume or shape of body and thus exert a restoring force. All bodies can more or less, be deformed by applying forces on them. In the simplest case of deformation, an equal compression applied in all directions to a body may produce a change of volume but no change in shape or the forces applied may cause a change of shape keeping its volume constant. Under these conditions, the body is said to be constrained.

Chapter 10

Special Theory of Relativity

THE special theory of relativity, formulated by Albert Einstein, revolutionized our understanding of space and time. Its postulates challenge classical notions, asserting that the laws of physics are the same for all observers in uniform motion. The Lorentz transformation equations mathematically describe the relationship between different observers' coordinates. Consequences of this transformation include time dilation, length contraction, and mass increase. The Doppler effect in relativistic scenarios impacts observed frequencies. The twin paradox explores time dilation in a fascinating context. Transformation of velocity, mass-energy equivalence, relativistic momentum, and Lorentz invariance in energy further illuminate the profound implications of Einstein's groundbreaking theory.

In this chapter, we are concerned with only special theory of relativity. But before describing it, it is pertinent to first discuss transformation equations of coordinates and velocities given by Galileo for inertial frames of references. Only, then we will be able to appreciate the transformation equation given by Lorentz, which are in agreement with postulates of special theory of relativity.

10.1 Frame of Reference, Twin and Einstein Paradox

Question **Differentiate between special and general theory of relativity. What is frame of reference, discuss about inertial and non-inertial frame of references. Also, explain twin paradox and Einstein paradox.**

In equations of motion with constant acceleration, developed by Galileo and laws of motion given by Newton. Mass, length and time (which are base quantities of mechanics) are considered as invariants i.e., they have same values for all observers irrespective of their state of motion. Einstein argued that when objects are not moving with very high velocities comparable to velocity ofl ight or relative to each other. What happen when objects start to move with velocity close to velocity ofl ight? In this regard, he presented two theories:

1- Special theory of relativity, which deals with inertial frames of reference.

2- General theory of relativity, which deals with non-inertial frame of reference

Chapter 11

Simple Harmonic Motion (SHM)

EACH day we encounter many kinds of oscillatory motion. Common examples include the swinging pendulum of a clock and vibrating guitar string. Examples on the microscopic scale are vibrating atoms in quartz crystal of a wristwatch and vibrating molecules of air that transmit sound waves. The above cases are **mechanical oscillations**.

We are also familiar with **electromagnetic oscillations**, such as electrons moving back and forth in circuits that are responsible for transmitting and receiving radio or TV signals. One common feature of all these systems is the mathematical formulation used to describe their oscillations. In all cases, the oscillating quantity, whether it is the displacement of a particle or the magnitude of an electric field, can be described in terms of sine or cosine functions. In this chapter, we concentrate on mechanical oscillations with their description and applications.

11.1 Simple Harmonic Motion (SHM)

Question **Define simple harmonic motion. Solution and calculation of equation of motion.**

The repetitive motion of a body in a specific interval of time about its mean position is called SHM. **To and fro motion of a body in which acceleration is directly proportional to the displacement and always directed towards the mean position is called simple harmonic motion.** The body executing simple harmonic motion is called simple harmonic oscillator. Motion of simple pendulum and motion of spring mass system are the examples of SHM.

Conditions of SHM

- **Restoring Force**: Necessary to move the object towards mean position. According to Hook's law, $F \propto -x$.
- **Inertia**: To move the object away from the mean position to extreme position.

Derivation of the Equation of SHM

Consider an oscillating system consisting of a particle of mass m attached to a spring of spring constant k (see Fig.(11.1)) that is free to move over a frictionless horizontal surface subjected to restoring force F such that:

$$F \propto -x \qquad \Rightarrow \qquad F = -kx \tag{11.1}$$

$$(\text{K.E.}_1)_{\text{rot.}} = \frac{1}{2} m_1 r_1^2 \omega_1^2$$

The rotational kinetic energy for particle of mass m_2 is:

$$(\text{K.E.}_2)_{\text{rot.}} = \frac{1}{2} m_2 r_2^2 \omega_2^2$$

The rotational kinetic energy for particle of mass m_n is:

$$(\text{K.E.}_n)_{\text{rot.}} = \frac{1}{2} m_n r_n^2 \omega_n^2$$

Now, the total rotational kinetic energy acting on the rigid body is described as:

$$\text{K.E.}_{\text{rot.}} = \text{K.E.}_1 + \text{K.E.}_2 + \cdots + \text{K.E.}_n = \frac{1}{2} m_1 r_1^2 \omega_1^2 + \frac{1}{2} m_2 r_2^2 \omega_2^2 + \cdots + \frac{1}{2} m_n r_n^2 \omega_n^2$$

Since the body is rigid, so all the masses will rotate with the same angular velocity ω i.e., $\omega = \omega_1 = \omega_2 = \cdots = \omega_n$, we have

$$\text{K.E.}_{\text{rot.}} = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2)\omega^2 = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2 \tag{7.11}$$

Where, $\left(\sum_{i=1}^{n} m_i r_i^2\right) = I$ is the moment of inertia of the rigid body.

Example 7.2.1 A solid cylinder of mass m and r roll without slipping down an inclined plane of lenght L and height h. Find the speed of its centre of mass when the cylinder reaches the bottom.

Solution:

Consider, a solid cylinder of mass M and radius r. The length of inclined plane is L and height h.

$$\text{Loss in P.E.} = \text{Gain in } (\text{K.E.}_t + \text{K.E.}_r) \quad \Rightarrow \quad mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

Moment of inertia of solid cylinder is $\frac{1}{2} m r^2$ and $\omega = \frac{v}{r}$, so

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} m r^2\right)\left(\frac{v^2}{r^2}\right) \quad \Rightarrow \quad m(gh) = m\left(\frac{v^2}{2} + \frac{v^2}{4}\right) \quad \Rightarrow \quad v = \sqrt{\frac{4gh}{3}}$$

7.3 Parallel Axis Theorem

Question **Define and proof the parallel axis theorem by using the concept of rigid body.**

Statement

The rotational inertial of any body about any axis is equal to the sum of rotational inertia about a parallel axis through the center of mass and the product of mass of body and square of distance between two axes. Mathematically, it is described as:

$$I = I_{\text{cm}} + M h^2$$

Where,

I = Rotational inertia about an arbitrary axis.

I_{cm} = Rotational inertia about the parallel axis through the center of mass.

M = Mass of body.

h = Perpendicular distance between the axes.

1.16 Short Questions with Answers

1.1 Can a scalar product of two vectors be negative? If your answer is yes, give an example. If not, give proof.

Answer: Yes, if vectors are oriented at an angle of 180°. Example: work done by forces of friction is negative.

1.2 A plane is moving at 100 km/h at an angle of 60° with ground. What is speed of its projection on ground if the sun is just above plane at noon?

Answer: It is given that projection of plane is moving on ground, say along x-axis, then we have to find x-component of its velocity:

$$V_x = v\cos\theta = 100\,\text{kmh}^{-1}\cos 60^\circ = 50\,\text{kmh}^{-1}$$

1.3 Consider a vector $\vec{A}$ with components A_x and $A_y < 0$. Find possible directions of vector $\vec{A}$.

Answer: The vector $\vec{A}$ lies in the third quadrant.

1.4 Under what conditions would a vector have components equal in magnitude?

Answer: A vector has equal rectangular components if it is inclined at 45° to coordinate axes.

1.5 Does the value of vector quantity depends upon selected coordinate system?

Answer: The value of vector quantity does not change with change of reference axes since direction of vector is specified by angle made with reference axis.

1.6 Whether gradient of a scalar field depends upon any particular system of coordinates?

Answer: No, gradient of a scalar field is entirely independent of any particular system of coordinates. Such quantities which are independent of coordinate system are called invariants.

1.7 Suppose $\vec{C} = \vec{A} + \vec{B}$, does it follow that either $C \geq A$ or $C \geq B$? If not, why? If yes prove it.

Answer: Suppose $\vec{C} = \vec{A} + \vec{B}$, then $C = A$ if $B = 0$ and $C = B$ if $A = 0$, otherwise $C > A$ or $C > B$.

1.8 Suppose $\vec{A}$ is non-zero vector. It is given that $\vec{A} \times \vec{B} = 0$ and $\vec{A} \cdot \vec{B} = 0$. What can you conclude about B.

Answer: From vectors, we know that

$$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = A^2B^2$$

Using given values:

$$0 = A^2B^2 \quad \Rightarrow \quad AB = 0$$

Since, $A \neq 0$, showing that B is a null vector.

1.9 If all components of vectors $\vec{A}$ and $\vec{B}$ are reversed, how will this affect $\vec{A} \times \vec{B}$?

Answer: By reversing vectors, we have

$$\vec{A} \times \vec{B} = -\vec{A} \times -\vec{B} = \vec{A} \times \vec{B}$$

This shows that vector product is unaltered if vectors are reversed.

1.10 A vector $\vec{A}$ lies in xy-plane. For what orientation will both rectangular components be negative? For what orientation will its components have positive signs?

Answer: Both rectangular components of $\vec{A}$ will be negative, when they lie in 3rd quadrant. Both rectangular components of $\vec{A}$ will be positive in sign, when they lie in 1st quadrant.

1.11 Scalar triple product is cyclic. Can you give such a statement for vector triple product?

Answer: No, vector triple product is not cyclic.

1.12 Can we say that cylindrical polar coordinates are orthogonal?

Answer: Yes, we can say that cylindrical polar coordinates are orthogonal.

1.13 What is the physical significance of derivative?

Answer: Consider, a curve given by equation:

1.17 Solved Problems

Problem 1.1. Show that vectors $\vec{A} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{B} = \hat{i} - \hat{j} - \hat{k}$ are perpendicular.

Solution

Two vectors are perpendicular if their dot product is zero. It is given that

$$\vec{A} = \hat{i} + 3\hat{j} - 2\hat{k} \quad \text{and} \quad \vec{B} = \hat{i} - \hat{j} - \hat{k}$$

Now, taking the dot product of $\vec{A}$ and $\vec{B}$, we get

$$\vec{A} \cdot \vec{B} = (\hat{i} + 3\hat{j} - 2\hat{k}) \cdot (\hat{i} - \hat{j} - \hat{k}) = 1(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) + 2(\hat{k} \cdot \hat{k}) = 1(1) - 3(1) + 2(1) = 1 - 3 + 2 = 3 - 3 = 0$$

Hence, $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Problem 1.2. If $\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$, find divergence and curl of vector $\vec{A}$ at $(1, -1, 1)$.

Solution

We know that:

$$\vec{\nabla} \cdot \vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}) = \frac{\partial}{\partial x}(xz^3) - \frac{\partial}{\partial y}(2x^2yz) + \frac{\partial}{\partial z}(2yz^4)$$

$$\vec{\nabla} \cdot \vec{A} = z^3\frac{\partial x}{\partial x} - 2x^2z\frac{\partial y}{\partial y} + 2y\frac{\partial z^4}{\partial z} = z^3(1) - 2x^2z(1) + 8yz^3 = z^3 - 2x^2z + 8yz^3$$

Now, at $(x, y, z) = (1, -1, 1)$, we get

$$\vec{\nabla} \cdot \vec{A} = (1)^3 - 2(1)^2(1) + 8(-1)(1)^3 = 1 - 2 - 8 = -9$$

Also,

$$\text{Curl } \vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^3 & -2x^2yz & 2yz^4 \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(2yz^4) - \frac{\partial}{\partial z}(-2x^2yz)\right]\hat{i} + \left[\frac{\partial}{\partial z}(xz^3) - \frac{\partial}{\partial x}(2yz^4)\right]\hat{j} + \left[\frac{\partial}{\partial x}(-2x^2yz) - \frac{\partial}{\partial y}(xz^3)\right]\hat{k}$$

$$\vec{\nabla} \times \vec{A} = (2z^4 + 2x^2y)\,\hat{i} + 3xz^2\,\hat{j} - 4xyz\hat{k}$$

Now, at $(x, y, z) = (1, -1, 1)$,

$$\vec{\nabla} \times \vec{A} = (2 - 2)\hat{i} + 3\hat{j} + 4\hat{k} = 3\hat{j} + 4\hat{k}$$

Problem 1.3. If $\phi = 2x^3y^2z^4$, find div(grad) ϕ.

Solution

$$\text{div(grad)}\phi = \vec{\nabla} \cdot \vec{\nabla}\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi$$

$$\vec{\nabla} \cdot \vec{\nabla}\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}\right)$$

$$\vec{\nabla} \cdot \vec{\nabla}\phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial^2}{\partial x^2}(2x^3y^2z^4) + \frac{\partial^2}{\partial y^2}(2x^3y^2z^4) + \frac{\partial^2}{\partial z^2}(2x^3y^2z^4)$$

$$\vec{\nabla} \cdot \vec{\nabla}\phi = 2y^2z^4\frac{\partial^2}{\partial x^2}(x^3) + 2x^3z^4\frac{\partial^2}{\partial y^2}(y^2) + 2x^3y^2\frac{\partial^2}{\partial z^2}(z^4)$$

$$\vec{\nabla} \cdot \vec{\nabla}\phi = 2y^2z^4(6x) + 2x^3z^4(2) + 2x^3y^2(12z^2) = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$$

Problem 1.4. Find a unit vector which is perpendicular to both vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}$.

1.18 Multiple Choice Questions (MCQs)

1. The direction of a vector in space is specified by the:
(a) One angle (b) **Three angles** (c) Two angles (d) All of above

2. If a vector $\vec{A}$ makes an angle θ° with x-axis, then its x-component is:
(a) A **(b) A cos θ** (c) Asinθ (d) All of above

3. If position two aeroplanes at any instant in given by $(2,3,4)$ and $(5,6,7)$ from origin in kilometres, then distance between position of two aero planes will be:
(a) 4.8 km (b) 5 km (c) **5.2 km** (d) zero

4. If resultant vector lies in 2nd quadrant, then its direction is:
(a) $\theta = \phi$ (b) $\theta = 180^\circ$+ϕ (c) $\boldsymbol{\theta = 180^\circ - \phi}$ (d) θ=270°-ϕ

5. Two forces act together on an object, then magnitude of their resultant is maximum when angle between forces is:
(a) θ**=0**° (b) θ=180° (c) θ=270° (d) θ=45°

6. The resultant of magnitude of two forces 6N and 8N acting at right angle to each other is:
(a) 14 N (b) 2 N (c) **10 N** (d) 8 N

7. The magnitude of resultant of two forces $\vec{F_1}$ & $\vec{F_2}$ acting at right angle to each other is:
(a) $F_1^2+F_2^2$ **(b)** $\boldsymbol{\sqrt{F_1^2+F_2^2}}$ (c) $\sqrt{F_1+F_2}$ (d) Zero

8. When two forces of equal magnitude make an angle of 180° with each other, then magnitude of their resultant is:
(a) 10 N (b) Maximum (c) **Zero** (d) 8 N

9. If resultant of two vectors each of same magnitude is also of same magnitude, then angle between them is:
(a) 30° (b) 90° (c) 45° (d) **120**°

10. The example of vector product is:
(a) Torque (b) Angular momentum (c) Linear velocity (d) **All of above**

11. The vector product of anti-parallel vectors is:
(a) **Zero** (b) Maximum (c) One (d) All of these

12. Vector product of $\vec{A}$ with itself is:
(a) A^2 (b) $A\sin\theta$ (c) A (d) **Zero**

13. The magnitude of $\vec{A}\times\vec{B}$ is equal to area of $\cdots$ with $\vec{A}\&\vec{B}$ as its adjacent sides:
(a) Square (b) Triangle (c) Hexagon (d) **None of the above**

14. If $\vec{A}\times\vec{B}$ points along x-axis, then$\vec{A}\&\vec{B}$ must lie in:
(a) xy-plane (b) xz-plane (c) **yz-plane** (d) All of above

15. A particle starts from center O towards OA, then moves along AB and stops at B. If $R = 100$ m, the displacement of particle is:
(a) $100\sqrt{2}$ m (b) $\frac{100}{\sqrt{2}}$ m (c) **100 m** (d) None of the above

16. Mark the wrong statement:
(a) $\hat{i}.(\hat{j}\times\hat{K}) = \hat{i}.\hat{j}\times\hat{k}$ (b) $\boldsymbol{\hat{A}\times(\hat{B}\times\hat{C}) = (\hat{A}\times\hat{B})\times\hat{C}}$ (c) $(\hat{i}.\hat{j})^2+(\hat{j}.\hat{j})^2+(\hat{k}.\hat{k})^2$ (d) $\vec{A}.\hat{i} = A_x$

17. Three coplanar forces acting on a body keep it in equilibrium. This should therefore be:
(a) **Concurrent** (b) Non-concurrent (c) Parallel (d) All of the above

18. Three unit vectors r , $\hat{\theta}$, $\hat{\phi}$ are:
(a) Concurrent (b) **Perpendicular** (c) Parallel (d) All of the above

19. The direction of $\nabla\phi$ is always:
(a) **Perpendicular to surface ϕ= constant** (b) Parallel to surface ϕ = constant
(c) Perpendicular or Parallel to surface ϕ= constant as per geometry of problem. (d) None of the above


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